

**Financial Mathematics**  
**Prof. Pradeep K. Jha**  
**Department of Mechanical and Industrial Engineering**  
**Indian Institute of Technology – Roorkee**

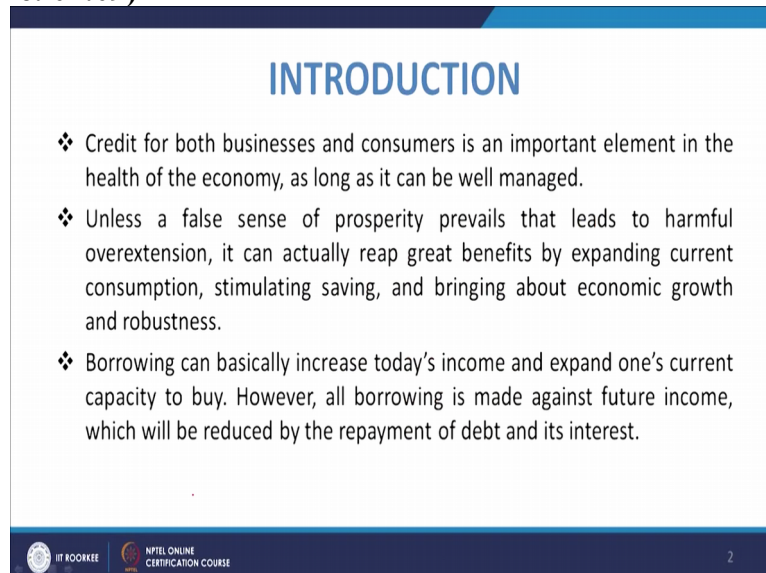
**Lecture – 26**  
**Analysis of Credit and Loans**

Welcome to the lecture on analysis of credit and loans. So, this is an important area which financial mathematicians must know because we deal with the payments you know payments at different times normally we are concerned about how much because this concept of loan or mortgaging all that came after Second World War. And then when people had to buy something which they cannot buy at that time so they had to take a loan.

And then how much loan they can be they can be getting what is the credit you know earning for any person or you know how much debts you know limit will be there? How the loan should be paid. So, all these things are you know important to know and it is important to you know study although also in the sense that many a times these credits and loans are you know useful you know in the perspective that when you take loan you try to save more and more.

So, you are basically you know investing in something until you try to save more and more or you try to earn more and more so that way it has you know there are contrasting beliefs and we will discuss somewhat introduction about these aspects in this lecture.

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**INTRODUCTION**

- ❖ Credit for both businesses and consumers is an important element in the health of the economy, as long as it can be well managed.
- ❖ Unless a false sense of prosperity prevails that leads to harmful overextension, it can actually reap great benefits by expanding current consumption, stimulating saving, and bringing about economic growth and robustness.
- ❖ Borrowing can basically increase today's income and expand one's current capacity to buy. However, all borrowing is made against future income, which will be reduced by the repayment of debt and its interest.

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So, now credit for both business and consumers is an important element in the health of the economy as well as it can be well managed. So, as we know that for both business and consumers for the health of the economy it is very important but the thing is that it should be

well managed we must be able to manage it properly unless a false sense of prosperity prevails that leads to harmful over extension many-a-times.

We have the credit we take some debts and then we have false sense of you know prosperity prevailing. So, that basically will be harmful because that will we will be under dipped and then there will be cases of bankruptcy and all that so that is one aspect which needs to be alluded to. But otherwise it can actually reap great benefits by expanding current consumption.

Accumulating saving and bringing about economic growth and robustness that is what we discussed that in normal case you know it has it can give you large benefits you know you can expand the current consumption and you can stimulate you know yourself for saving more and more and also it will bring the economic growth and robustness unless you are going to invest how can you think of you know reaping the benefits from the investment because it is going to give you the return in future.

So, and that investment for that you have to have the; you know loans or credits so for that you have to get that money from somewhere. So, that is how the you know this economic growth will be coming up it will be experienced and that is why these credits are very important to be studied. Now when we talk about borrowing so when we borrow something to meet our need at our end then it can basically increase today's income and expand one's current capacity to buy so what happens that you your capacity to buy is basically increasing.

And all borrowing is made against future income so future income you know which will be reduced by the repayment so certainly when you are paying that so that will be reduced because by the repayment you know that is what is there and also what you do is when you repay now you repay the principle amount as well as the interest so both you know both things are to be paid so that also is to be you know kept into mind.

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- ❖ A person's future income will not be able to maintain the level of consumption unless it is increased by what can compensate for all debt repayments and other variables, such as inflation and changes in one's needs and wants.
- ❖ The bottom line is that borrowing and saving involve the choice of spending more or less today versus less or more tomorrow.
- ❖ Making the right choice requires a good understanding of what credit is all about and how debt can best be managed. It is important to understand the types of debt and loans, the process and calculation of repayment, how debt is amortized, and how the cost of credit is determined.



Now if persons future income will not be able to maintain the level of consumption unless in it is increased by what can compensate for all the pre payments and other variables. So, certainly you have to increase the income and you have also to keep in to mind this inflation and also change in one's needs and wants all these things are to be kept in mind and accordingly you will have to know you know income you have to adjust you have to increase you have to earn more and more so that you can meet you can manage your debts by looking into the aspects of inflation or changing once you know needs and wants.

So ultimately the bottom line is that the borrowing and saving involved the choice of spending more or less today versus less or more tomorrow. So, what we do is that we are spending more when we borrow or and we are basically we are you know so if we spend more and then tomorrow we have to spend less so that whatever you I borrowed so you have to you know repay that so that way you have to keep that in mind you have to keep a balance of the two things. How much you are receiving how much you are spending all that.

So, in this chapter we are going to discuss about the understanding of what is credit you know how these debt can be managed and what are the different types of dates and loans and you know how these what are the schedules by which the debts are paid then how these debtss are amortized so all these aspects we are going to discuss in this lecture. Now coming to the types of debts you know there are two types of debts basically.

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## Types of Debt

❖ Noninstallment debt: also called **open-ended debt**. This category includes:

✓ Single-payment debt, where the full balance of both principal and interest is paid at a certain agreed-upon time.

❖ Installment debt: also called **close-ended debt**. This is the most common type of debt and is also known by its association with amortization.

➤ An amortized debt is any interest-bearing debt on certain borrowed money that, by contract, is to be paid off in a series of equal payments at regular intervals, usually months, for a certain period of maturity.



One is non installment debts and there is and another is the installment debts. So, non installment debts also is known as the open-ended debts so what we do in that is that it is a single payment debts where the full balance of both principle and interest is paid at a certain agreed upon time so that is what is the single payment or non installment there is no installment in between.

Here you have a maturity time you have an agreed-upon time so you have a lender and a borrower's so in between them there will be an agreement that the borrower will return the money after certain time and the balance will full balance of both principle and interest is paid at a certain agreed-upon time so this is what the single payment debts is all about and the example is normally in the case of credit cards where you know you have many business you know cards are there.

So, they charge you and you take that and in the end basically you are paying that you know the principle and the interest whichever is there. So, that is an example of the non installment of debts so you have many kinds of you know businesses like you have service credit or you have postal lines of credit are there. So, these are the example of non installment debts and here actually you are giving a minimum you know payment of 2 to 3% of the full balance and there are even other type of mode of payments optional payment is there.

So, you can have apart from the usual minimum payment you can have other kinds of you know conditional payments which can be followed for you know for these kind of non installment debts then is coming the installment debts now installment debts is also known as the closed-ended debts. So, this is the most common type of debts and also is known by its

association with amortization so this call amortization basically comes into picture when we talk about the installment debts.

As the name indicates that here you know you know you have to repay the instant the principle plus the interest in a number of installments in certain time that is your maturity time and there is a contract you know and that is to be followed and in that case this payment is you know made and normally you have you know you pay in the term of months in these cases of these amortized loans.

And what we do also is that we pay off these you know amounts at a regular integer intervals so normally it is in the you know term of months so you certain interest rate is there and then you have a period which is fixed so based on that so when you are paying in terms of so you have certain periods on which you have to do the payment so it will be  $12 * 1$  and suppose some percentage is there annual percentage rate is fixed in that case being 12 months in a year you have 12 into then n years.

So, that will be your times for which you will be paying. So, that way we are going for these you know installment debts types and in that you have the rules which are to be you know followed and we will see that how using these rules we will find that how much is to be paid by the person or the borrower. So, in that when we talk about these installment debts then what we do is so what we do we have studied the non installment debts as well as the installment debts or the amortize debts.

Now we will talk about the how the interest and principle proportions are maintained how these are calculated when we talk about the loans or the debts.

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Repayment of loan

$r \rightarrow$  rate of interest,  $n \rightarrow$  maturity time

Level Method: Both interest and principal proportions remain same during entire maturity period.


Ex: loan of Rs 1600, 12% Int. for one yr. (break into 12 monthly payments)

$n = 12$ ,  $I = P \cdot r \cdot t = \frac{1600 \times 12 \times 1}{100} = 192$

$\downarrow$

$PYT = \frac{P+I}{n} = \frac{1600+192}{12} = 149.33$

$149.33 = MIP = MIP + MPP = 16 + 133.33$



So, when we talk when we take a loan that loan is to be repaid so that repayment of the loan will be normally subjected to the interest rate. So, when we have the repayment of loan so you have certain you know parameters which are to be kept in to mind and in that you have  $r$  that is rate of interest and another parameter or variable is the  $n$  that is maturity time so normally it is in years. So, based on that you know these interest will be calculated and the you know what we do is that you have principle amount and then you have the interest amount which will be computed based on the formula that is your  $P$  and  $t$  that is time of maturity when what is that time you know.

So,  $r$  will certainly be calculated based on the how you are paying so and then you have the time so based on that we calculate this interest. So, there are different methods and one of the method what we will know is the level method. Now this level method in that basically the principle and the interest both remain the same you know throughout the entire maturity. So, in that both interest and principle proportions remain same during entire maturity period.

So, this is known as the level method so what happens that in these cases if suppose for example if suppose you have taken a loan of rupees 1600 suppose loan of rupees 1600 suppose it is taken and the interest rate is say 12% interest for one year and break into 12 equal payments 12 monthly payments. So, what is done is that you have normally the calculation of the interest in these cases.

And what we do is that in these cases first of all you have and as 12 so now first of all we will find the interest so if you find the interest, interest will be  $P r t$ . So, it will be 1600 and then multiplied by  $r$ , so  $r$  is if suppose we take .12 and then it is 1 or you can have 1% and then you have 12 periods so that way now you will have 192 rupees. Now what this level method says that your both interest and the principle proportion they have to be same during the entire maturity time.

So, as  $n = 12$  so your periodic payment  $PYT$  it will be  $P + I$  by  $n$  so whole of the principle plus the interest and that will be divided into 12 equal payments so it will be  $1600 + 192$  and by 12 so it will be 149.33 so this monthly you know the payment will be broken down into the monthly principle proportion and the monthly interest portion. So, now you will have to you know, you have 2 portions only is there so you have monthly payment of 149.33.


And you are going to divide this monthly payment into 2 portions one is for the interest portion another is for the principle portion so you will have monthly interest portion and monthly principle portion so your monthly interest portion is basically  $192/12$  that is 16 and

monthly principle portion is 133.33. So, that is how these monthly you know monthly payment of 1149.33 you know managed.

What we see is that in all these months you are getting the same you know the same amount of interest will be there or the same amounts of principle proportion which will be paid you know that will be there in such cases.

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PYT No	PYT fraction	MP	MIP	MPP
1	$\frac{1}{12}$	149.33	16	133.13
2	$\frac{1}{12}$	149.33	16	133.13
3	$\frac{1}{12}$	"	"	"
4	:			
...	...			
...	...			
11	$\frac{1}{12}$	149.33	16	133.13
12	$\frac{1}{12}$	149.33	16	133.13



So, now if you look at the schedule how it is paid so if your payment schedule if you see if it is there so if it is 1, 2, 3, 4 and it will go to 11 and 12 so you will have PYT fraction so and that we have 1 by 12 you know this is this is again also 1 by 12 in all the portions you are going to do have the 1 by 12 portion of the payment and if you look at the monthly payment it will be 149.33 and this is all going to be 149.33.

So, that is there and if you look at the monthly interest payment so as we know that we have 192 is the interest which is earned are to be paid so it will be 1616 and all that it will go so it will be 16 and 16. Similarly you have monthly principle portion. So, it will be 133.33 like that so what we see is that this proportion is the same for the you know monthly income portion and the monthly you know this income payment this interest payment and this is the principle payment and its proportion is always the same that is known as the level method.

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Rule of 78 method :



Interest & principal proportions are in reverse relationship .  
 Throughout the maturity period, interest portion will decrease & principal portion will increase .

<sup>fraction to find</sup>  
 The amount of interest for any payment :

$$K_j = \frac{j}{D} \quad j: 1, 2, \dots, n$$

$j$ : no. of payments placed in reverse order

$$D = \frac{n(n+1)}{2}$$


Next method is the rule of 72 methods so this is rule of 72 now this is 78 methods. Now in this case what we do is that you know in these cases as we do the monthly payments. So, in this case the interest in these principle proportions are in reverse relationship. So, interest and principle proportions are in reverse relationship now what happens that in these cases you have this 78 is coming because you are doing the 12 payments and the sum of these 1 to 12 is  $12 * 13 / 2$  that is 78.

So, basically if you have the proportions like 1 by you know 78 and  $2 / 78$ ,  $3 / 78$  like that so it will be added to and completely give you one so that is why these 78 is coming. If it is 2 for 2 years then you have 24 months then it will be  $24 * 25 / 2$  so it will be 300. So, they will be rule of 300 similarly for 3 years it will be 36 months or  $36 * 37 / 2$  so that is you know 666 like that this rule comes into picture.

Now in these cases the interest portions will decrease and the principle portions will increase throughout the maturity period. So, throughout the maturity period interest proportion will decrease and now you know and so this portion basically interest portion will decrease and the principle portion will increase it means that initially the interest is charged more and the interest will be slowly charged lesser and lesser.

And the principle amount which will be deducted will be lesser initially and that injection will be more of that principle amount. So, it will be lesser and lesser as we go as the time is increased. So, this as you know that this 78 is increased because of the 12 number of monthly payments and that will be basically you know different for the different you know monthly payments and interest for each payment is determined by a certain fraction of you know so that will be calculated based on these 78 method.



Now this 78 will be in the denominator and you will have only in the numerator you will have the one two three or something like that it will be moving gradually. Now what we do is that you know the amount of interest so the amount of interest for any payment so when we try to find the fraction you know to determine the amount of interest for any payment there used up formula  $K_i$  will be  $j / D$ .

So, this  $j$  will be 1, 2 and  $n$  so that way these you know this  $K_i$  will be calculated so this will be fraction basically so the fraction to determine the amount so this will be basically that fraction to you know find the amount of interest. So, that will be  $K_i$  will be  $j / D$  and the  $j$  is the number of payments placed in reverse order. So, now what we do is that normally when you are going for the first payment will be 12 by 78, second 11 by 78 like that it will be going.

And these basically the you know some of these you know terms like you have a monthly payments are 12 so that way you will have  $D$  and  $D$  will be  $n * n + 2$ . So, we have number of payments are  $n$  so  $D$  will be  $n * n + 1 / 2$ . So, as we discussed that depending upon the number of payments you know you will have the calculation of  $D$ . Now what we do is that normally when we deal with this method how we are going to calculate that we have to see.

So, suppose we are using this rule of 78 method for the you know problem which we have discussed earlier where you have the amount of 1600 which has and you are doing in you know one year payment you have the normal level method that interest was 192 and in the case of this rule of 78 method if you apply.

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Applying rule of 78 in earlier problem:



$$PYT = 149.33$$

$$MIP_6 = \frac{7}{78} \times 192 = 17.23$$

$$MPP_6 = 149.33 - 17.23 = 132.10$$

$\frac{12}{78} \times 192 = \frac{384}{13} = 29.6$   
 $MPP = 149.33 - 29.6$   
 $\approx 119.8$



So, applying rule of 78 in earlier problem if you look at so now if you use this rule of 78 well how that will go so we know that we have PYT is 149.33 that is what we have calculated. Now suppose you have to get the interest portion in the 6th you know payment so in the 6th payment as we know that in the case of level method your values are the same every time. But if you are talking about the rule of 78 then in this case it will be coming from the reverse order so the 6th one will be so 12, 11, 10 and then 9, 8, 7.

So, seven will be the 6th value from the in the reverse order so it will be  $7 / 78$  so it is since it is one year so you have D is 78 so 7 by 78 and you have a total amount of interest was 192 so it became a 17.23. So, now if you look at the value of the you know principle portion for the 6th payment. So, it will be 149.33 then you have 17.23. So, it will be 132.10. Now what you see that in the case of level method you had this as 133.33 and this has  $\times 16$ .

Now in this case the MIP the monthly income your interest portion is 17.23 and this so there is a increase in the interest portion and there is a decrease in the principle portion in these cases if you take the first value it will be  $12 / 78 * 192$  and so that will be you know so if you take the first you know  $n = 1$  that way so in that case it will be  $12 / 78 * 192$ . So, it will be  $384 / 13$ . So, it will be if you look at 29 and then you have around 6.

So, if you look at as compared to the you know the level method the interest portion is quite high and so your monthly principle portion will be so this will be MIP and this will be  $149.33 - 29.6$ . So, something like it will be you know 119.8 or something like it will be coming. So, the thing is that what we see is that in such cases the interest charge will be larger. In the earlier you know periods and the principle proportions will be larger in the later years.

So, that way these things you know change in the case of the rule of 78. So, we studied about these two methods level method and the rule of 78 and also there are cases of the you know penalty in the case of premature payments so that again depends upon how much interest is paid or so that is about this lecture we will discuss all aspects of it in our next lecture, thank you.