Financial Mathematics
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Lecture – 24
Project Balance

Welcome to the lecture on project balance. So, in this lecture we are going to discuss we have already discussed about the alternatives comparison methods and to introduce this project balance what it is? It is one of the methods of measuring economic worth of alternatives so when we are talking about different alternatives then it is said to be one of the method by which you can measure the economic worth.

Now what it does tip typically that we must know so it describes the equivalent loss or profit of a cash flow as a function of time. So, basically what we know that in a in a typical financial transaction cases you will have the recipient disbursements at different times. So, in a typical investment you know for by a for a typical investment company something will be invested at zero time and then certainly so that will be a disbursement and then in future you are going to have the receipts.

So, you are going to get the you know whatever you have invested so that will give you that will give you certain fruits like you will be getting some income profit and all that so there will be received you know in due course of time. So, now what happens that now if you are to ascertain that at any particular instant where you are what is the status of you know the company so that basically will be measured by finding this project balance and they will be present value diagrams also that will tell us about it. So, it describes the equivalent loss or profit of a cash flow as a function of time.

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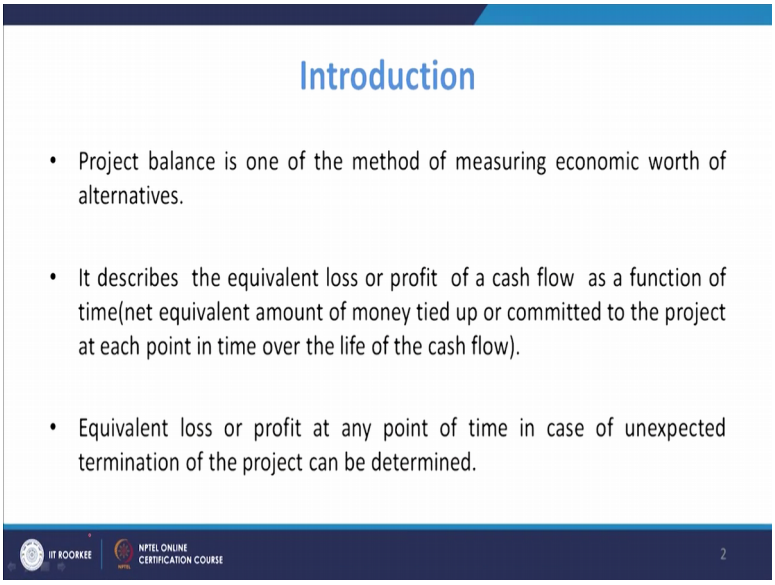

So, what it will do is the net equivalent amount of money tied up or committed to the project at each point of time over the life of the cash flows. It will talk about the you know equivalent amount of money which is tied up or committed to the project at each point of time. So, what happens that many a times there may be you know that a company may be terminated the project may be terminated at unexpected time.

So, you will be able to know that what is the status of equivalent you know a profit or net equivalent to IO you are in the state of profit or you are in a state of loss so that basically will be you know computed that will be known to you by now finding by using this finding the project balance at that particular time.

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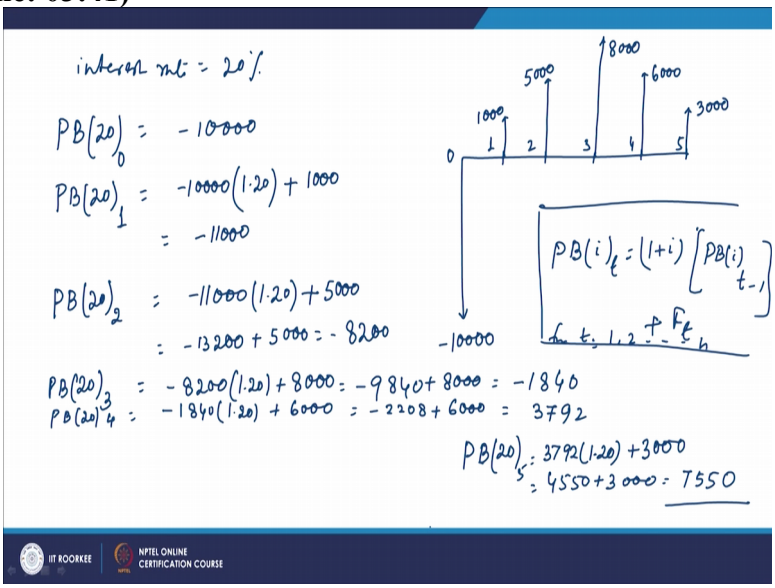

For example suppose that you take you know one cash flow and this cash flow typically you have invested at zero times this is zero time and then you have 1, 2, 3, 4 and 5 so you will have 1, 2, 3, 4 and 5. So, now suppose you have invested 1000 rupees now here you know

10000 rupees initially so that will be basically in the negative with negative sign and then you have the cash flow that is in the first year and you are getting 1000.

Second year you are getting 5000 then further third year you are getting 8000 and then fourth year you are getting 6000 and then further you are getting 3000. Now suppose you see that this is the cash flow for a particular investment. Now you need to know that at particular time what is the status whether if suppose the project is terminated at any stage then at what stage it is whether you are at loss or whether you are at again or you are at a position of no loss or no gain so like that that will be you know ascertained by calculating this project balance in that case.

Now suppose for this case suppose interest rate is taken as 20% per annum so compounded annually. Now what we do is that when you are taking you know this project. So, before so at present if you take the project balance at present if you assume that now the project is terminated you have the company has already put in 10000 of rupees or the person has put in 1000 of rupees and the project is terminated in that case what is the project balance?

So, project balance at rate of 20% interest rate at time 0 so this is time 0 and if you look at that that will be basically 10000 so it will be minus of 10000 that is what the meaning of you know this project balance is. Now suppose you are you have invested and now you are coming to this time one. Now at time one what will be the project balance now if the rate of interest is 20% in that case at this time the value of this 10000 will certainly not be 10000 because there is interest rate attached to it.

So, if you find the project balance at 20% for time equal to one year in that case what will happen does this 10000 it will have the value so it will have the value at this point you will have to multiply with F by P i n. So, it will be minus of 10000 into 1.20 so that will be the factor as you know so that will be F by P i 1 and then at this point you are getting a receipt of 1000 so if you give the receipt of 1000 in that case so you are getting minus of 11,000.

So, what we see that you were having you know the deficit value you know net you know net present value net present you know possible represent to balance value as 10000 at 0 time but after an year although there was a receipt of 1000 but the project balance if you calculate this project balance will be further decreased. So, it will be going to -11000 so instead of 10000 has gone to a -11000 and it is all because of the value of this 10000 which has become 12000 after an year.

Then you go to the project balance for 20% at time equal to 2. Now at this point you know at time equal to 1 your value was -11000 and then you are further calculating that project balance at a time one year ahead. So, that will be you know at this point you had 11000 at this point you had 11000 - of 11000 then it's value will be further that is to be multiplied with 1.20 and then you had the receipt of 5000 at an equal to 2 so 11000 *1.20 so it will be 13200 so it will be minus of 13,200 and +5000 so you will get it -8200.

So, what you see that the project balance has now decreased because of the receipt which is there at the end of second year equal to 5,000 rupees. So, this way project balance has changed. Now you can then further you can calculate project balance at n = 3. So, it will be again - 8200 * 1.20 and then at n = 3 you have 8000 rupees. So, 8200 *1.20, so it will be 9840 and so it will be -9840 + 8000 so it is still coming out to be minus of 1840.

Then you can similar way you can calculate PB you know 4 and PB 5, so if you calculate this PB 4, so PB 20 at 4 time it will be again you know you have minus of 1840 and that will be multiplied by 1.20 and at n =4 you have the receipt of 6000. So, 1840 * 1.20, so it will be 2208 so it will be minus of 2208 and +6000. So, it will be you know 3792 so it has gone to now it has changed its sign.

Now what you see that if you continue the project upto 3rd year discontinue then what you see that you are still at the net present balance value that is minus of 1840 but to go to 4th year in that case the project value is changing its sign from negative to positive and now it has gone come to 3792 so basically that much time it must run so that it gives you profit otherwise you are going to have the loss.

So, you know this is the time which is very important further you can also calculate PB for 5 and then that will be 3792 then again multiplied by 1.20 plus then you are getting 3000 further so 3792 * 1.20 so it should be something like 4550 something like that and then +3000 so it will be 7550 so you see that at the end of the period n = 5 what you see that your project balance value is coming out to be 7550 and up to PB 3 you did not cross that demarcation line from negative to positive.

But once you have gone to the 4th year coming taking into account the transaction of 4th year you are coming to a positive sign indicating the profit and then that profit has further increased as you go to the 5th year. So, this way you can have a generalized type of you know expression even for that and you can write that for PB i and PB i for any time t it will be now

you have what you have done is you have got the project balance at you know PB i of t-1 and then you have added this you know transaction which is there at time t.

So, this way for t = 1 to 2 and you can find these project balance values and what you do is that you use these positive and negative values and then interpret that how that is going to be you know going to be useful for coming to a decision that how long the project should run so that you are at a stage of profit or a loss. So, what happens that you can draw the project balance diagram basically for presenting it.

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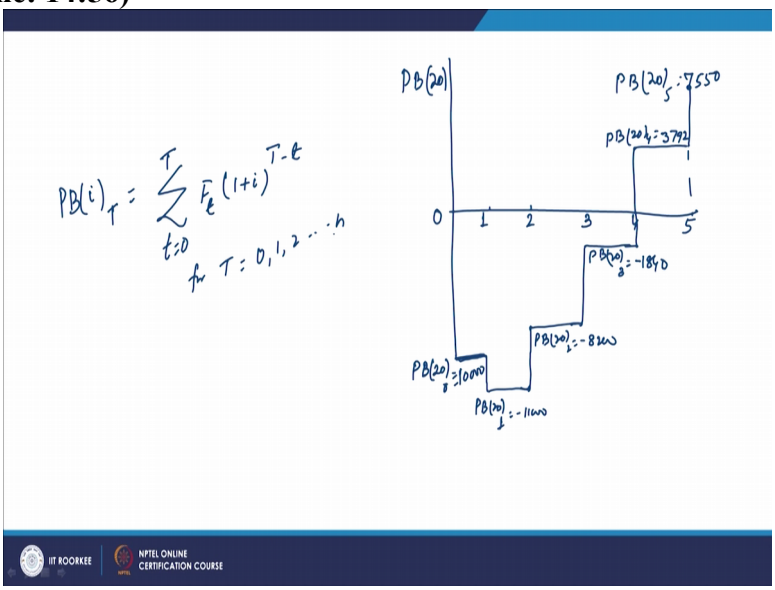

And what you see is that you calculate this project balance values and what you see that what you have seen in this case this is a project balance with 20% interest rate. So, this is your zero and one side you have profit one side you have lost and now what you see is that you have this is you know this is 1, 2, 3, 4 and 5 so now as you see that for 1 you get -10000 so this is how this line will move so this will be your PB 20 at 0 will be minus of 10000.

Then now once you come to 1 then after that you further deep to 11000 so PB 20, at 1 will be -11000 then add to once you have come you have come to 11000 then you know at 2 once you come you are coming to minus of 8200 so here you are coming to minus of 8200 then you are further moving to -1840. So, it will be at this point you will be writing PB 22 will be -8200 then you will be having a PB 23 so it will be minus of 1840 and at four you simply move up and you are coming to you know PB 24.

So, that is equal to 3792 and then further at once you reach at 5 so you go here and this is PB 25 so that will be your 7550. So, that is how now you are going to get the diagram and this diagram is known as the project balance diagram and this is useful for you know for finding that when this project should be terminated and in this case what you see that when n = n

prime that is when taken as equal to you know 4 this project can be you know if it is terminated you will not be in a loss mode.

So, that is how this status of the project can be ascertained in such cases. You can also find this project balance so if you are using another formulation you can also find for you know t-time so PB for any particular interest rate at any time t that you can find by you know summing this t = 0 to t and then whatever transactions you have at whatever time t will start from 0 and then 1 + i raised to the power T – t.

So that way for T = 0 1 to n, so for this you know you know you can use these transactions and you can find the positive valence value. So, if you are taking all these transactions into in one go and try to find the positive balance so that way you can use this formula and also find it.

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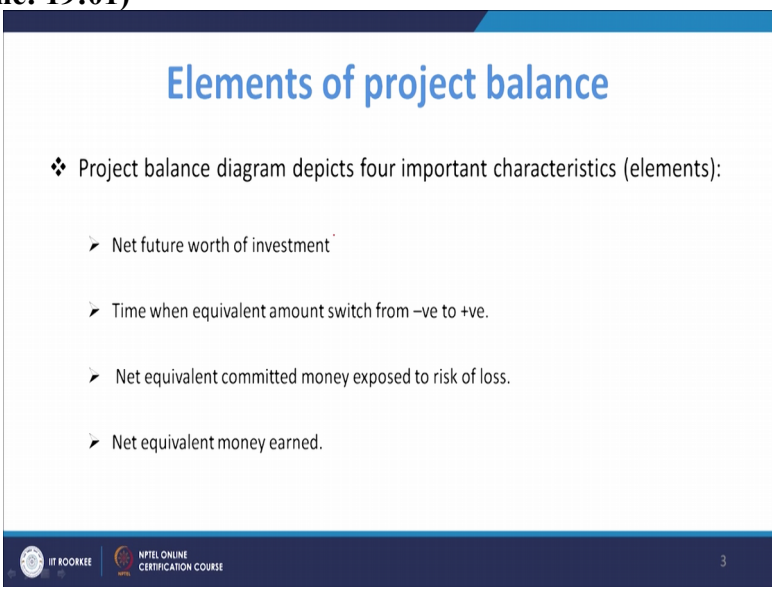

Now we will also discuss about the other traits of this project balance and as we discussed that project balance basically you know this diagram depicts four important characteristics. Net future worth of investment so basically when you are finding the value at the end so when you are finding the project balance diagram then in that case you know the value which is there in the end that will be shown as the net future worth of the investment then we will see that.

Apart from that you also know that when the equivalent amount will be switching from negative to positive that is what we have seen that when this equivalent amount which we are calculating that is whether the when that will be switching from the negative value to the positive value. So, in the in that case you are getting this at t = 4 then you also know net

equivalent committed money exposed to risk or loss basically this is risk or loss and also in the end you have you know a net equivalent money earned so that also you can find.

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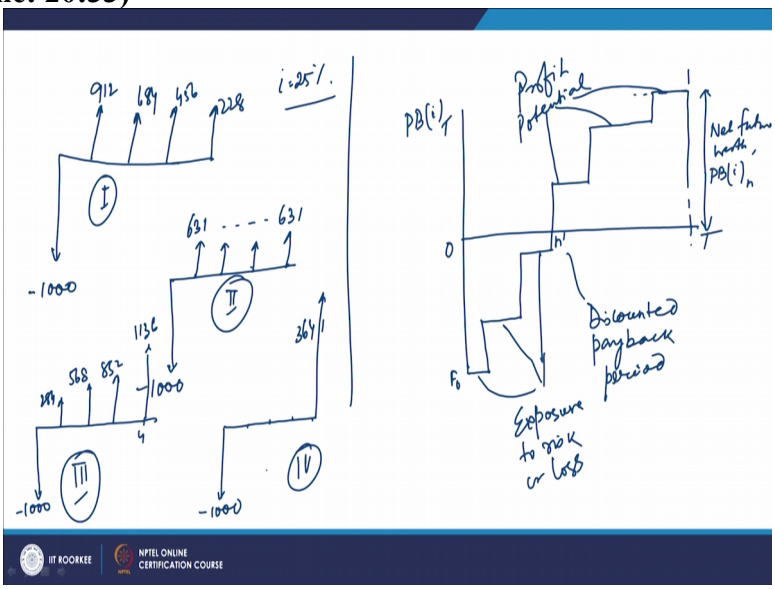

So, what you can see that if you draw this any particular project balance diagram so what you see that you have this F0 and then a typical project balance diagram will go like you know this and then it will move so like this and you know then it will move like that. So, what you see that in a particular so you have 0 and this is T, so this way your T is there and this is PB i at the interest rate i and this is T.

So, when we talk about this net future worth so basically what you find in the end this one so this is basically the net future worth. So, this will be in a PB, i, n so basically that can be so this is one of the element of the project balance then you know your time that is your time when the equivalent committed money switch from negative to positive and in this case this is the time and in the earlier case as you have seen that this was basically the time when your equivalent money from it turned from negative to the positive. So, that is how this time is known as n prime and this is known as the discounted payback period.

So that is how this period is you know calculated when there is no profit or all this is changing its sign so that basically is known as the discounted payback period. Now these are the reasons where which is exposed to risk or loss so this reason this reason or this reason so these are the reasons where you have exposure to risk or losses there. So, you are likely to be under loss if your project is terminated at this stage in that case you are under the you know position of risk.

Whereas if you are and in these you know situations then in this case you have the chance of profit so it is profit potential. So, this way this project balance you know diagram is going to

be you know understood and as we know that you have you must know these when you are at the risk or loss so this area with which is talking about where the PBi is negative that is the exposure to risk or loss and when you have profit potential then you have the chance of you know in that case this but as value will be positive and you are going to have the values you know positive in such cases.

Now as you know that you may have you know different cash flow diagrams and then you are supposed to basically find many times you know this n prime the discounted payback you know period and that is basically typically what you are expected to know and if in the case of different you know cash flow diagrams you will have the different cash flows and in that case you will be calculating the project balance at different times and then you are going to calculate that what will be the time at which this project violence value that is basically changing the you know direction.

So for example you may have different types of you know situations like you may have one type of you know cash flow where you can see that you have minus of 1000 and then you have different amounts coming here like you have 912, 684 then 456 and 228. Similarly you may have another cash flow where you are getting this minus of 1000 and then you can have the cash flows at the 4 times you know like 631 each.

So, that will be another you know cash flow the third type of cash flow may be there and that third type of cash flow may be like you have minus of 1000 and then you have 284 in the first then 568 in the second then you have you know 852 in the 3rd and in the 4th you have 1136 so that may be another type of cash flow and you may have another cash flow where you have -1000 and at the end of 1, 2, 3, 4 you have a receipt of 3641.

Now what we need to know using these type of cash flow diagrams that what will be your n prime which is required to be calculated. So, for example if you look at the first case if you look at the first case now you see that if for a particular interest rate say suppose I = 25% is given now what will be the project balance values at this time. So, we know that at the time zero the project balance value will be minus of 1000.

Now if you go to the second value at n = 1 or T = 1 now what will happen to this now at so suppose this is cash flow 1 and this was cash flow 2 and this was cash flow 3 and this was cash flow 4 now if you look at and try to compare these values.

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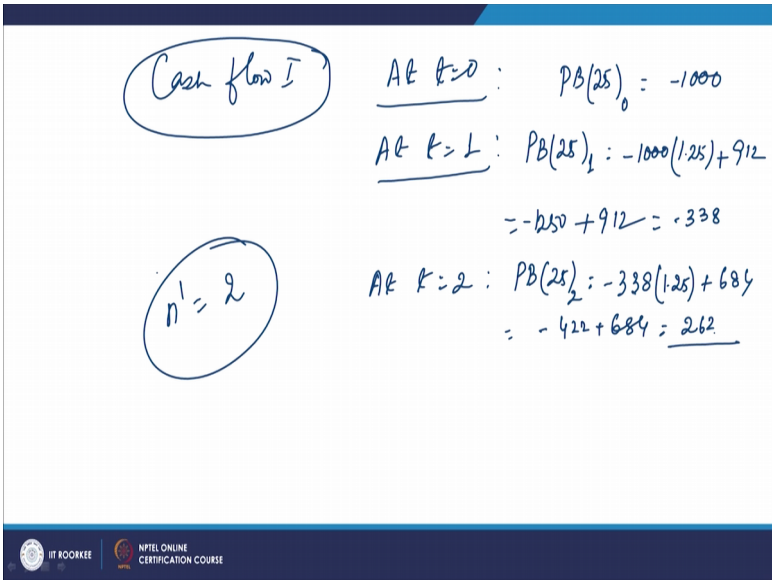

So, suppose you are talking about cash flow 1 so in the cash flow 1 what you see is that at t = 0 your project balance value because you are talking about 0 time so the investment is minus of so 1000 so project balance for 25% at 0 time will be -1000 then if you go for t = 1, so now what you see is that for t = 1 what will happen PB 25 1 it will be -1000 and then it will be multiplied with 1 + i so it will be 1.25 and then you have receipt of 912. So, it will be 1250 plus so it will be minus and then +912 so it will be -338.

So, similarly if you go to you know at t = 2 in that case PB 25 2 will be -338 *1.25 and then you have the receipt of 684 so it will be something like -422 and +684 so it will be minus of you know 422 + 684 so it will be 262 something like that. So, basically this way you can calculate these values and you can come to this at t = 3 you can calculate t = 4 you can calculate and then you can see that how these;

So you have seen that in this case you know at n = 2 so n prime what you see is that you are getting at second time period. Now this time period basically will be changing for the different types of transactions and that we can see. So, in the next lecture we can see when we try to solve the problems based on this then we will see that for different cash flows how can we find these values and we can get more you know you can have more clarity about the questions, thank you very much.