Financial Mathematics
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Lecture – 29
Analysis of Amortization Schedule

Welcome to the lecture on analysis of amortization schedule. So, we had the introduction about the amortization concept where we have mortgage debt is there so in those cases we deal with these principles. So, we know that under these amortization methods we have long term of maturity may be from 15 to 30 years and that determines basically the number of you know payments which are usually monthly.

So, if you are taking for suppose 30 years in there in that case the number of payments will be 30 * 12 that is 360 so like that you have these payments which are made a monthly basis.
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As a certain interest rate is applied a sequence of equal payments is set to so how the entire loan earnings and its interest are to be paid off by the end of maturity. So, in these cases you know there is certain interest rate which is applied and that will be known to you or that may change in between so either you have the fixed rate of interest or maybe variable rate of interest and then you will be depending upon that basically you have the equal a sequence you know equal payments sequence will be set.

And you know in that whole loan and interest are to be taken care of so we will see that how they are now you know being set. And the sequence of payments they break down between principle and interest and their reduction in the outstanding balance that is called the amortization schedule. So, that is what the name of amortization schedule means that you know how you are paying what is a breakdown between principle and interest means in every; you know payment you are paying a fixed amount every month.

And you know what part of it is basically the principle part what part it is the interest part and how it is you know going to change the outstanding balance of your loan. So, that basically you are going to you know see that. So, let us see that if you take the example of one you know loan.

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And suppose you are taking a loan of say 1 lakh rupees now suppose so you have a loan of rupees 1 lakh and you are given that you are told that the term is 30 years and you know interest rate is you are telling that it will be fixed so it interest rate is 8% fixed. Now you have to find it is amortizing schedules how this loan is to be paid you know every month. Now what happens that you have as we have discussed you have two portions is one is MIP monthly interest portion another is your, you know MPP monthly principle portion.

And that will make you that will tote in total make PYT that is a monthly payment while we are repaying redoing the repayment of such loans. So, if you take this interest rate 8% in that case we know that you have a formula by which you can make this A so some something has been paid now so that is your CV or also you can say it will be well sorted to take as P as 1

lakh we have to find A what is A? What will be the equal monthly payment which has to be paid to the person but we have which has to be paid by the person you know to the lender.

And you we know that your interest rate so interest rate basically is given 8% fixed so if you are doing the monthly payment it will be 8 / 12% so that that is how you know the monthly interest will work so it will be .08 / 12 and if you take n and will be basically 30 years so 13 *12 so it will be 360 so if you use the formula where we have to find the A provided P in that case we know that we have to find A by; so a will be P * and then you have A / P i n.

So that is what we have understood earlier and that formula A/ P i n will be I * 1 + I is 2 power n / 1 + i t to the power n - 1 something like that. So, that can be basically you know changed so basically in these cases will be CV * r / 1 + r raised to the power -n so if you see this formula what will happen that this will be 1 /1 + r raised to the power n that will go up so it will be CV * r * 1 + r raised to the power n / 1 + r raised to the power n -1 it is the same formula but you can use it in a different way and you can use even that formula also.

So, if you put all these values it will be coming so if you put CV as 1 lakh then r as .08/12 and n as 360 now in this case it will be coming as 733.76 so if you do the computation you will get this it means you have to pay rupees 733.76 every month for 30 years if you have to repay the loan of 1 lakh rupees. Now let us see how this how your outstanding balances are changing as you proceed.

So, let us say that you are we are talking about the term so that will be you know years so we are talking about the 1st year only now let us say in the 1st year we are talking and we are talking about the number monthly term non-payment term number so we are going to talk in the that first payment second payment third payment fourth payment fifth payments experiment seventh eighth ninth tenth eleventh and twelfth. So, that is how your monthly payment will be there in the first year.

So, this way it will go in the second year then third year fourth year then it will go till thirty years. Now in this case we know that your monthly payment PYT it is coming out to be 733.76 and it is going to be same by you for all the month. So, 733.76 will be the monthly payment by you in that now what we have to find we have to find the monthly principle portion we have to find the monthly interest portion.

So, how much you know interest we are paying and we will be talking about total you know PYT total monthly payment what we are paying every time so that we will see and then we are also going to talk about loan balance. So, what is the balance at present so at present initially without paying anything we are starting with 1 lakh rupees and then we have to see that how this payment is going to vary.

So, you know as we know that you have 8% of you know fixed-interest. So, the monthly income portion monthly interest portion at first time will be nothing but you know based on this 8% fixed in you know interest for the 1 lakh rupees of principle amount. So, it will be 1 lakh and multiplied by you know the interest rate so that will be .08 / 12 so it will be basically if you look at it will be 8,000 by 12 and if you look at this it will be 666.67 so that is what is you know this is your monthly interest portion this will be for the 1st month. So, for 1st month the interest portion which has to be paid will be 666.67 that is what it is here so you will have better we provide these lines to see that how they are changing.

So, in the first month your interest portion has come out to be 666.67. Now since the monthly payment is 737.76 rupees it means the monthly principle portion will be in the monthly payment minus the interest portion. So, that is the interest which is charged by the lender by the bank. So, the principle amount which was 1 lakh out of that we have you know we have paid only an amount which is the subtraction of this minus this.

So, if we do the subtraction it will be 67.09 it means we are only subtracting this 67.09 and we are total payment is only 733.76 rupees by this time by first month and your loan balance will be 1 lakh - the 733 point know so it will be only you know principle portion will be reduced so since 1 lakh – 67.09. So, it will be 99932.91 rupees. So, that is the loan balance at the end of first month.

Now in the second month what you will do is that you are going to first calculate the monthly interest portion and monthly interest portion will be 8% on the basis of these 8% of this amount. So, for that you will calculate 99932 * .08 /12 so it will be somewhat lesser than the what we have paid in the first month and this will come out to be 666.21 and anyway we are paying amount of 733.76 so, your monthly principle portion what you are going to pay now it will be little more from 67.09 and it will be 67.55 and your total monthly payment which

total you know PYT basically to date to up to date this date that what we have shows you have paid the second installment it will be 2 * 733.76 so it will be 1467.52.

So, and your loan balance will be again 99932. 91, minus this 67.51 so it will be 99865.36 so this is how the terms will move so in the third year now we will get it as you know 68 and then your PYT will be coming out as 665. 76 and this will be 3 times 733.76 so it will be 2201.28 and your loan balance at the end of three month will be 99797.36. So, this way your you have; you will see that your payments are gradually moving in the downward direction and at the end of 12 months so you can feel all these data.

And just for example if you look at you know after 12 months if you see you can check it that it is anyway 733.76 and your in the 12th month after the you know in that case your monthly principle portion is coming out to be 72.19 and your this portion is 61.57 and total PYT will be 12 * 733.76 so total PYT will be coming out to be 8805.21 and your balance at the end of 12 payments will be 99164.63.

So, this is you know your loan you have the balance of 99164.63 at the end of 1 year of payment. During that year of payment you have paid you know the sum of all these values as the monthly principle portion and that will be nothing but 1 lakh minus this it will be 835.37 so you have only paid in whole year you have basically paid 8805.12 rupees altogether but out of that the principle portion which has been deducted from 1 lakh rupees it is only 835.37 rupees.

So, that is what is the only principle portion and rest is only rest is all the interest portion the interest portion is charged by the lender and in the initial periods because our standing balance is more they are charging more and more interest from you. So, that is how these loan schedules move in you know in the case of these mortgage loan payments where the interest rate is fixed.

So this way the interest you know this payment will go on and if you try to see that how his payments will be moving how his you know balances will be changing so that you can further see.

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Term (yrs)	PYT No	PYT Amount	MPP	MIP	Total PYT	Loan balance
2nd yr	24	733.76	78.18	655.58	17610.24	98259.94
10th yr	120	733.76	147.95	585.81	88051.20	87724.70
20th yr	240	733.76	328.38	405.38	176102.40	60477.95
25th yr	300	733.76	489.25	244.51	220128.00	36188.09
30th yr	360	733.76	728.90	4.86	264,153.6	0

So, if we further see the you know interest herbs and this will be PYT number and then you have the PYT amount and further you have the principle portion and then you have the interest portion and then you have total PYT and your this is your balance finally loan balance. So, for example if you further continue so we have seen that in the first year it was this one if you go to if you go in the second year and this is your at the end of the second year this is which is your 24th payment.

So in the 24th payment you are PYT anyway will only 733.76 now in the 24th payment your monthly principle portion that will be 78.18 and the monthly interest portion will certainly get lesser because the monthly principle portion will increase as the time proceeds and monthly interest portion will decrease as the time proceeds. So, monthly interest portion in the 24th you know payment will be 655.58 and the total will be 24* 733.76 so it will be 17610.24 and your loan balance after 2 years is 98259. 94.

So, it means in 2 years basically 1740.06 that much of only principle you know amount has come down from 1 lakh. So, we can further see that if you go to 10th here suppose so in the 10th year it is nothing but the 10 * 12 the 120th payment so in the 120th payment your PYT will still be 733.76 and the monthly principle portion has further increased so that has gone to 147.95 monthly interest portion basically has gone down it will be has become 585.81 and then you have till date 120 payments so 120 * 733.76.

So, that becomes 88051 so it will be for the 10th year it will be 88051.20 your balance is 87724.70 so this way you go to now 20th year and this 20th year means this is a 240th

payment this is a 240th number of payment in that anyway 7 this will be 733.76 this will further increase monthly please you can you can have a Excel program and from there you can directly calculate this there is no you know not much of the trouble in that because you have you know the calculation of MIP first.

So, that will be from the 1 lakh rupees for the first time and then once you get that once you get 733.76 by a formula then rest things will be all based on the Excel programming. You can very well do it now in the 20th year the monthly principle portion which will be deducted it will be 328.38 and in the interest portion in that 20th year it will be 405.38 it was further on down 240 * 733.76 so that will come out to be 176102.40.

So that will be your total yearly payment what you have made total you know monthly payment which you have made and your balance will be 60477.95 so that you can basically calculate and in the end of you see 25th year if you see further so it is 25 to 12 so it is 300 this will be again 733.76 then you have 489.25 then you have 244.51 and further your this becomes 220128.00 and finally you have the left of 36188.09.

And in the 30th year if you see this is the final payment that is 360th payment and in that again you are paying only 733.76, monthly payment now it has come to 728.90 and your this is coming as 4.86 and this is 164153.60 and this comes out to be 0. So, that is how you know these what you see if you look at these 2 these slides you see that in the first month you have paid MPP as 67.09 and in the event in the 1st year you are paying the monthly principle portion is 72.19, 25th year it is 489.25 and the last you are paying out of 733 you are paying basically 728 is between deducted from your principle portion.

So, interest portion is very, very less towards the end because your outstanding balance is quite less in those cases and finally at the end your balance becomes 0. So, that is how these dues are being calculated. So, you can solve different you know problems based on such schedules and get more and more confidence by solving this in such problems.

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For example suppose you may be told that someone has is interested to buy a house for himself and he will be getting you know the house for $98000 you know somebody wants to purchase a house for that and he has been told by the you know mortgage officer that the best interest he will be getting maybe 5% interest if he takes the loan for 15 years maturity time. So, for that you know how you know how much he will be the monthly payment you know if you are told to calculate about it then you can simply use that formula and you can get you know the value like A will be 98000.

And then you will be into r so, r will be 0.05 / 12 then 1 -1+ .05/12 and then there is you know 15 years with 15 * 12, 180 payments it will be -180, so if you solve that you will be getting 774.98 you know dollars. So, this way you can get you know you can know what will be you know the payment for any particular in a maturity time at particularly interest rate. Now what happens that when you are taking the you know this loan now what happens that depending upon the interest rates basically the monthly payment will be changing.

Now if your interest so what you see that in this case the interest rate is 5% and only for 15 years you are getting 774.98 whereas in the earlier case similar type of amount was there something like 1 lakh but your interest rate was 8% and your time of maturity was 30 years in that case you are getting 733.76 so that is the difference in this case you are going to only complete your you know whole payment in 15 years whereas in that case you have to complete in and 30 years and interest rate is little bit higher.

So, how with this interest rates your monthly payments vary that also you can calculate and if you calculate that if you if you look at what is your annual percentage rate of interest so in that if you have a different you know percentage rates then so what will be your monthly payment for maturity of you know years so maturity here is suppose 1 or maturity here is basically maybe 2 or suppose say 5 years or say suppose 10 years.

So, in those cases if your, APR annual percentage rate is changing. Suppose if you have the percentage rate of 1% in that case if your maturity is only 1 year or you are making only 12 payments then you can use the formula and what you see that for a rupees 1000 loan, for 1000 loan you have to pay 83.79 rupees. So, you are if the interest rate is only 1% in that case for the one year of maturity you are paying 83.79 rupees that is 83.79 * 12, 83 * 12 is 996.79 * 12 it will be 9.48 so it is a 1005.48.

So, you are only paying 5.48 rupees as the interest overall but anyway this is how because a very small interest rate you are paying very small amount as the interest portion finally. So, if you take 2 years maturity time or 24 you know months in that case it comes out to be 42.10 so look at it for the 42.10 you are paying for 24 you know payments so 24 means it will be 1008 + 2.40 so it will be 1010.40 so, 1040 is the extra payment which you are making at only 1% of interest so this way you know then if you go for 5 years maturity it will be 17.09 and if you go for 10 years maturity it will be you know you know if for the 10 year maturity it will be 8.76 so this way you are depending upon the interest rates it will be changing.

Now let us say if you take the 5% interest rate in that case this amount will be coming as 85.61 and at 5% interest rate for 2 years maturity it will be 43.87 at 5% interest rate for 5 years maturity it will be 18.87 and at 5% interest rate for 10 years maturity it will 10.61 so that is how what you see the things vary if you go for you know the 8% case what you see that for you know 1 year maturity it will be 86.99.

Then for 2 years maturity you know it will be 45.23 then for the you know 5 year maturity with 8% it will be 20.28 and for 10 years maturity it will be so no this is for 8 years so for 8 years it will be 45.23 and then it will be 21.28 and then it will be 12.11 so it will be there for the 10 years so what we see that when we are charging you know the this APR will increase. Now what we see that with that this value is also increasing when the APR is increasing and

when the if you look at the maturity you know periods now suppose you have the 5 years you know or 1 year itself we have seen that.

When you are going for 10 years maturity it means it is 120 months and 120 will be multiplied by 8.76 so now it will be something like 2628 * 4 so it will be 10512 so go out 51 rupees 51.12 rupees extra we are paying here it is 5% interest rate is 51 rupees we are paying extra in such cases so how these you know interest rates changing how they are going to change the monthly payment that you can see from this table.

And this can also be programmed on excel you know programs or even normal you know calculators also you can purchase then you can have a schedule and have a feeling you know that how you are going to calculate this. And in this case that is how we are you know doing these calculations and getting that. Now you can also get the balance so when we have to we can also calculate that after some you know payments what is the you know balance remaining?

So, that also can be purchased suppose somebody has given 12 payments so again in that you have to get the you know current value and if suppose in the earlier problem where we had 360 payments if suppose somebody has given 12 payments then what is the balance remaining that can be purchase that in calculated and that can be seen so that will help us to calculate what is the balance remaining at particular date.

So, there in place of 360 if you put 348 so that will give you the balance remaining and that will tell you the balance from the table also you can you know clearly see and reassure yourself, thank you very much.