Financial Mathematics
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Lecture-03
Progressions and Series, Growth and Decay Curves

Welcome to the lecture this is lecture number 3 and the title is progressions and series growth and decay curves. So, as we discussed in the last class that last lecture we discussed about the terminologies like exponents, exponential functions, then we also discussed about (()) (00:47) functions and all. So, we will further I get quanted with few more mathematical terms and particularly those which are typically used in the financial mathematics. So, we will discuss one by one the which we will deal with will be the progressions and we know that there are different types of progressions.

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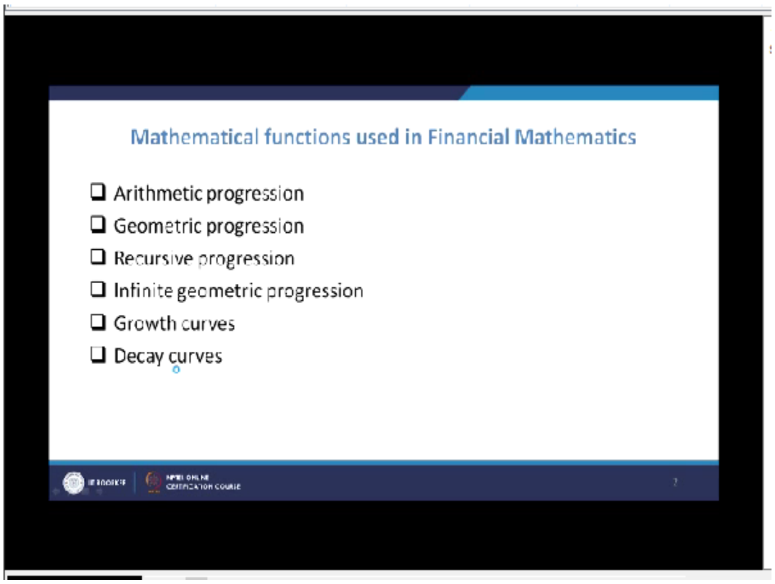

We have arithmetic progression we have geometric progression we also know about harmonic progression which is normally not used. So, we will discuss a review some of the trades of these progression series then we will also discuss about other kind of progression like recursive progression which is somewhat newer. So, will know about the trades of this recursive progression, we have infinite geometric progression that is also many of you might be knowing.

And then will also of the growth and decay curves, so will switch to the you know different types of progressions. So, as we know that first is the arithmetic progression.

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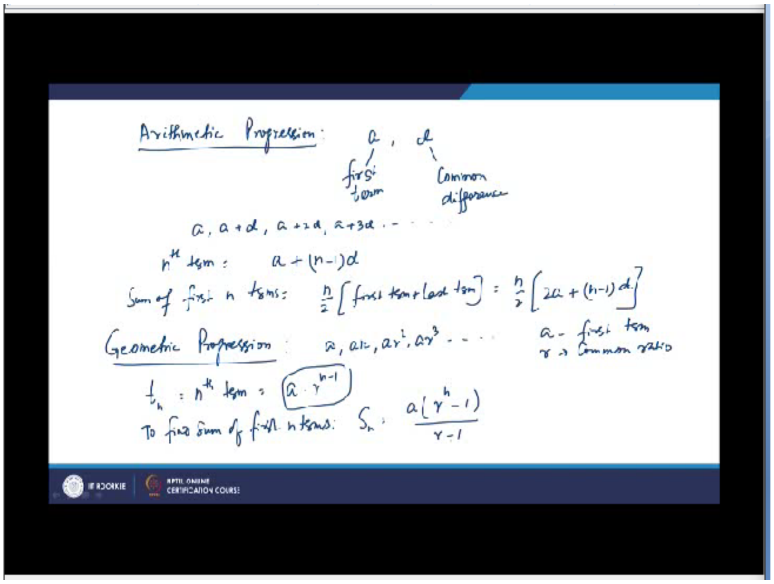

And arithmetic progression as we know that this way here you have a series of numbers and in all these progressions basically you have certain you know rules are followed. So, that you can predict you know once you know certain things about the this series or progression where the number progresses in a particular manner. Then you can predict about the number which will be coming at suppose certain position or you have to find the some of these numbers.

So, these are typically you know used and they are of use when we deal with such analysis. So, in arithmetic progression basically the important thing is that you have 2 terms which should be known to you and that is a and d. So a is known as the first term and d is the common difference, so basically the arithmetic progression is defined as that progression where the successive terms or the consecutive terms differ by a constant number that is known as common difference.

So in the first term is a the second term will be a+d, so you know that the difference between a+d and is d. So, then the next term will be a+2d then you have aplus3d like that, so it will go. So, in these you know type of progressions series we find we can find any terms suppose one to find the nth term of the this progression. So, for nth term basically the formula is aplusn-1*d as you know that the second term is a+d third term is a+2d.

So, this way you can get the nth term as a+n-1*d and we also need to sometimes know the you know some of the terms and some of the suppose n terms. So, if you are required to know the sum of first n terms, so again you know you can find it and you must have study many of you how **we** find these some of the n terms and as you know that it will be n/2*first term+last term.

So, you have if you know the last term you can have it otherwise if you know the first term in the common difference then it will be n/2*2a+(n-1)2d. So, this way we try to have the some of the first nth terms, so if this is these are the formulas which are used in you know such kind of progression. And many a times you will require to use it one deal with the financial mathematical you know concepts like while if you will see that how the payment is made.

Suppose you have taken a loan and you have to pay it, so you want to pay it in a way that every time you are going to increase by certain amount. So, basically that is in a arithmetic progression forms, so suppose in the first month or first year you have paid 1000 rupees, second year suppose you are paying every year 2000 more. It be second year it will be so first year if it is 5000 suppose, second year it will be 7000 then it will be 9000.

So, this way you know you can have you can know that in particular year what will be the amount of you know that will be paid to repay the loan amount of so. So, this way these are the places where they are arithmetic progressions are used, then similarly you can also at any particular point also if you need to add many terms which are in progression arithmetic progressions specially then you can use these formulas.

And you can get those values, so you may have different types of series and then you can solve them. Now the next progression which is very important will be your geometric progression, now as you know that the geometric progression. In the geometric progression in plus of common difference which is their arithmetic progression you have a common ratio here. So, basically the first term will be a and the second term will be ar that is your common ratio.

So, second term to first term is having a ratio that is fixed, so next term will be ar square then it will be are cube and all that. So, you have a at the first term and r is the common ratio, so in the case of such cases where they increases in terms of a certain factor that is r, so that is basically ratio is there is from a or 2a ratio is r or ar square to ar is ratio is again r. So, like that, so you have a common ratio which is maintained between the consecutive entities.

And that way this is known as geometric progression and if you talk about the you know geometric progression then in this also you have suppose you have to find the nth term of a geometric progression. So nth term the same way as you see there if you have second term it will ar, third term is ar raise to the power 2. So, similarly in the nth term it will be a*r raise to the power n-1, so this is how you get the nth term.

And this is basically now denoted by tn, so that will be your nth term then I hope that you must have the idea about how to add these terms. So, once you add them, so after adding, so if you want to find the sum of first n terms. So first n terms you are adding and you have sum of the calculations which are performed. So, you must have done it in the algebra courses and that is defined as Sn and that is basically a*rn-1/r-1.

So, this is how the you know in the geometric progression you find the sum of the first n terms and this will be the case normally when the r is more than 1 and if it is less than 1 you can have you may have also in case like a*1-rn/1-r. So, you know in that case you will have because once r is less than 1, in that case both will be negative. So, any way we can use the that also, for in that but otherwise you use this formula a*1-rn/1-r depending upon the value of r as more than 1 or less than 1.

So this way you will have the formula for the Sn , now you may have sometimes to find out for the geometric progression which is having there is a infinite series and what will be the sum of that series and normally that sum is defined as a/1-r. So, basically r has to be less than 1 in that case, so what happen is in that case this will be tending towards 0. And then that is why it will become as s=a/1-r.

So, this is for that infinitely infinite geometric series and the sum of the first n terms, so you maybe many a times you may required to finds suppose you are given one geometric progression.

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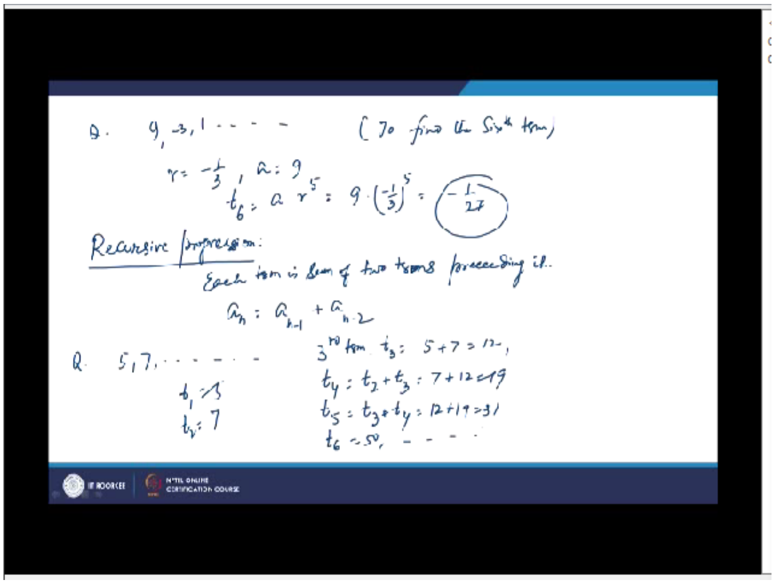

And the progression is something like 9, -3, and 1 and suppose this is a type of series which is given to you and you are to find the so question is that to find the 6th term of the this series. So as you know that if I looking at this progression you can see that you have a common ratio that is -3/9 that is -1/3. So your r becomes -1/3, you can also get from 1/-3, so it will be -1/3. So, **7** and a is basically 9.

So, if you have to find the 6th term, so 6th term will be t6 and that you can will be having a*r raise to the power 5. So, r will happen that it will be a* if that is 9 and then -1/3 raise to the power 5. So, 6- is there an it is raise to the odd number exponent, exponent is odd integer, so it will be – and it will be 3*3*3 raise to the power 5. So, 3 2 is anyway cancel, so you will have 3 raise to the power 3, so that will be -1/27.

So, this way you can use such kind of you know progression formulas to find any particular term or even you can find the sum of the first n terms in that case you have the formulas like a*1-rn/1-r. So that way you can have this values, then now next type of progression which is also very popular in the case of financial studies is the recursive you know progression. Now as you know we will discuss about the geometric progression.

And that is also many a times used especially you can say that many a times it will be increasing by certain factor or the payment which you are making every times, so that is basically a factor way it is increasing. So, in those cases you can see that it is having geometric progression type of you know pattern and in this formulas maybe useful under such circumstances.

There is another type of progression which is also very much common, very much popular and that progression is known as the recursive progression. And in the recursive progression what is there that the third term for knowing the third term, the third term will be known as the sum of the first 2 terms first term and second term. So here basically each term is sum of 2 terms preceding it.

So in these cases if you know the 2 terms then you can predict the third term and rather than you know use of the common difference or the common ratio here basically you have to know the first 2 terms. And this way if you find the next terms, so the standard formula will be a and if you have to find it will be a and -1 and a and -2 it is sum. So, you have to know the n-1th term and you have to know the n-2th term.

Then you can add these and you can get the you know nth term and this way you are getting the any term you can predict. So that type of progression is known as the recursive progression, now suppose you have you are told that there is a progression known as, so if you have a question like if progression is a recursive progression and you are your series is going like this. And you have to find the different terms in this case.

So, as you know that the first term and second term is known, so you have your third term, so your third term that is t3. So, it will be 5+7 12 because t1 is 5 and t2 is you know 7, so as you know that recursive progression then your third term t3 will be 5+7 12. Similarly t4 will be t2+t3, so t2 is 7 and t3 is 12, so it will be 19 similarly t5 will be t3+t4. So, t3 as we know the t3 we have got 12 and t4 is 19, so it will be 31.

So, similarly t6 will be 50, so this weight will be going on, so such as the you know such kind of progressions where you find these values they are known as the recursive progressions. So you may have any question like you find the 10th term of certain type of you know recursive progression where first and second term is given. So, in those cases you can find that particular value of the term which is to be required.

So there such progressions are known as the recursive progressions , so these are the normally the kind of progressions which are required in our you know financial you know mathematics courses. Now we will discuss about certain other terminologies which is required in a case of while we study in this course. So, there is growth and decay curve.

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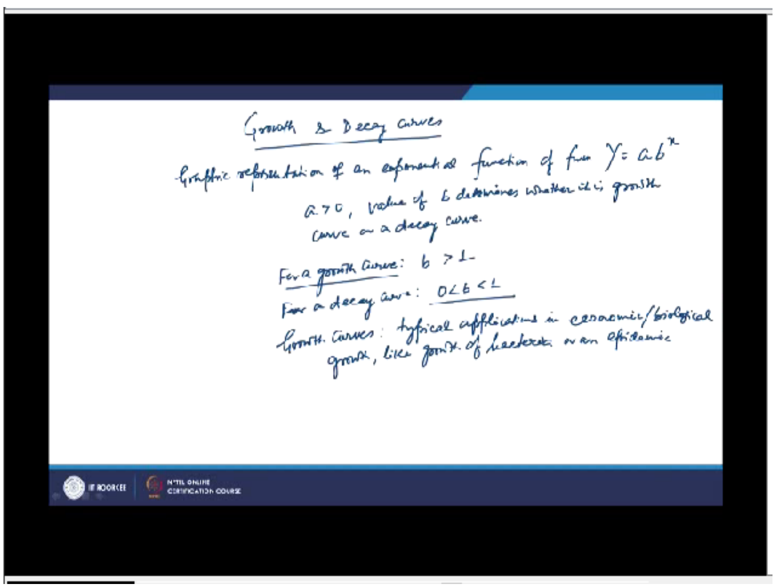

So you have growth and decay curves, so basically these curves are the graphic representation of the exponential function of the form. So you know, so these are basically graphic representation of and exponential function of form Y=a b raise to the power x. So, we have come across such kind of functions and they are basically used to represent these growth and decay curves. Now in this case depending upon the value of this parameters ab and x you have either growth curve or decay curve.

So when you know now you have a is so a has to be positive and the value of b basically and value of b determines whether it is a growth curve or a decay curve. So a has to be 0 I mean more than 0 and the b the way b is, so that only will determine that whether it is a growth curve or it is a decay curve. Now for a growth curve, now as you know that first of all the terminologies like growth or decay.

It is because of you know certain type of pattern where something is growing, something is increasing in certain way. So, that is why it is a growth curve and because many a times the epidemic you know grows, so that is kind of you know how it is growing, so that is why on that line this term growth curve is being used. Now in the for a growth curve basically b has to be more than 1 and for a decay curve now this b value.

They have has to be larger than 0 but it has to be less than 1, so your value will be going like 0 to 1. So, this way if the function if you know the function where a anyway has to be positive number and if the b is more than 1 then it is a kind of growth curve. And if it is you know in between 0 to 1 then it is a decay curve, now this growth and decay curve they are basically used for predicting many times the you know economical or biological growth.

So, many a times you have growth of bacteria or a spread of the epidemic, so they are you know used for the so this growth curves are used. So, growth curves use, so typical applications in economic or biological growth like growth of bacteria or you know an epidemic **,** so many a times you know if we see that epidemic has spread this it will spread in a very exponential way or it will be a different way that is basically a type of growth curve.

It will not grow in a constant you know by constant increment or so. So that is why in that line it is done. Similarly you may have the application of decay curves also, example of the decay curves also, now many a times you used the decay curves like you have economic decline or infant mortality. So on all these you know cases we normally use these growth or decay curves which is very much you know used in a practice in economic cases.

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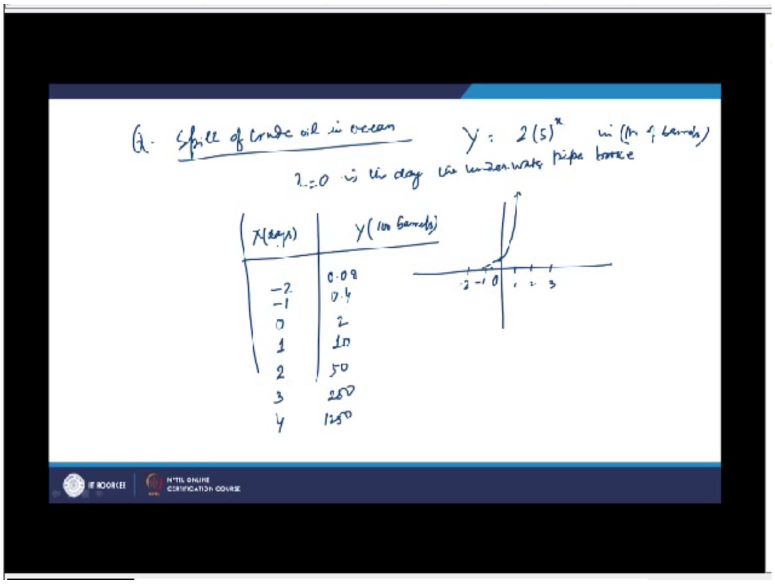

Now if you look at certain examples suppose you have the, so if you have suppose an example like you have spill of crude oil in ocean. And suppose you are estimating this a spill of the crude oil in the ocean and the function is Y equal to you know 2*5 raise to the power x. So, when you have x as 0, so it the day you know that under which is pipe has broken. So, x=0 which is representing the day the underwater pipe broke.

Now negative x will represent the you know the laws or the crude oil spilling before today and the positive x will tell you about the time in the advanced that is maybe next day or day after tomorrow. So, **so** that way, so negative will represent the time in the past and the positive will be represent the time in the future. So, the amount of spill which will be growing it will be following this way.

And if you look at this graph now from this you can predict that what will be the laws you know laws in on a particular day. And then putting that value x at that point you can have the prediction of the you know spillers of the crude oil into the sea. So, you can predict it now if you try to draw the graph of such you know spillers, so what you see is it will have -1, you have -2 here 1 and 2 and these are the days and if you look to the you know further the how it is growing.

So, for 0 it x 0 you will have certainly here it is 2 and then it is so and then it will be going negatives it will be coming out to be 0. And then it will be further moving like this, so this way this is example of growth and rate and if you look at the values. So, if you have x as days and if you have y as you know 100 of barrels. So, you can use this you know formula and you can find the what is the spillers.

So, what you see is that once you have and this is also in terms of 100s of barrels. So that is why in 100s of barrels, so what you can do is for the today that is 0 you will have 5 raise to power 0 1, so it will be 2. The first day it will be you know 2*5, so that is your 10 second day I mean 5*5 25*2, so that is 50, third day it will be 5*5*5, so 125*2, so that will be 250. Similarly 4th day it will be 625*2 1250 like that it will go.

If you go to the previous day -1, so it will be 2/5, so it will be 0.4, next to that if you take -2, so day before yesterday. So in that case it will be -2, so 1/25* 2, so it will be 0.08, so this way if you go it will moving towards 0. So, that way it will move and move towards 0, so these are the type of the growth curves which we normally come across in the you know in this financial cases and you can predict that what will be the spillers and you know as the time progresses. Then you may have also the example of the decay curves, so suppose for the decay curves one example is about the infant death in the country.

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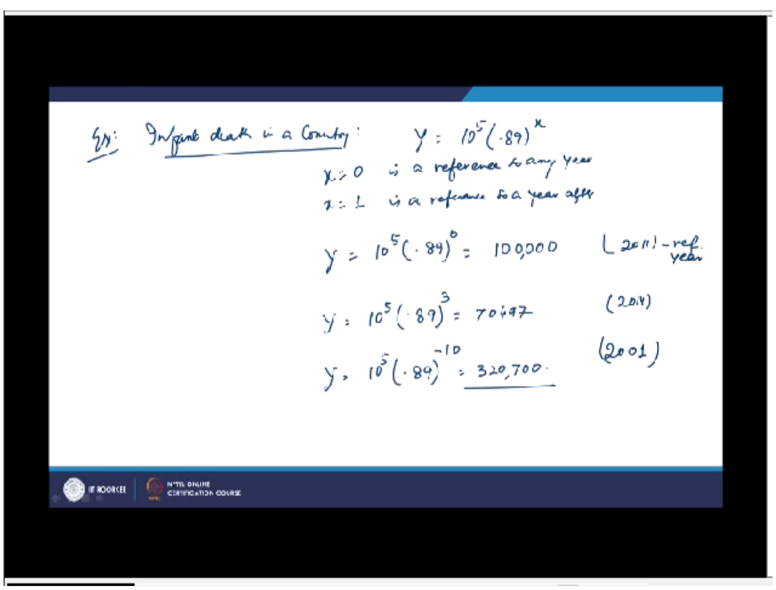

So, in those cases if that is being basically predicted by or estimated by the function, so the function becomes 10 raise to the power 5*0.89 raise to the power x. So this is you know, so x-x is 0 it is the difference year, so this is a reference to any year. Now in such cases **x** if x is x=1, so that will be year after, so that will be x=1. So, that is a reference to year after similarly if you take the negative values it will go the year before.

So, what we can see that here if suppose the x is 0 then it will be 10 raise to the power 5 and o. So, otherwise if you have x raise to the power 1. So, it will be 0.89 so what you see is if you take any year as the reference and then if you try to find the values, so suppose when the you are calculating in the reference year, in that case Y=10 raise to the power 5. And then you will have 0.89 raise to the power 0.

So it will be 10 raise to the power 5, so you will have 1 on that you have 00000**,** so it will be that now if you try to have if suppose in this is in a particular year suppose say in 2000 element. Suppose you have calculated this as the you know infant death in the country, now if you want to find suppose in 2014 if you want to calculate in 2014 then for that the x will be, so from 11 onwards you will have +3, so it x will be 3.

So, for 2014 if you have to predict it will be 10 raise to the power 5*0.89 raise to the power 3. So, this will be basically coming as 7497, so this way you can have the prediction of this decay curves. So, if want to go you know in the back if suppose you want to go towards the back if the this is the reference year and if you want to go towards the back. And if want to predict in 2001 that case you are going 10 years you know back.

So, x will be -10 and then in that case y will be 10 raise to the power 5 and into 0.89 raise to the power -10. So, it will be divided by 0.89 raise to the power 10 and in that case if you look at we calculate it will be 320700, so what you see is that this value is quite larger as compare to the value which is now, so basically the value which is in the past it is larger and in the reference year from here if we move onwards you will have the decaying the value.

So that is the decay curve similarly you have so as this basically depending upon the you know value of this v you will have the growth or the decay curve. And you can predict these values now when we talk about this growth and decay curves it is further worth mentioning to tell you that you may have the this growth and decay curves.

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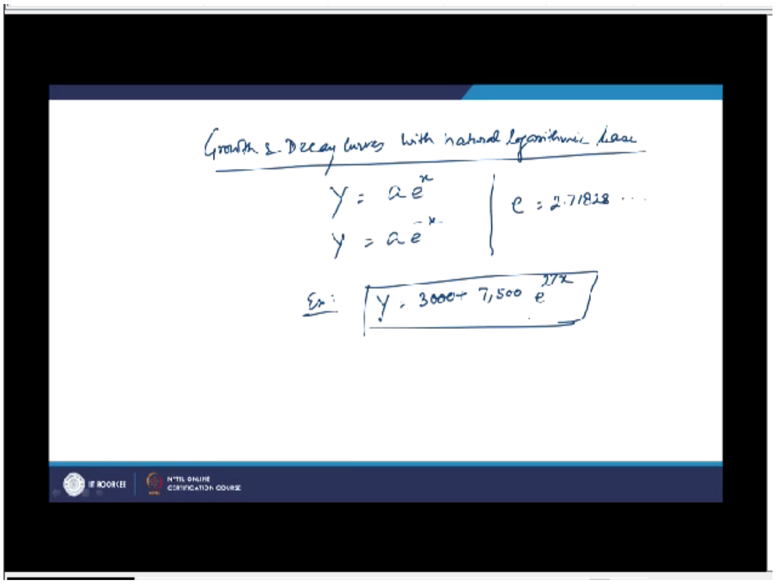

So, you may have the growth and decay curves with the natural logarithm as base, so with natural logarithmic base. So, **so** in those cases what the function becomes a e raise to the power x and otherwise Y a e raise to the power –x, so in this cases you have the use of these natural logarithm. And this e is used and this e value as we know that it has particular value 7, 2.71828, so this way we use that and based on that you can find these values.

So, these are the functions, so maybe you may have some examples like suppose you are having the annual profit for a firm and the annual profit is basically going as if it is like 3000+7.5, so 7500 and then exponential raise to the power27x. Now in this case x is how long the firm has been selling the product, so based on that you can have this x values and you can get the you know calculations you can do the calculations and find the results.

So, as you see that here you are this will be a type of growth curve and you can find these values accordingly you know as the function comes out to be and suppose you are going for finding some values you can have the value here. And find those values that this may be there may be negative sign here and that may have the different you know values in those cases. So, that way you will have depending upon the different functions you will find these values. And they represent these growth end, decay curves which will be further you know discussed in our lectures to come, thank you very much.