Financial Mathematics
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Lecture – 25
Problem Solving on Alternatives Comparison And Project Balance

Welcome to the lecture on problem solving on alternatives comparison and project balance. So, in the this lecture we are going to deal with some problems based on these topics which we have covered in this week and we come to straight way to the problems of different type. So, first of all we will deal with question number 1.

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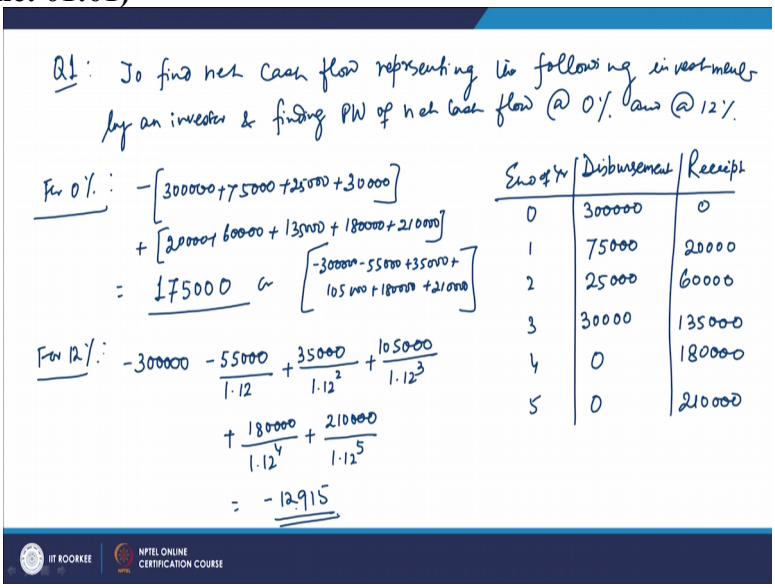

And in question number 1 we are basically to find the net cash flow resulting representing the following investment by an investor and finding present worth of net cash flow. So, actually the transactions, his transaction histories are given at the end of different years and you have to find the present worth of net cash flow and you are given 2 you know interest rates at 0% and at 12%. So, now this kind of problems are very much you know common and what we see that we have the data given and the data is like this that you have end of year is here so it will be moving from you know you have 0, 1, 2, 3, 4 and 5.

Then you have the disbursement, so disbursement and then you have receipt So, in the end of 0 the disbursement is basically 3 lakh and the receipt a certainly 0 so the there is disbursement of 3 lakh in the year and of one you have disbursement of 75000 and you have the receipt of 20000. Similarly you have in the end of year 2 disbursement is 25000 receipt is 60000 then you have disbursement of 30000 in year 3 and here you have 135000.

Then here you have 0 and 0 and this is 180000 and 210000 so this is the you know cash flow which is given to you. You have to find the present worth of net cash flow at 2 interests at 0% and 12% so you have to calculate it. Now what we see that when you have 0% interest rate then the time value of money is not coming into picture and in that case when you have to find the present worth.

So, all these things will be simply added up so you have to take all the disbursement into account and you have to take all the recipient to account and then accordingly you will have to find the present worth value. So, now for 0% if you have to calculate then you have the disbursement values are all negative. So, you will be adding all these negative values 3 lakh + 75000 +25000+30000.

And then you have positive of so receipt and receipt is 20000 + 60000 and +135000 + 180000 and then +210000. So, this is your receipt and if you add them so you are seeing that this is 430000 here and this is coming out to be to like 200000 + 18390 + 135525 and then this is you know this is 80 so this is 605000. So, you know basically now if you see that if you take its summation it is coming out to be 175000.

So, this is your net present worth value of this cash flow if the interest rate is taken as 0%. Now if you take an interest rate of as it is shown to be 12%. Now for 12% what will be the net present worth so you know what you will do is that now at this point of time your; and also again you can find this value also like 3 lakh + so it will be -3 lakh then it will be again here it will be net disbursement.

So, -3 lakh and so it can further be calculated or you can have alternate methods like - 3 lakh and then you have here -55000 then here always will go to + 35000 then here it will go to 105000 then 180000 and then 210000 so it will give you the same thing from here it will be giving you the same thing so that way you can find for 0%. Now when your interest rate is in to in picture then you have to take into account that value so time value of money will be coming into picture.

Now what you see that the present worth value will be so this -3 lakh is anyway you have this is at present time so it's value is same then you are coming to the end of year one. Now end of year one you have net you have net disbursement here and that is 75000 minus 20000 that is your 55000 so it will be -55000 and if you are trying to say you know it's equivalent value at present time then you have to multiply it with factor P by F i n and that is 1/ 1 + i raise to the power n so n is being 1 so it is 1/ 1 + i, i is .12.

So, you are going to divide it by 1.12 then coming to the next now here you have net receipt so you have 25000 and here it is 60000 so ultimately all together you have net receipt of 35,000. So, this will be +35000 and now this has if you have to map it to the present time then you will have 1 /1 + i t to the power n so 2, so it will be 1. 2 raised to the power 2. Then you have here again net receipt of 105000 so it will be 105000 / 1.12 raised to the power 3 then you can further come to the value of 180000 and that will be divided by 1.12 to the power 4 and then 210000 / 1.12 raised to the power 5.

So that is how you can calculate these present worth values and if you compute these values so this value will come out to be about 49,107 so this will be coming out to be 27901 so that way you can do the basically computation and you can get the value of this net present worth and if you calculate that it will be coming out to be some something close to -12915. So, what you see that as the interest rate is basically increasing as an interest rate is changing then what you see that this present worth value is changing.

Now you can have a look at how these there was receipt now in this case you have receipt although you have receipt of larger values at the later time but its value at present becomes very low so that is why I will tell all together you have net you know disbursement value you are completely you know net value is in a negative value and that is because of this these values. So, but had these transactions been at the you know at the top at the initial periods this situation will have been different because its contribution will be more at this point because this is at a future time.

So, that is how you can you find these you know cash flow patterns you analyze and you find these present worth values.

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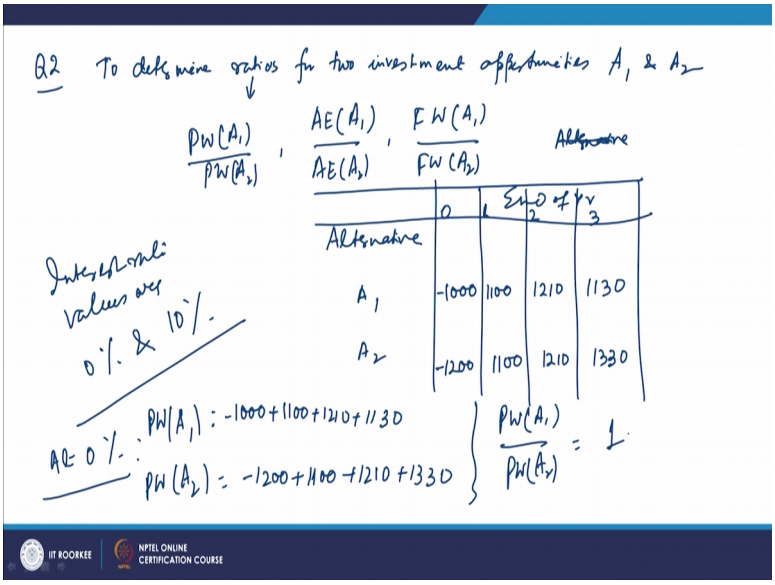

So, next we are going to discuss the question number 2, now in question number 2 basically we have to find the ratio so to determine ratios and the ratios we have to; so we have basically you know two investment opportunities ratios for two investment opportunities A1 and A2 and the ratios what we are to calculate so these ratios are like present worth A1 upon present worth A2 or annual equivalent A1 to annual equivalent A2 and then future worth A1 to future worth A2.

So, now you have two opportunities to investment opportunities A1 and A2 and the cash flows are provided and the recipient disbursements are provided and you have to find these ratios for the; you know interest rates given. So, first of all you will have the alternatives so if you take the alternative so you will have alternative given and so this we can write here you have alternative A1 and A2 and then you have the end of year.

So, you have end of year and end of year will be 0 you know 1, 2, 3 so you this is your end of year and for alternative A1 this is coming to be -1000 and this is for 1 it is 1, 2 and 3 so, for 1 it is 1100 this is 1210 and this is 1130 and for A2 it is -1200 this is 1100, 1210 and 1330 now what do you see that in this case you have to find basically the you know the ratios of these PW1, PW2 then you have AE A1 and AE A2 then you have future worth A1 and future worth A2.

All these ratios are to be calculated and for that you are given 2 interest rate values, interest rate values are 0% and 10%. So, for 0% and 10% you have to find these you know ratios now when you have the 0% as we know that in the case of 0% you can see that the present worth value will be same so you have to add them. So, at 0% now if you look at for A1 so, PW for A1 will be -1000 + 1100 + 1210 + 1130 and PW A2 will be -1200 +1100 + 1210 + 1330.

Now what you see is that here it is -200 more and in this case here it is + 200 more so I will do altogether their sum is same. So, basically PW A1 upon PW A2 it will be same so it will be 1, so for 0% interest when there is no interest rate provided in that case there is no matter PWA1 / PW A2 will be 1. Now we know that if you calculate AA 1 and AA2 or FW A1 and FW A2 these ratios are same as we have understood their principles of economic equivalence.

So all these rituals become same we can calculate it and then in all these cases these values will come out to be you know 1 itself, now if you take the interest rate of 10% in that case how to find these present worth values ratios.

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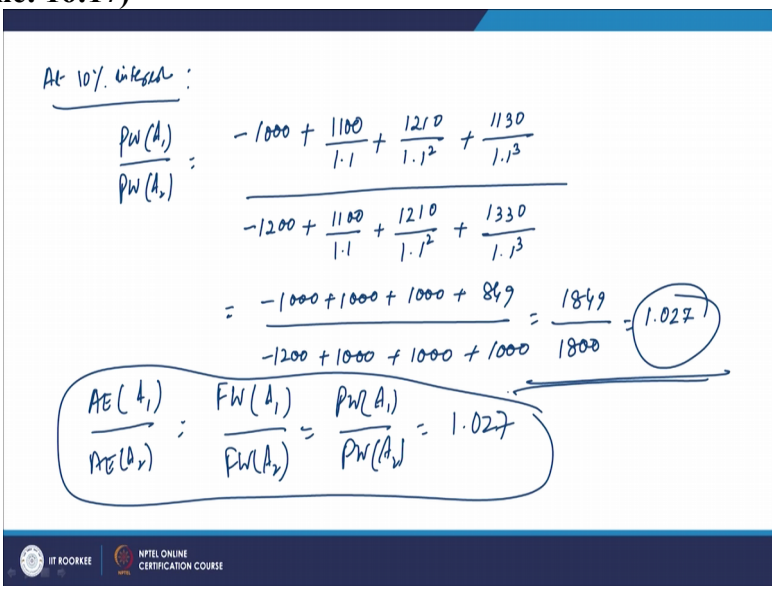

So, if you take the you know at 10% at 10% interest now this ratio PW A1 upon P W A2 now this will be basically so you will have in the first case you have -1000 so it will be -1000 plus now in this case you have 1100, 1210 and 1130 so this 1100 its present worth for that it will have to divide it with the 1 + i so it will be divided with 1.1 then you have 1210 it will be divided with 1.1 square.

And the next is 1130 it will be divided with 1.1 raise to the power 3. Then you have to divide it with the 2. So, in that first 1210, 1200 then if we have 1100/ 1.1 + again 1210 / 1.1 square and then you here you had a 1330 / 1.1 cube so that way you have these PW A1 and PW A2 can be calculated and if you compute this will be -1000 and this will be also 1000 so and this will be also 1000 and then it will be 849.

So, -1000 so it will be +1000 + 1000 and +849 and then divided by it will be -1200 + 1000 +1000 and then again it will be 1000. So, that way you will have the values so in this case here the value will be this 1000 will cut so it will be 1849 and in this case you will have

3000-1200 so it will be 1800 so this way 1.027 something like this ratio will be coming as the ratio of these and at present what values for the different interest rates.

So, that is how we calculate the ratio now if you so this ratio will be same so your annual equivalent also for A1 annual equivalent A2 will also be same as the future worth A1 then future worth A2 that can be computed basically and that will be PW A1 / PW A2 and that can be equal to 1.027. So, this way you can calculate these net present worth values and find it's ratios for the 2 types of basically the you know for two types of investments you can calculate what will be the value of the ratios of this PW A1 and PW A2 similarly to AA1 to AA 2 and all that.

Now next we are going to discuss about a problem which we are discussing in the last lecture we could not complete. So, it was related to the project balance and as we discussed that in the case of project balance you may have the different types of cash flows and you need to know the value of n prime that is your discounted payback period and how that is going to change if the transactions are changing. So, that is to be seen in this example.

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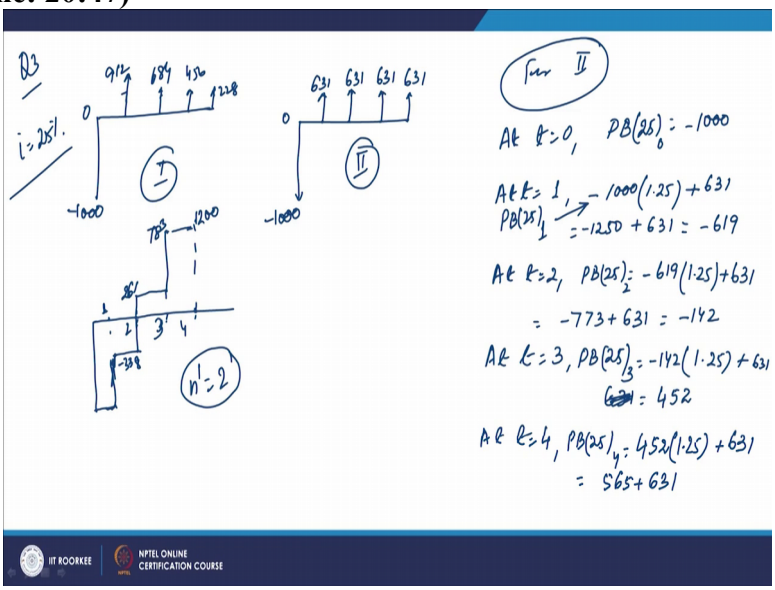

So, what we have seen that which we saw that in the in that case you had 2 to 4 types of cash flows and as you know that you have -1000 and 1, 2, 3 and 4 so in the first case we had computed it was 912 then you had 684 then you had 456 and then you had 228. So, that was your first case of the cash flow one then and for that we had computed the project balance at all the you know different times and we saw that it was -1000 at 0 time.

And then we computed for this so we found the project balance to be you know we found the project balance going like this so at 0 time it was -1000 and then it went to at one time 1 year time it was the 338 then so it was 1 then at two it went to 261 so it was 261 then at 3 it went

to 783 and then further finally so it was 3 and then in the 4th one it was 1200. So, that was so in 3rd and 4th we could not we had not calculated at that point of time but it can be computed and it can be seen that you know it will be -1000 and you do know that the interest rate is basically.

So, it will be question 3 and here i is 25% now if you look at so what we saw that here n prime was coming out to be 2. Now let us again further discuss that we have to discuss about 4 types of cash flows diagrams and we have to find this project balance for these 4 types of cash flows. So, this is cash flow 1. Now the second type of cash flow we will talk about basically this is your -1000 and then you have the 1, 2, 3 and 4 and this is all these 631.

Now in this case when we talk about finding the project balance values so you have -1000 so at 0 time now for 2 if you look at so for 2 you can calculate the for 2 at t =0 the project balance at 25% with 0 time it will be -1000 so that is obvious so when we try to draw this you know project balance diagram it will be -1000. Then if you go to the first term time at t = 1 now in this case it will be minus of 1000 and it will be multiplied with 1.25 and then add with 631.

So, it will be 1250 + 631 so it will be - 619 then at t = 2 it will be project balance will be computed so it will be PB 25 1 so PB 25 2 will be -609 into 1.25 and then +631 so 619 and multiplied by 1.25 so it will be something like 773 or so and you know 773 so it will you - 773 + 631 so it will be -142 so you are getting now what we saw that in the earlier case at n = 2 itself at t = 2 you got these direction changed the sign change from negative to positive what here is still it is negative and that is basically because of the difference of the initial transactions you see.

In the first year you had the larger amount of transactions receipt 912 where you as you have here only 631 so that has created the difference here. Now go to at t = 3 if you compute PB 25 at 3 so it will be again -142 * 1.25 +600 you know 31 so 631 will be a further added now if -142 +631 you can write so now it is -142 * 1.25 it will be something like 177.5 + 631. So, that will be basically now coming out to be 452.

So, that is how you can calculate that and then at t = 4 so this PB 25 4 will be again 452 * 1.25 and then you have + 631 so it will be 565 + 631 is this close to you know 1200 so that way what you see is that in this case in the case of 2 this n prime has come out to be 3 years.

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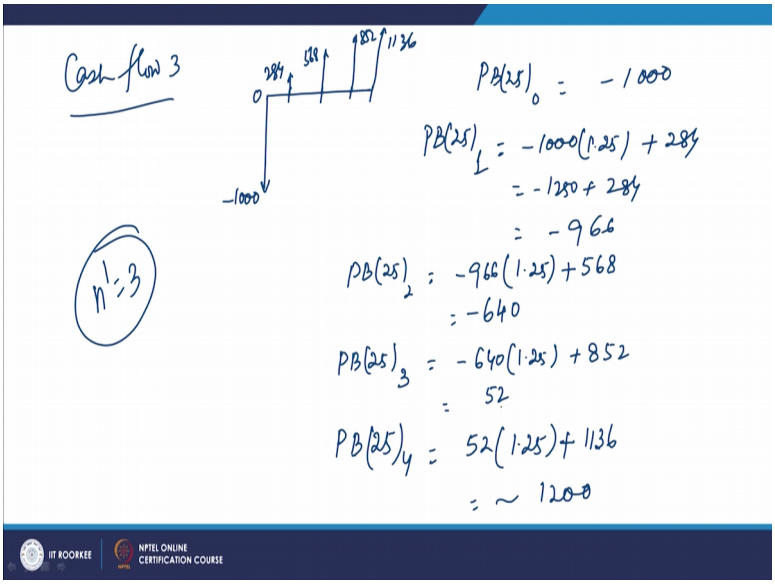

Now you can further check for the next type of cash flow 3 now in the cash flow 3 we have seen that you had power -1000 here at 0 time and you have 4 years and in that you have 284 then you have 568 then 852 and then you had this 1136 so as you see here that you have small amounts you know receipt in the initial times and how it will be going to effect this project balance let us see that.

Now in this case again PB 25 0 will be -1000 then PB 25 1 will be -1000 * 1.25 and then + 284 so it will be -1250 + 284 so now it will be 966 what with negative sign. Similarly you go to PB 25 2 so it will be -966 * 1.25 plus now you in the second year you have the receipt of 568 so - 966 * 1.25 it will be something like 1206 or so. So, once you do that once you do it is you know arithmetic computation you are coming out to be 640.

So, it will be twice into 1208 or so and then that multiplied by that +568 so it will be 640 then you come to PB 25 at time 3. So, it will be again negative sign then you come to 3 so again it will be -614 to 1.25 plus now you have 852 so with 852 now at 640 * 1.25 that will give you 800 + 852 so it will be giving you - of so it will be giving you 52 so it has basically gone to positive sign and that is why n prime has come out to be 3.

If you compute for you know 4th year so for 4th year 52 * 1.25 and + 800 so this will be 1136 and then again this is 65 so it is coming out to be close to 1200 so present balance and the net value is coming out to be the same one like 1200 at the end but the n prime is coming out again here as 3.

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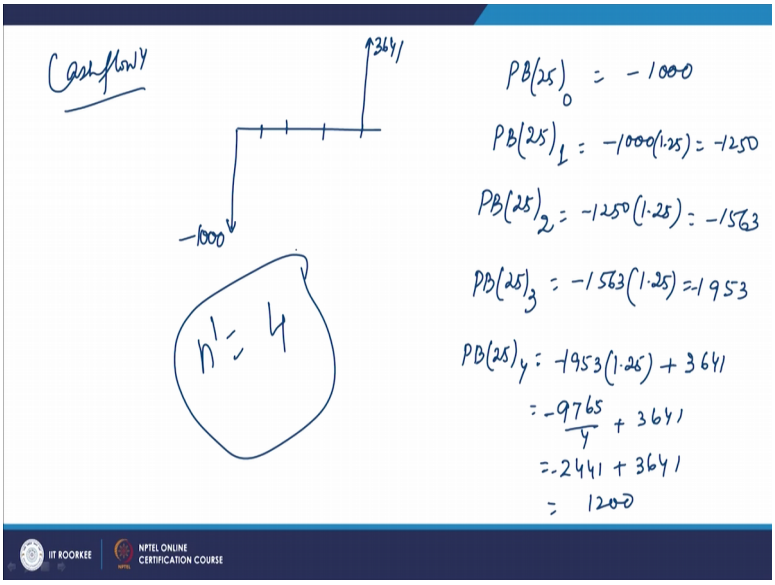

Now in the end you know for the cash flow 4 was like you have 1000 here and in the end of 4th year you had the 3641 so for this what you see is that for certainly with this 0 you have -1000 then at 1 because there is no receipt or disbursement so at one it will be -1000 * 1.25 so this will be + 0 so minus 1250 then you go to the second time now in this case you have again 1250 * 1.25 so it will be -1563.

And in the case of 3rd it will be -1563 * 1.25 so it will be 1953 so it will be basically this 1563 Multiplied by again 1.25 then once you go to 4 then in that case you will have so it will be minus, -1953 * 1.25 and then you have added to 3641 so now if you look at this it will be mean to you know 5/ 4 so -1953 into 5 so it will be 9765 / 4 and +3641 so it will be you know 2441 it will be minus and +3641. So what you see is that you are giving getting at 1200.

So, in all these cases what you see that ultimately and the end you have 1200 as coming as the project balance at the end of 4 year but what you see here that in this case your n prime comes out to be 4. So, n prime basically is going to change in this case in the last two cases it was in the first case for 2 years second and third case it was three years in the 4th case it was 4 years because of this typical type of cash flow transaction.

So, that is how the meaning of cash flow you know you know meaning of project balance is you can find at what time there will be a situation of you know signing that present to net value from changing from the loss to profit and so and that can be computed and that is being utilized for these investment you know purposes thank you very much.