Financial Mathematics
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Lecture – 27
Assessing Interest and Structured Payments in Loans

Welcome to the lecture on assessing interest and structured payments in loans. So, we discussed about the different methods you know how we charge the proportion of the interest and the principal proportion when we talk about the loans and we studied about the methods like level method and the; you know another method which in which there was the increase the increased value of interest that is rule of 78, method.

So, for 1 year the rule of 78 method so based on that you know number of years maturity time you will have rule of 72, 78 or maybe for 2 years you will have rule of 300 or so. So, all that we have seen now there may also be cases when you will have the premature you know pay off. Now what we are going to discuss in this lecture that we are going to discuss about the premature payoffs and also when you have the payments of loans in the structured manner then how it is you know to be represented?

How they should be represented that is what we will discuss in this lecture. So, many times when we are talking about these you know known loans then there is a premature payoff and you know somebody has to you know hand over somebody has to prematurely pay off the loan so what will be you know what will be the consequences you know what we see is that rule of 78 what we have seen will allow him to give you give more interest at any time if you look was in a year he has after 6 month.

If he feels that he should be you know he is prematurely pay off the loans in that case he has paid more interest for the coming you know 6 months for the for the past 6 months as compared to the level method. So, that can be basically calculated and it can be seen that how what will be you know the balance due in all these cases like if you talk about the level method. So, if suppose somebody is thinking of you know thinking of pay of the entire loan in 4 months.

After 4 months only after 4 payments then what is due so if you talk about the level method that deal will be 133.33 *8 so that will be 1066.4 or so. Then if you talk about the interest you know the interest how much he has paid that will be 16 * 4 that is 64 so that is what is there is a case of the level method. But if you go to the you know rule of 78 then in that case your

balance do will be different it will be 1600 minus the 4 times what he has paid and he has paid basically less.

So, if you calculate using the rule of 78 method he will be having the balance due of about 1106 so instead of 1066 rupees he has to still have the principal portion of 1106 rupees and certainly the interest he has you know paid more in those cases. So, that is how what we see you the calculation of these you know interest and if you find the interest what he has paid in 4 times in those cases we have seen that it is 12 / 78 * 192.

So, it was 29.54 in our 1st month similarly 27.08 in the second month 24.62 in the third month and 22.15. So, altogether you are paying the interest of 103.39 rupees instead of 64 rupees what you have paid as interest in the case of level method. So, if you look at the difference basically it will be 103.39 - 64 about 40 dollars so 40 rupees of extra interest you have paid in the 4 months using this rule of 78 methods.

So, basically in that case you can see that you know these you are paying more interest by the rule of these 78 methods in such cases. There are methods to basically find the balance due so when you are paying prematurely in the payment in the case of rule of 78.

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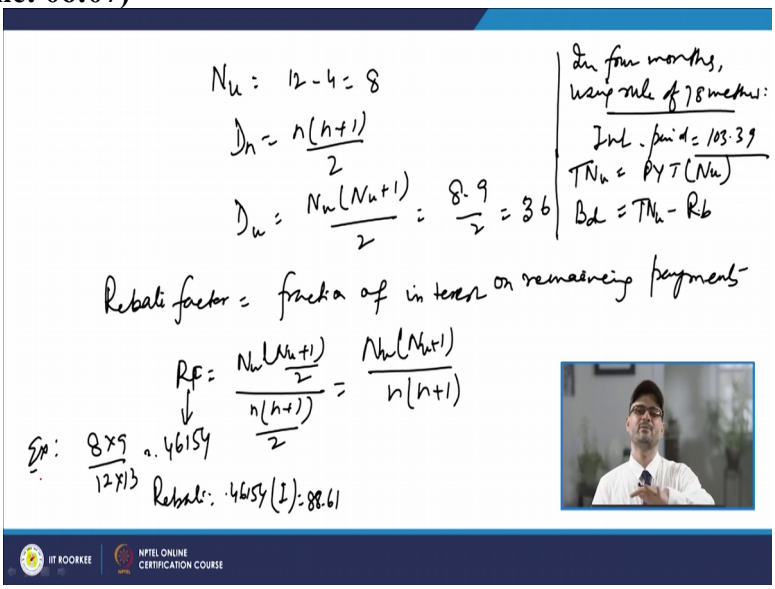

In that case suppose you have you have to find the number of unpaid in you know payments so if you have paid 4 payments to is that will be 12 - 4 that is 8. So, now what is done is that normally you have Dn as n * n + 1 / 2 similarly you have d you we get as Nu * Nu + 1 / 2 so in this case it will be 8 * 9 / 2 that is your 72 so it is 36. So, what happens that the fraction of the interest or remaining payments so that is also known as a rebate factor.

Rebate factor is nothing but fraction of interest on remaining payments so this will be calculated using the formula so that will be you know rebate factor so that is RF it will be Nu * Nu 1 + 1 / 2 / n * n + 1 / 2 so it will is nothing but Nu * Nu + 1 / n * n + 1 so now what happens that if suppose in this case if you look at in this example it will be 8 * 9 / 12 * 13 so that way you will have a .46154 so the rebate what you get is rebate factor into interest paid so your rebate will be basically a .46154 that is rebate factor.

This is rebate factor into interest so that is 192 and it will be 88.61 so this is how these rebate is being computed and this is the amount of remaining interest which should not be paid that is what the debate is. If you calculate that interest it will be coming as this one this is nothing but interest if you recall in 4 months using this rule of 78 methods you have paid the interest as interest paid is a 103.39.

So, that is what you have not paid 88.61 so you are getting a rebate of 88.61 one has interest so that is what you are calculating from here in such cases and you have you and from here you can get the balance amount due you also. So, what you get is that you get the amount of interest which is to be from the remaining payments which is due so you have the formulas like TN will be PYT * Nu and similarly you have a balance due will be T Nu – rebate.

So, this way now you can calculate these you know balance due or other values using these rule you know in the case of the premature the payments of the loans.

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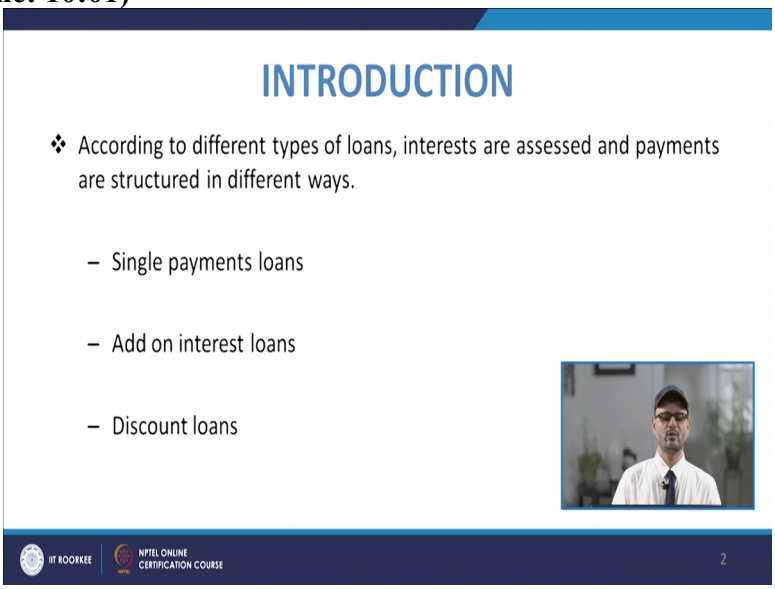

Now what we see is that when we talk about the different types of loans then we have the interests which are assessed and the payments are structured in the different way like you have single payment loans or add-on interest loans or the different you know discount loans.

Now when we talk about the; you know single payment loans as we know that we are going to pay the principal plus the interest earned in a single payment.

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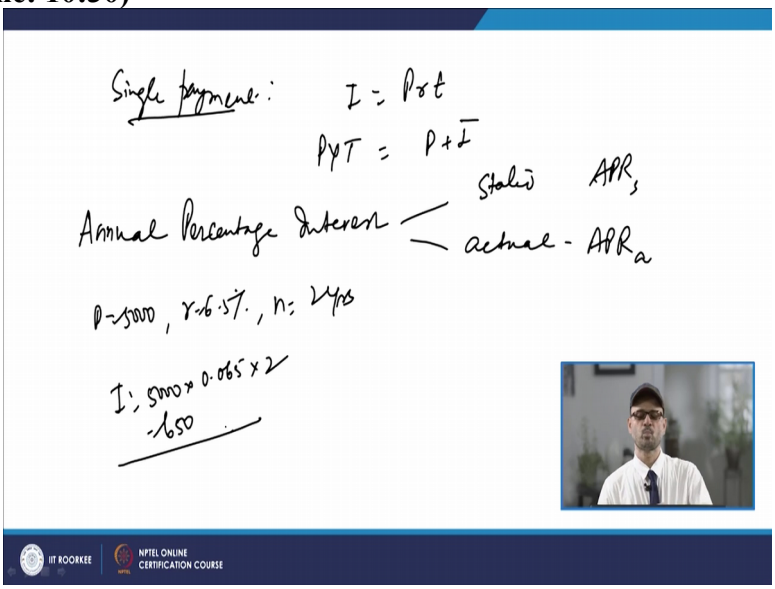

And in that case in case of the single payment you know loan when we talk about single payment we have already discussed that we have I is a PrT and then PYT they yearly you know the you know the payoff amount it will be P + I. So, there that is what singly you are going to pay it back. Now there may be examples like somebody has borrowed rupees suppose 5000 and you want to you know paid back full at the 6.5 interest after 2 years so in that case how much he has to pay.

So, it will be suppose he has taken 5000 multiplied by rate of interest that is .65 * 2 so it will be something like 650 rupees so, 5650 he has to pay back at the end of 2 years. So, in that way you can you know get what amount he has to pay. Now there are things like you know the you know annual percentage interest. So, annual percentage interest that is you know defined and they are basically two types one is stated and one is actual.

So, when we talk about stated it will be S and it will be APR a. Now when we talk about the single payment loans in those cases the annual payment annual percentage interest or rate interest rate basically so this will be same as the actual one. Because if you look at this case like you have P as 5000 and r as 6.5% and n is your you know 2 years. So, I you got as 500 * 0.065 * 2 so it will be 650.

Now if you look at this case now in this case the stated annual percentage rate is 6.5% and if you find the actual you know percentage rate also animal percentage rate. So, now he is giving he is basically paying 325 rupees every year as the interest amount so 325 he is paying

for the 5000 so it is also 6.5%. So, in these cases of single payment the annual percentage rate interest rate for the stated and for the actual case will be same.

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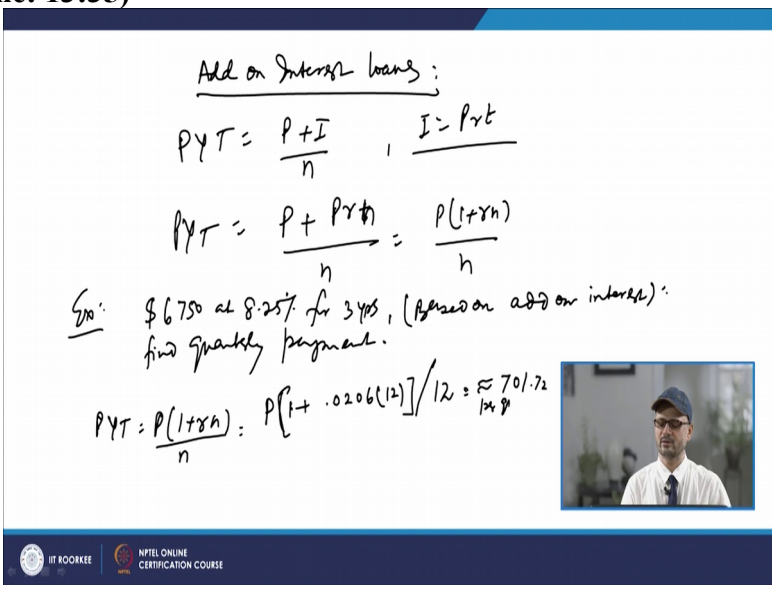

Now when we talk about the current types of these loans like you have add-on you know interest loans. Now this is another category of the interest loans and they are you know add-on as a method to calculate these interest rates. So, now in these cases whatever is happening so most of the commercial banks and loan firms they basically you know use this method of you know add-on loans.

And what we do is normally in these cases the interest or what is the you know assessed and they are charged up front and they are not basically received immediately. So, that is what the difference between them up front. So, it will be calculated based on the whole you know principle and for the entire you know maturity type. Now the whole interest basically will be added to the principal and then total will be divided with the maturity time you know to you know to determine the size of the periodic payments.

In these cases so in these cases what happens that your periodic payment will be P + I / n so now you know so PYT is nothing but the periodic payment P is the principal amount which is borrowed n is the maturity time you know measured in equal intervals like you can have the terminal you know these intervals as months and I the total interest. So, I basically you know that I will be PrT.

So, you will have PYT as P plus you know I is as you know PRT upon you know or Pr n you can say. So, Pr n / n so that way P * 1 + rn /n so that way you will have the you know these periodic payment amount can be computed. Now what you can say that if somebody has so

suppose somebody has taken the loan of suppose dollar 6750 at 8.25% interest rate for 3 years so, based on the add-on interest so in this cases find quarterly payment.

Now in such cases what happens that you have to find the I first so your PYT will be P * 1 + rn / n now in these cases you know r will be you can have this is quarterly payment you have to find so you will have 12 payments so it will be P and this will be also by 4. So, 8.25 / 4 it will be 2.0625 so r will be 0.625 so it will be 1 + .0625 and then you will have 12 payments so it will be P * 1 + rn and then you will be dividing it by 12.

So if you take that P is being 6750 it will be; you will be getting something close to 70 you know 1.72 per quarter. So, this way you can have these you know periodic payment values can be calculated.

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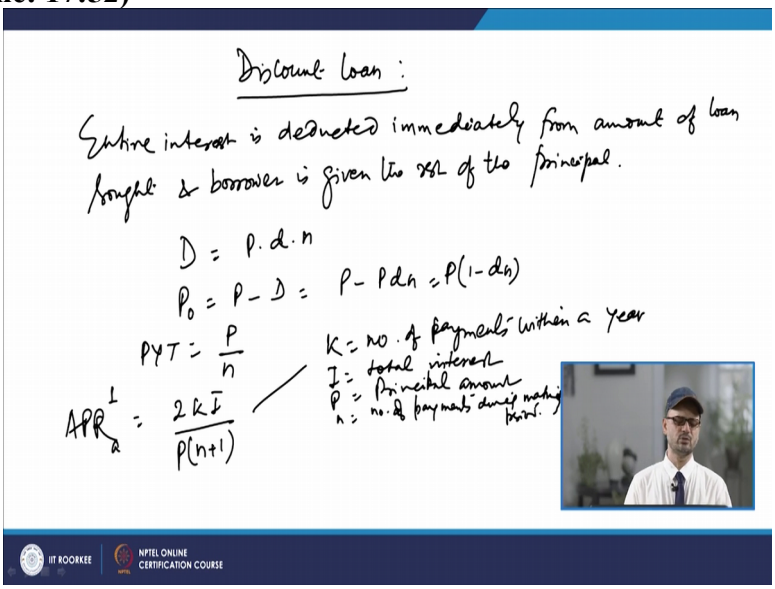

Now there is another type of loan that is your discount loan now what is discount loan we have also studied somewhat in our earlier lectures that in the case of the you know discount lone the interest is assessed up front but the entire you know interest basically will be deducted. So, in those cases is not deducted it is charged but in these cases the entire interest is basically deducted and the rest amount will be you know borrower will be given the rest of the principle. So, that way this is known as the discounted loan.

So, in this case you know entire interest is deducted immediately from amount of loan sort and borrower is given the rest of the principle. So, now that is what the case is there in the case of you know discount loans and then you have these rest amount will be basically divided you know you know in equal payment mode and then it will be divided by the maturity time.

So that way the interest which is you know rate which is there this is not a discount rate in these cases and the total interest which will be calculated on a total discount that is capital D and actual amount received by the borrower is you know P0. So, in these cases what we do is that you have in case of interested you have a discount rate so D will be P * d * n so this is the discount rate and P0 will be actually P – D.

So, that is what you are getting P - D and this will be you know P -P dn so it will be P - * 1 -dn so your periodic payment will be you know P * P / n so this will be your PYT. Now what we see in these cases that you know basically he is not getting the you know the amount which he was supposed to get in these cases and he is paying the interest. So, basically in these cases the actual annual percentage rate is different than the stated value.

And for that the actual percentage rate in these cases are calculated by a certain method. Using first method basically you calculate this method as 2Ki / P * n +1, so in this case K is the number of payments in a year then you have I as total interest and P is the principle or original amount and n is certainly the total number of payments throughout the maturity period. So, n will be number of you know payments during maturity period.

So, basically based on that you can calculate the actual you know APR annual percentage rate which will be different than the stated because you are deducting the interest upfront and then you are giving that P0 that is smaller than P so that way and then PYT will be P / n so that also is there so based on that so if you take the example of the earlier case of giving the loan of rupees 6750 for 3 years in that case if you try to find the you know actual annual percentage rate.

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So, now in those cases actual annual percentage rate now using the first method will be basically 2Ki / P * n +1 so now K is here you have so as we know that K is the number of payments within a year and since you are doing quarterly payments K will be 4. Now I will be PRT so as we know I will be P r T so it will be 6750 r is known to us so r will be .0825 for 3 years so it is coming as 1670.62 so I will know P we know that P it is 67.50 and n we know that it is 3.

And n is the number of payments so it will be 12 so if you put all these values 2 * K is 4 and then you have I as 1670.62 /6750 and then you multiply it with 13. So, if you do that it is coming as 0.152 or 15.2% so what we see is that the actual annual percentage rate differs

from the stated you know percentage rate in these cases of the discount loans and there are other methods also there are second method also to calculate the APR.

So second method in the second method we are further having another formula and what we see that normally you know you have the different estimation of the APR and the it is larger than the stated you know APR what has been said and that is why we can say that they overestimate the you know the interest they are over they are larger than the stated one and so that is why they overestimate the interest charged.

So, the second method so that also you should know the second method which is actual which is being calculated this is the second method and this uses the formula K * 95 n + 9 * I and divided by 12 and * n + 1 * 4 P + I so this if you use this formula also you can calculate the actual annual percentage rate and K as we know that it is you know 4 and then you have 95 * 12 + 9 and then interest is 1670.62 and then further you have 12 * 12 * 13 * 4 * P so it is 4 * 4 6750 so it will be calculated like 4 * 6750 and plus interest is 1670.62.

So, this can be calculated and this comes out to be the .143 or 14.3% so from the first method you are getting 15.2% is the actual annual percentage rate from the second method you are getting the actual annual percentage rate as 14.3% so these are basically the always larger than the stated you know percentage rate invested in annual percentage rate and that is why they are said to overestimate the you know charges or the interest amount charged they are overestimating it.

So, this is the you know aspect which should be known to the persons who are you know into the business into these you know who are subjected to such kind of you know transactions you when you take loan you must know that what is the you know an actual annual you know percentage rate you are you know giving or subject me or your any of your you know you know loan which you are giving what is that actual value that can be calculated using methods.

And what we see that in these methods since the interest charges more so that is how you want to assess and you can compare the different you know offers or different options which is available with you. So, that is about these you know you know different you know structured payments of laws in a different way and how so we will discuss more and more about other aspects in our coming lectures thank, you very much.