Financial Mathematics

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Lecture – 30
Graduated Payment Mortgage and Sinking Funds

Welcome to the lecture on graduated payment mortgage and sinking funds. So, you know in the last lecture when we talked about the mortgage debt and we also talked about how when you take any loan then how what is it will be the mortgage sojourn how we determine the monthly payment we also had discussed about the balance due after certain you know instalments being paid so in that case of also again we will use the formula to find the present value of the loan after paying certain you know instalments.

We can also you know find the you know remainder of the maturity so that also can be found so suppose you have to find the remainder of the maturity.

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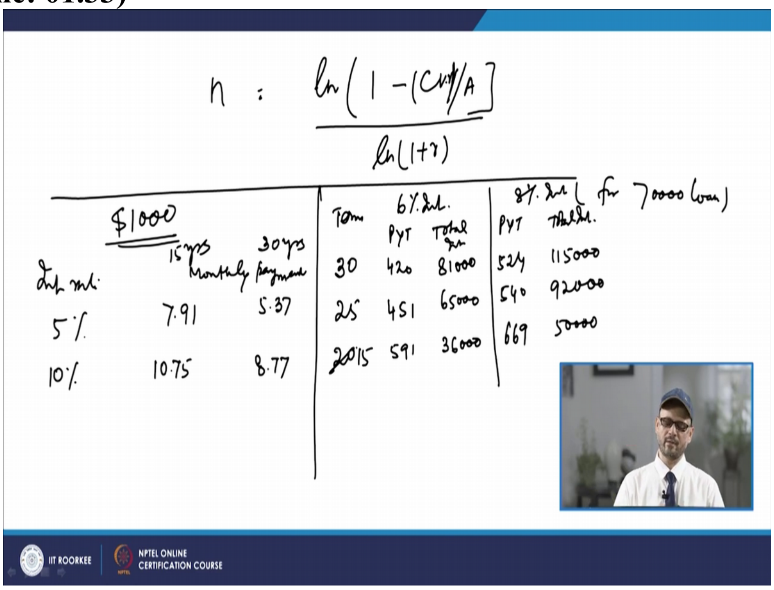

In that case what you do is you have to in the same formula you have to find n so that can be found by using this formula ln of - Cv by Cv *r and that divided by A and that will be further divided by ln of 1 + r, so this is a something which you can get from the formula itself. So, there if you are given some data suppose you are given the present value you are given the annual payment or the monthly payment A and the rate of interest is known in those cases simply you can find the number of payments.

And by using this formula now what we have so far seen that there are certain variables affect these you know monthly payment and these you know factors are or these variables are the interest rate because the interest rate will be certainly affecting the monthly payment then your term of maturity how much in how many instalments you are going to pay the entire loan and also what will be the down payment.

So, that down payment is also important because down payment will ultimately you know because your interest what you are going to pay from the start that will be smaller if your down payment is the larger because the down payment is more than the principle amount on which the interest calculation will be there so that will be you know less. So, that is why they can be studied you can have the different types of studies.

And for example if you look at the suppose we have seen that the monthly interest monthly you know payment will vary with the interest rate because interest rate when it is increasing then for the same year of maturity your monthly payment will differ like when you have $1000 you know payment is to be carried out and if you take the interest rate so if you take interest rate of 5% in that case your; for 15 years of maturity time your, at 5% you will have to pay 7.91 and you know dollar for every month payment.

And if you go for 30 years in that case you have to go for 5.37 so that way your, you know monthly payment will be varying and if you take 10% in that case this 7.91 will go to 10.75 and similarly this in 30 years it will be 8.77 so, this way you depending upon the interest rate effective interest rate your these monthly payments basically will be varying this is you know the monthly payment.

Now we can further see so this is the effect of the interest rate on the monthly payment. Similarly you can have you know different maturity rates and then you can have the different you know interest rates maturity time may be different and you have different interest rates and in that what will be your monthly payment and what will be the total interest which you are going to you know pay for that.

So that will be also another subject of study which you can you know find and if you see in that case if your term is there if suppose term of maturities very suppose 30 and 25 and 20 now in that case if you take the 6% interest suppose. Now in that case your monthly payment you know either this instalment will be 420 for 70000 loan. Now in this case you are pivoting is coming as 420 and your total interest is coming as 81000.

While if you take 25 years of time then this will be 451 and this will be 65000 and if you take the 15 years of maturity time then it will be 591and this will be 36000 so this is for; not for 20 years this is for 15 years and this will be 36000 so that way you know what you see is that your total interest which you are paying that will be very;

Now if you increase the interest then these amounts will vary. Suppose you are taking 8% interest in that case the monthly payment and the total interest if you look at so in that case for 8% it will be 524 and this will be 115000 so that is what you see that for 8% interest you are paying for 30 years you are paying about 115000 of loan. Similarly for 25 years you are paying 540 and you are paying here 92000.

And for 15 years you are paying 669 and you are paying 50000 as interest. So, what you see that if you change the maturity time then with the change in the interest you can see that your monthly payment is going to change with decrease of maturity time certainly your monthly payment amount will be increasing. But certainly the interest which you are paying will be decreasing.

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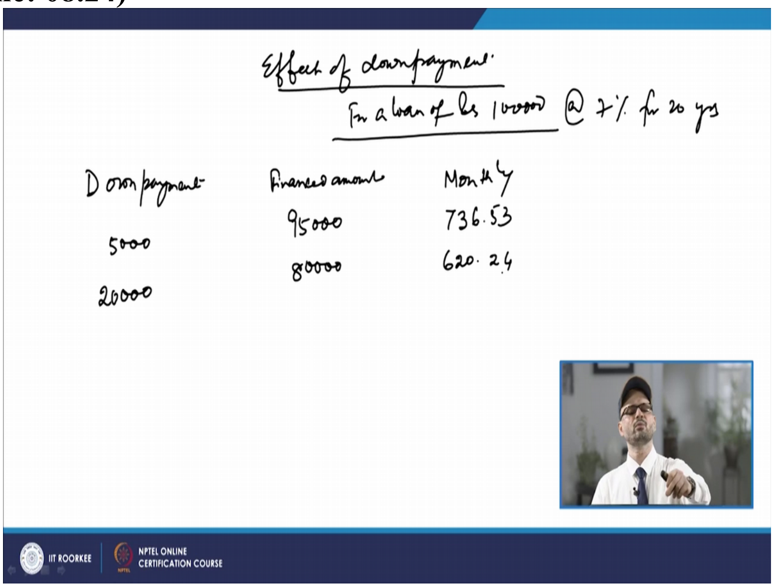

So, that is how we see that and when you know you can see the effect of down payment also so effect of down payment is also there on the you know payment schedule or mortgage schedule of the loan and suppose you have taken a loan of for a loan of 1 lakh so now what you see that if you take the down payment of; you can have the down payment of different you know amount and if you take these at 7% interest in that case if you take these or down payment is 5000 it means you are financing you know finance the amount will be 95000.

And in that case your monthly payment we will be you know so this is you are taking for 20 years. Now for 20 years if you take down payment of 5,000 your monthly amount will be coming at 736.53 but if you give larger down payment suppose you are giving 20000 down payment then in that case your finance amount is 80000 because you are taking the loan on only 80000 rupees and in that case it comes down to 620.24.

And if you further increase then finance amount will be further less and monthly you know instalment amount will be further less. So, that way your these interest you know these factors have the effect on these instalments schedules how you are paying how much interest will be paying and you know all these are having some interconnected you know effects on each other.

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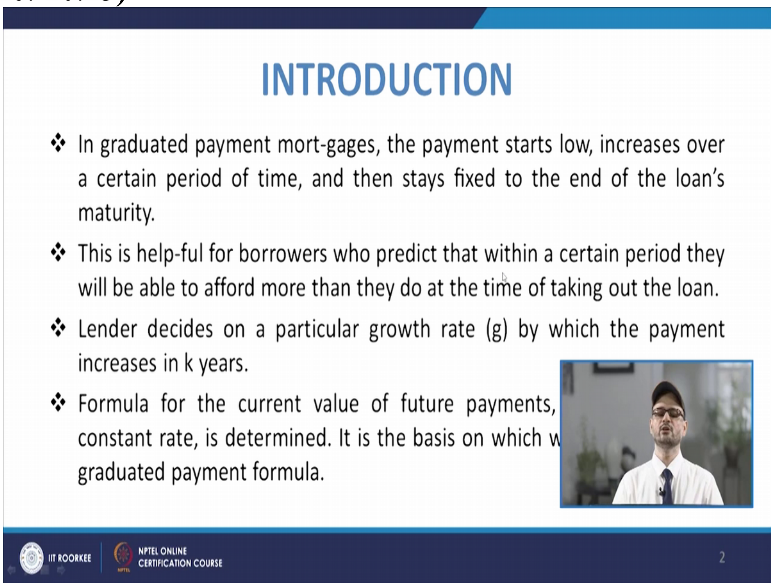

Now we will move to the you know the introduction to these graduated payment now in that case one of the example is that many a times we try to see that we should pay this loan initially smaller amount and then that should increase over you know the time so that is known as the graduated payment mortgages. Now in these cases what we do is that we anticipate that our income will grow with time.

So, when we take a loan we feel that initially we should pave; now so we fix that we will pay start paying from some amount and then every successive payment will be increased by a certain factor certain fraction or certain percentage. So, that way this type of payments are known as the graduated payment mortgages and so in this payment the payment starts low increases over a certain period of time and then stays fixed to the end of the maturity.

So, you know it can be that it is starting from some amount and then it will be increasing by certain you know percentage slowly and then after sometime it maybe that will be fixed to that or that may further continuously increasing so like that. So, basically we have studied for such you know payment series we have studied about them in you know compounding interest, that factor and that was the geometric gradient Series factor.

So basically that series factor will be used under those conditions and when you know the P and when you know the yearly payment I mean the first instalment and then it will go for the increase. So, in those cases you can calculate using that so what will be the current value of the principle. So, for the current value of the amount what will be A or vice versa you can use these factors to find that.

So, basically coming to the concept of these graduated payments this is helpful for borrowers who predict that within a certain period they will be able to afford more than they do at the time of taking out the loan. So, that is what your concept is that you with as the time progresses you will be able to pay more you and you expect that your salary will increase or you will get some additional income then you can increase.

So, the lender will decide on a particular growth rate by which the payment increases in K years. so, basically there will be a growth rate g will be decided and on that your payment will be increasing and there will be formula for this current value of future payments which grew at a constant rate. So, that basically in that term this is a graduated payment formula and we know that it is basically the present value P if you recall those formula P will be F1 / 1 + g and then it is multiplied by the factor P / A g Prime.

And so g prime is nothing but 1 + I by 1 + g - 1 so that way we calculate these values and in this case if you take the r as the interest rate and g as the growth rate so what we get is that you can get the current value.

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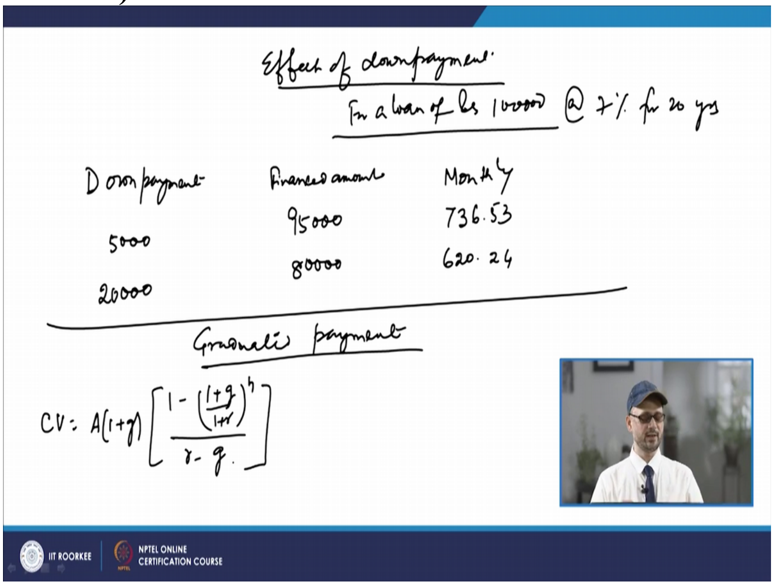

So, for graduated payment series you know what we you know graduated payment you know we apply that formula what we get is you can get Cv as A* 1 + g and then 1 -1 + g / 1 + r raised to the power n / r – g. So, you know basically using this formula you can find the current value of the loan and for any current value of the loan you can even find the you know amount and annuity or the monthly payment depending upon that r will be changing.

And this way you are going to have the calculation of you know Cv the present value of the loan. So, this is known as the graduated you know payment.

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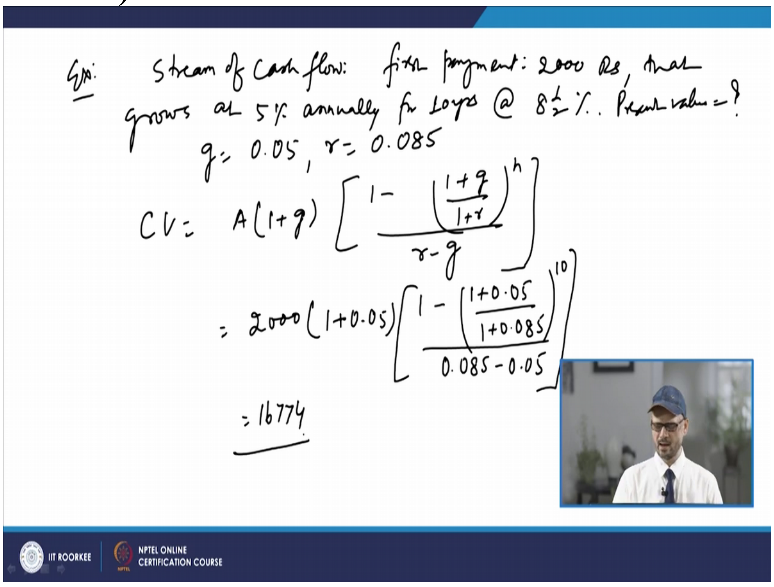

And you can have an example suppose a person is there so you have a stream of cash flow and you know in that your first payment is 2000 and then it will further go so that grows at some percentage so that grows at 5% annually and for 10 years. So, now and interest rate is 8 1/2 % so you know for such cases present value will be what? So, if you are told to calculate the values in some such problems we know that your first payment is 2000 and then it is increasing further.

So, that is increasing at 5% so g is 0.05 so g will be 0.05 and r is 0.085 so you can use the same formula current value current value will be A* 1 + g as we have seen and then you have 1 - 1 + g by 1 + r raised to the power n and divided by r- g. So, in this case you can use 200 so that is 2,000 and then you have to multiply that with 1 + g so g is .05 so 1 + 0.05 and then 1 - 1 + 0.05 / 1 + 0.085 and that raised to the power not 10 you are doing for 10 years and that divided by r - g so r is .085 and that g is .05 so you do the calculation on this and you will get 16 774.

So that way you can have the calculation of the present values using these formulas of the graduated payment in such cases. Now you can that that also can be basically put in a factor and there may be different table. So, when you go for monthly payment in those cases you may have the different table and this can be directly written as some factor values and that can directly be multiplied with A so that you can simply multiply with A and get the you know final Cv values.

So these are ready you know that can be used anyway and found the values. Now there are there is another thing which is coming while giving these mortgage loans and that is more mortgage points. Now many a times these mortgage loan points they are the additional charges which are imposed by lenders and paid in advance by the borrowers.

So, you have to pay the borrower that it was amount and it is just a way to increase the lenders profitability. So, it will be done by this lender to increase his profit and it will be increase the cost of borrowing for the borrower by pushing up the effective interest rate. So, now what happens that your effective interest rate will certainly increase because you are being paid less amount and so the effective interest rate will be you know larger and the points are normally set as 1% of the amount borrowed.

So, that is the standard practice but it can be different so you can just see that if you have an example.

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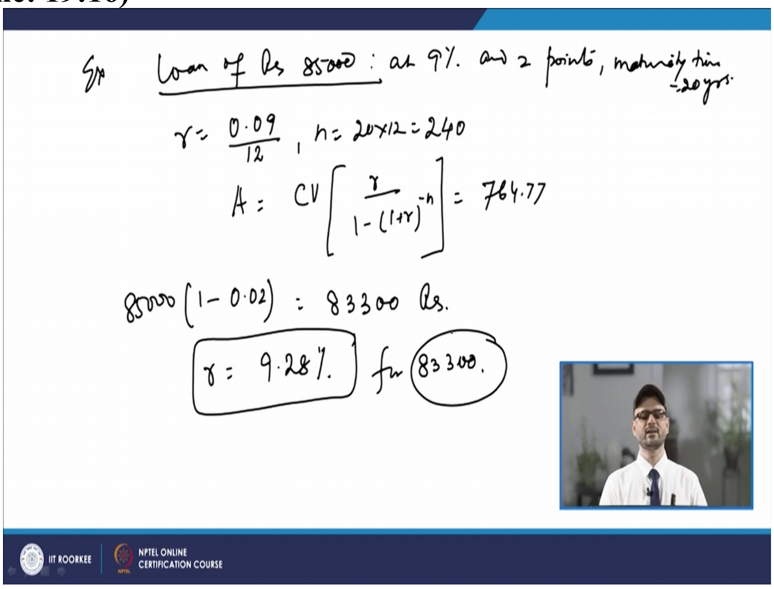

Suppose you have a loan so you have a loan of you know rupees 85000 and in that you have it is at 9% and two points so 2 points means 2% will be taken by the lender itself in advance so you will get only 98% of the this loan amount which you intend to get. Now if the maturity time is given as 20 years so in that case you are required to know that what will be the you know A and that is your monthly payment value.

So, what happens that your r becomes 0.90 / 12 and since the now there are 2 points now n is 20 years so 20 * 12 so it will be 240 and then you can get the you can calculate the value of monthly payment so we know that the monthly payment will be you know CV so CV is your 85,000 and it will be r/ 1 - 1 - r raised to the power 1 + r raised to the power - n so if you give all these values it will be coming as 764.77.

So, this is the your monthly payment now you know borrower has to pay this much amount for the whole of the life. However he will be getting only you know I mean 85000 - 2 points so you know basically it will he will be getting 85000 * 1 - 0.02, so he will be getting basically 83300 rupees only. So, basically he is getting 83300 and he is paying a 764.77. So, on that if you calculate the effective interest rate what is coming out because it is not 85000 it is basically 83000.

So, in that case if he calculates now in that case if you look at the r the effective r value will be coming as 9.28% for this 83300 because for paying this 83300 he is paying 764.77 rupees every month and that leads to the effective interest rate of 9.28% so what you see that the effective interest rate becomes more in such cases when your points are being charged by the lender and that is the case you know at many situations so you must be knowing that you are required to calculate in those cases the effective interest rates.

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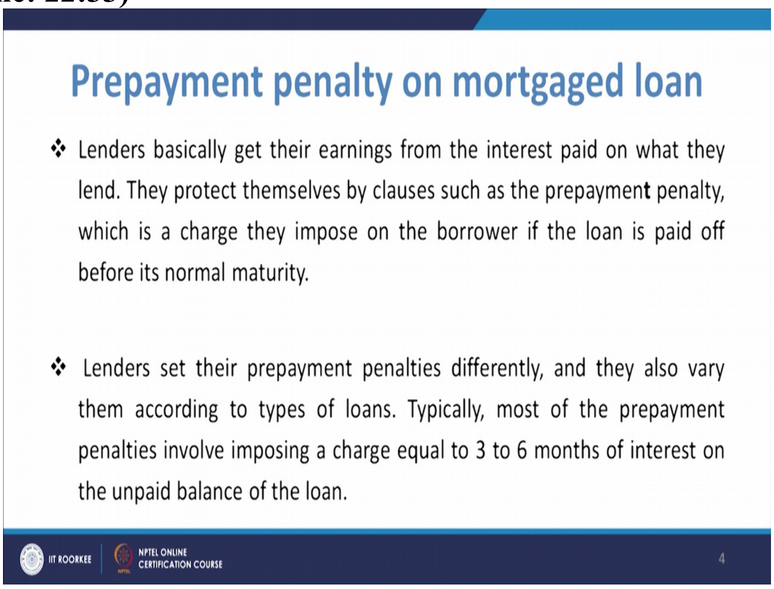

Then many a times there is a concept of you know prepayment penalty on all the mortgage loans. So, many times when you are trying to do the you know finalize this loan or you try to do the prepayment in that case they apply a penalty. So, basically lenders basically get their earnings from interest paid and for protecting themselves they put some clauses like prepayment penalty and it is nothing but a charge they impose on the borrower if the loan is paid off before its normal maturity.

So, when you go to bank to say you know to prepay me all that do the prepayment in those cases they will give you a penalty and this penalties are set differently and that will be normally according to the types of loans and they will be imposing a charge of equal to 3 to 6 months of basically the interest on unpaid balance of the loan so they will be charging that extra so that they feel that the interest day we are supposed to get for the rest of the maturity period which is not going to be finished.

So, they can compensate something out of that so there may be questions based on that and you can even solve them.

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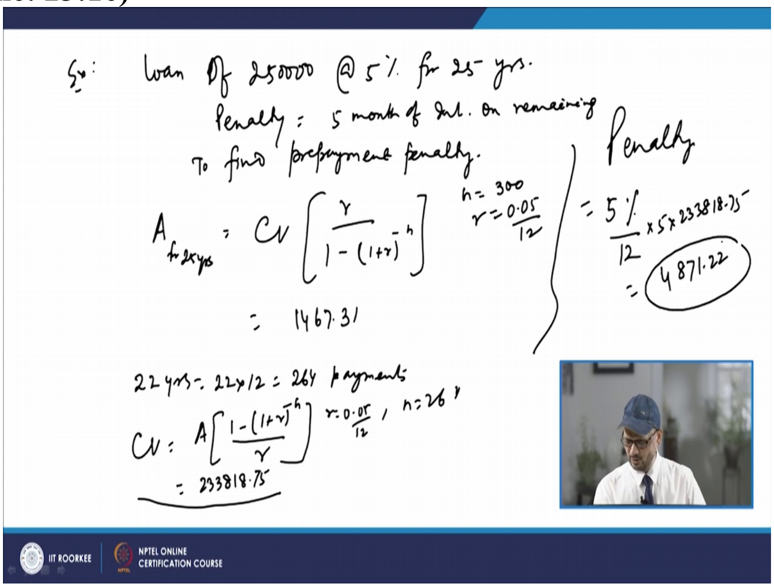

And for example suppose you are talking about this case where a loan was for 250000 and it was agreed that it will be at 5% for 25 years. Now after only 3 years it is you know plan to be paid off. So, know what happens that the mortgage penalty what has been set, so penalty has been set as the 5 month of interest you know that is set by the lender on remaining balance. So, in those cases what you need to do is now you have to find the prepare payment penalty.

So, in those cases you can use the formula for finding that prepayment penalty and if you look at the thing what you see is that for the y 250000 loan in that you have already paid for 3 years you have already paid. So, now what you do is that for initial you know for initial 25 years you will find A and A will be for 25 years if you calculate it will be Cv into again that formula. So, Cv * r / 1 - 1 + r raised to the power –n, so that way and will be 25 years it will be 300 and r will be 0.05 / 12.

So, that will lead to 1467.31 now this has been paid for three years so now after 3 years you have 22 years remaining. So, further that for 22 years it will be 22 * 12 so 264 payments now for 264 payments what will be the current value after three years. So, for that current value will be calculated with A * 1 - 1 + r raised to the power - n / r so here r will be same as .05 / 12 and n will be 264. So, this will be coming as 233818.75 so now this is the amount which is remaining at after 3 years.

And you are going to get penalty so your penalty will be 5% of 5 month of interest on this remaining so 5% / 12 and then you have to get you know these this is your a one-month interest and into 5 and then you have 233818.75. So, that way you have to calculate this value and that will come out as 487.22 so this will be the prepayment penalty which is to be given to you so that is how you know settle off this loan.

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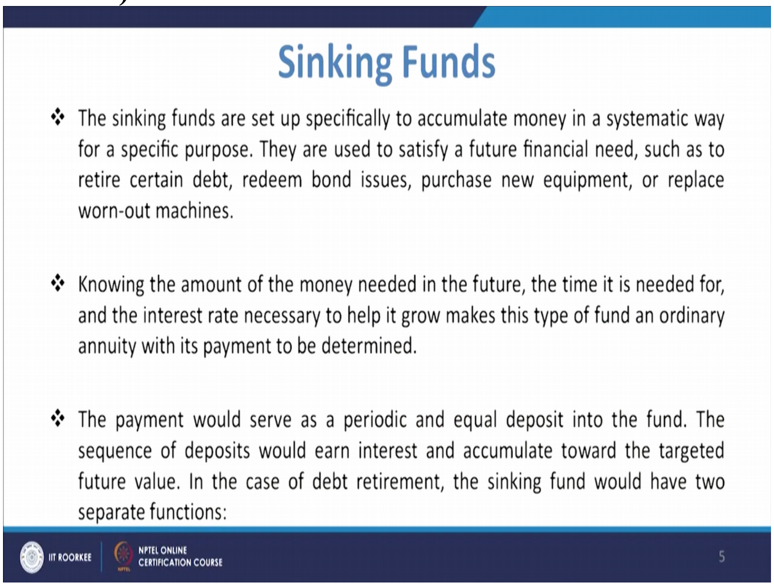

Then ultimately we are also talking about the sinking funds now sinking funds are basically set up specifically to accumulate money in a systematic way. Now what happens that we have studied about the sinking fund method also this sinking fund factors also there you have the you know calculation of A for a particular for a calculation of particular value of F so, that is your sinking fund factor.

Now in this cases the same thing is there that you have the setup of the fund to accumulate money in the systematic way to you know satisfy future financial needs. So, you basically try to serve deposit you know at periodic time to satisfy your future financial need like you have to retire certain debt or a bondage which is redeeming or purchase new equipment or replace one of the machines so this way these sinking funds our calculations are done.

And knowing the amount of the money which is needed in the future and the time is needed for and interest when you rate when you know you can have you know the finding to find the value of annuity so that that sinking fund or that fund which you require at some future time may be calculated. So, you have you know you have in many cases like the retirement or you have many cases these funds are being used.

So you use these sinking of fund formulas for such cases so and in the case of debt retirement basically it has two separate functions.

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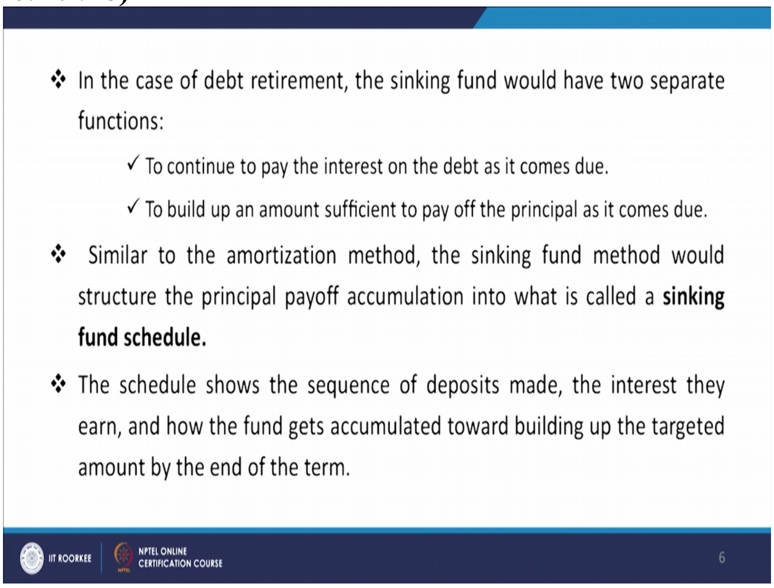

To continue to pay the interest on the debt as it comes to so many a times what happens that you know you think that you will be paying the interest on the debt you know as the time goes and are in between you are also paying you will be having that must you know fund at the end so that you can calculate you can pay the principle amount also. So, similar to the mortgage method you have sinking fund method also which will structure the principle payoff accumulation that is known as sinking funds to duel.

And it will be showing the sequence of deposits made the interest they earn and how the fund gets accumulated towards building of the targeted amount at the end of the term. So, that is how you have the simply the use of another factor that is in terms of A and FB which is used in the case of sinking terminates it can be basically you know related with the factors which we have studied in the case of compounding frequency.

And you know interest factors in those cases you can have the calculation and we can understand the help of maybe some examples.

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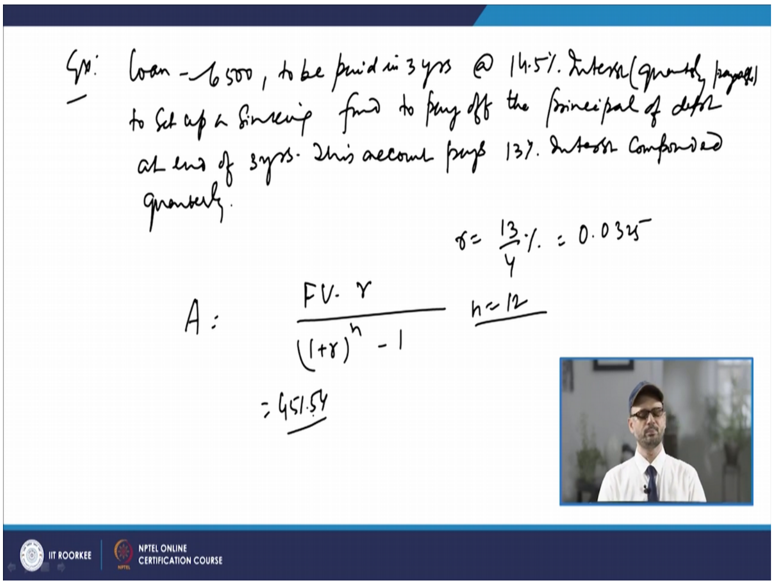

Like suppose you have taken a loan of suppose 6500 and it has to be paid in you know 3 years at 14.5% so to be paid you know in 3 years at 14.5% interest and payment is basically quarterly payable. So, now in this case somebody has to open the account of a sinking fund to pay off the principle of its debt you know at the end of 3 years. So, you know and this account is paying some interests about 13% of the interest which is compounded quarterly.

So in that case you have to calculate the you know interest which is being accumulated now so to set up a sinking fund and that on that fund you know to pay off the principle of debt at end of 3 years. So, now if you look at you know think that in 3 years something has to come to 6500 so how much quarterly he should be you know depositing and for that you will be using this interest rate and if you look at this account pays 13% you know interest compounded quarterly.

So, you may be interested to find the total interest on the loan and that will be basically 14.5% * 3 years * 6500 so that will be can be calculated. Now so based on that your quarterly payment so for the interest can be calculated but if you want to find the you know payment quarterly so that this 6500 should be paid in that case your quarterly payment will be something like FV * r / 1 + r raised to the power n – 1.

So in this case r will be your it is 13% and then compounded quarterly so it will be 13 / 4% so it will be 0.03 to 5 so you have to use .0325 here and your n will be for n as it will be 3 terms so if you use these and 12 terms you will get 451.54 rupees. So, basically if you pay this you are going to get rid of this 6500 in I mean principle amount in 3 years so that is how you are going to use this sinking fund formula for the calculation of these interest or defunding the annuity so that you can clear the loan, thank you very much.