Financial Mathematics
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Lecture – 23
Capitalized Equivalent and Capital Recovery with Return

Welcome to the lecture on capitalized equivalent and capital recovery with return. So, these are the two you know basis of comparison again and we will discuss one by one. So, what is capitalized equivalent amount? This is a special case of present worth basis of comparison.
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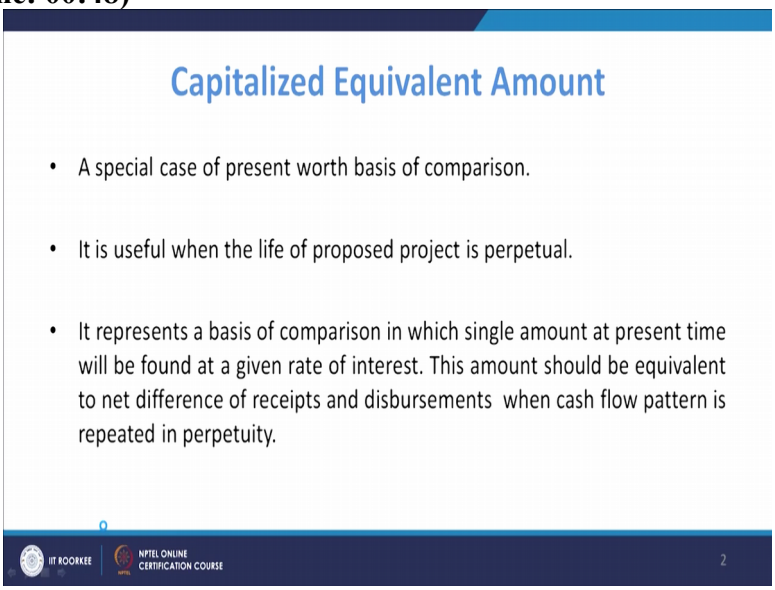

And it is useful when the life of the proposed project is perpetual so many a times we deal with situations when the life is perpetual and you are investing something you know at present and the life is considered to be very long so that is what it is perpetual. It represents the basis of comparison in which single amount at present time will be found at a given. Rate of interest so rate of interest anyway is defined and your the job will be to find the single amount which is required to be invested at present.

And these amounts will be equivalent to net difference of recipient disbursement when cash flow pattern is repeated in perpetuity. So, basically the cash flow pattern is required to be you know repeated in perpetuity in the long run and that is how you get it.
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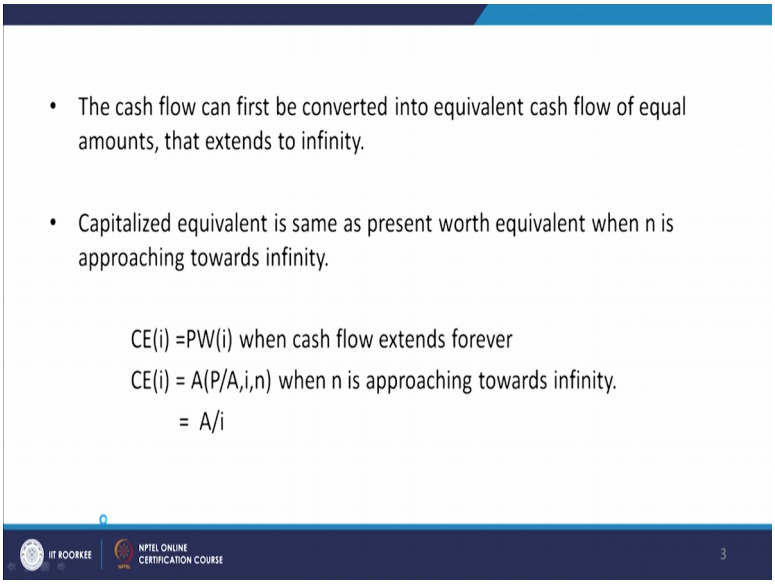

So, also the cash flow now first you know can be converted into equivalent cash flow of equal amounts that extends to infinity so you can have a cash flow which basically is you know going to infinity and you are getting basically the equal amount every year for the infinite time and it is the capitalized equality same as a present worth equivalent when n is approaching towards infinity so that is how the n is going towards infinity.

So, basically you are having a capitalized equivalent is nothing but the present vertical and when cash flow extends forever and it is coming at A/i so how that is coming that we will see.

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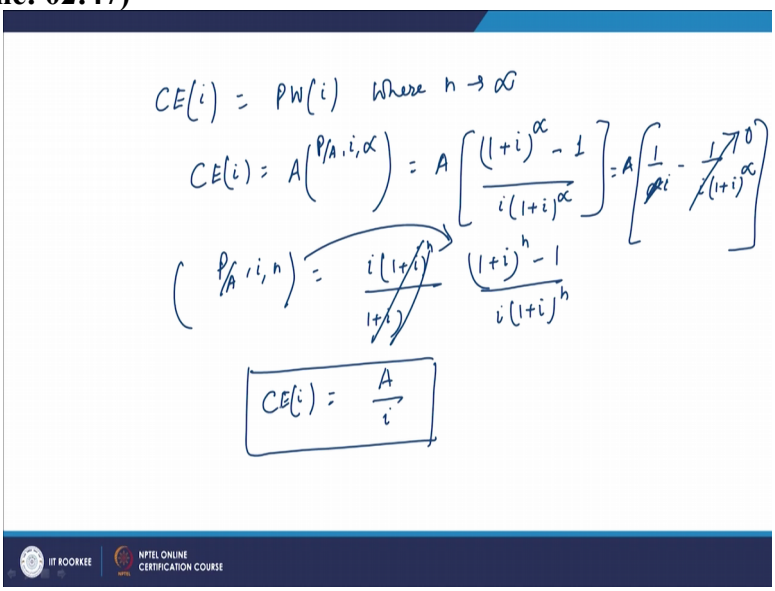

So, basically the capitalized equivalent is represented by CEi and it is as we know that it is the PE PWi and where n is basically approaching towards infinity. So, we know that we have to have the relationship between P and A. So, you know basically CEi we can write capitalized equivalent we can write as A * P by Ai infinity so basically it is the present

amount. So, A * P / A i infinity that will give you a so basically for finding P if you know A then A* P / A i infinity that will give you P.

So, that is how you calculate in the normal cases when you know n but here n is basically infinity. Now P/ Ai n factor is basically defined as you know it is 1 + i raised to the power n so it will be n is infinity - 1 / i *1 + i raised to the power infinity that is how it is defined because that P / Ai n is i*1 + i n / 1 + i no it is basically the reciprocal of it so it will be 1 + i n - 1 / i*1 + i n so that is what the P / A i n factor is.

Now this n is since it is approaching towards infinity so at in place of n we are putting the infinity value. Now if you take if you do that separately you put the terms it will be 1 by i -1 / i * 1 + i raise to the power infinity and when i is you know n is towards infinity so it will be approaching towards infinity and this term will be vanishing towards zero. So, this is this term is basically 1 / i. So, now what we see is that CEi it will be A/i.

So, if you know the annual amount which is required to be received by you then in that case you can have the capitalized equivalent amount and that will be found by dividing the A by the interest rate at which the cash flow pattern you know will be subjected to and once you know that then A/i gives you that so if you see basically if you look at it in a very rough way basically A will be CE*i. so, C basically is the amount which you are investing today and the CEi basically is the amount which will be you know amount of interest which will be generated in any year.

So, basically you have kept one amount in the investment as an investment and from that investment that interest amount generated you will be getting every year. So, since you are whatever interest is generated you are taking it out so the amount of investment which is their amount of investment amount that is anyway you know remaining the same and that is why the next year again you are going to get the same amount as the interest.

So, that will go till the infinity time so that is on that concept basically these capitalized the equivalent amount is basically defined. Now it is use will be mostly for you know for setting up off certain supposed municipal Gardens or you know for any park or so somebody wants to set a park and the maintenance of the park requires and other expenditures require every year you know you require one lakhs rupees for that.

In that case what should be the you know amount which should be put in the bank at certain rate of interest so that one lakh rupees you are getting every you know year-end so basically that did that kind of problems in those kind of problems we required to use such concept.

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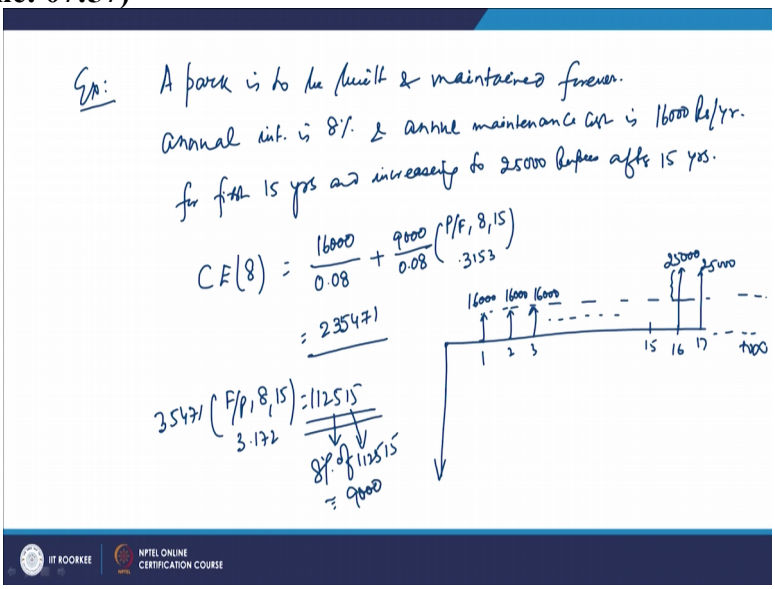

So now we can have one example suppose that you have so a philanthropic you know companies the foundation is there and they want to build a park they want to gift a part to someone and then it should be maintained forever. So, suppose a park is to be built and maintained forever. So, now for that you require to set up a fund. Now annual interest is you know which will be earned it will be and that to for perpetuity case is 8% and annual maintenance cost is something like 16000 rupees.

So, annual you know maintenance cost suppose is 16000 rupees per year okay. For first 15 years and further it will be increasing so and increasing to 25000 rupees for after 15 years now this may be very you know practical to see that because as the time progresses you will certainly see the increase in its maintenance cost. Now if you want to have you know a fund you know somebody wants to gift this park,

So, basically what should be the present amount of investment so that you are going to get the amount which is there required annually to maintain this park for building and as well as for maintaining that you know park forever. So, for that what we do is this is a case of the capitalized equivalent amount and you know for that you will get the CE8. So, what you see is that you are you know annual maintenance cost is 16,000 rupees for the for the first 15 years.

And then 25,000 rupees for you know after 15 years so the cash flow will be going like this so you are investing some pea and then you are getting 16000 for first you know 15 years it will go up to 15 and from 16 you are going to have you know to infinity so you are going to have the requirement of 25,000 rupees. Now this basically can be understood as a an infinite

you know a amount of 16,000 + also extra 9000 rupees from the 16th year onwards so that is how you can tackle this problem.

So, basically what you see is that for this 16000 getting get in finite time you know for perpetuity case you know that that amount will be A/i, so 16000/ i and i is the rate of interest is 8% so it will be 0.08 then this 9000 also you are getting for infinity but you only require from 16th year onwards. So, now you have to you know that amount so again so this 9000 will be also required then you know that will be for 9000 when you require from the initial period but then you only require you know the amount from 16th year onwards.

So, basically you are going to have it you know mapped with the factor by multiplying the vector with P /F 8,15 so you require you know here basically you do not require for these you know situations so for that this amount you want the equivalent amount from this point so that is why you have multiplied that with P/ F 8, 15 and that will be 0.3153. So, now basically if you so that can be seen P / F 8, 15 you can see from the table.

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Interest factor values for discrete compounding (i=8%)

i	n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	(A/P,i,n)	(A/G,i,n)
0.08	1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800	0.0000
0.08	2	1.1664	0.8573	2.0800	0.4808	1.7833	0.5608	0.4808
0.08	3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880	0.9487
0.08	4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019	1.4040
0.08	5	1.4693	0.6806	5.8666	0.1705	3.9927	0.2505	1.8465
0.08	6	1.5869	0.6302	7.3359	0.1363	4.6229	0.2163	2.2763
0.08	7	1.7138	0.5835	8.9228	0.1121	5.2064	0.1921	2.6937
0.08	8	1.8509	0.5403	10.6366	0.0940	5.7466	0.1740	3.0985
0.08	9	1.9990	0.5002	12.4876	0.0801	6.2469	0.1601	3.4910
0.08	10	2.1589	0.4632	14.4866	0.0690	6.7101	0.1490	3.8713
0.08	11	2.3316	0.4289	16.6455	0.0601	7.1390	0.1401	4.2395
0.08	12	2.5182	0.3971	18.9771	0.0527	7.5361	0.1327	4.5957
0.08	13	2.7196	0.3677	21.4953	0.0465	7.9038	0.1265	4.9402
0.08	14	2.9372	0.3405	24.2149	0.0413	8.2442	0.1213	5.2731
0.08	15	3.1722	0.3152	27.1521	0.0368	8.5595	0.1168	5.5945
0.08	16	3.4259	0.2919	30.3243	0.0330	8.8514	0.1130	5.9046
0.08	17	3.7000	0.2703	33.7502	0.0296	9.1216	0.1096	6.2037
0.08	18	3.9960	0.2502	37.4502	0.0267	9.3719	0.1067	6.4920
0.08	19	4.3157	0.2317	41.4463	0.0241	9.6036	0.1041	6.7697
0.08	20	4.6610	0.2145	45.7620	0.0219	9.8181	0.1019	7.0369

P / F + 8 + 15 so it will be point this 3152 so now 53 or 52. So, that way you can put it and what you are getting is that you get 235471 so now this amount will be required for that setting of the park. So, we can understand this in a different way also that if this amount basically the 2 lakh basically will be used for giving you 16000 every time then rest amount is 35471 this 35471 is basically you know deposited for 15 years.

So, if this 35471 its future amount will be now you know for that you will be multiplying with F / P 8, 15 so that will give you this factor will be 3.172 so that will give you something like 112515 and if you take 8% of this so 8% of this amount 8112515 so that one will be

coming close to 9000 rupees. So, this 9000 extra 16000 + 9000, 25000 you are getting from this time for perpetuity so for all the period you are getting.

So that is how you may have different you may look you may encounter with different cases and you have to solve such problems. Now we will come to the another you know point another terminology you know that is capital recovery with return.

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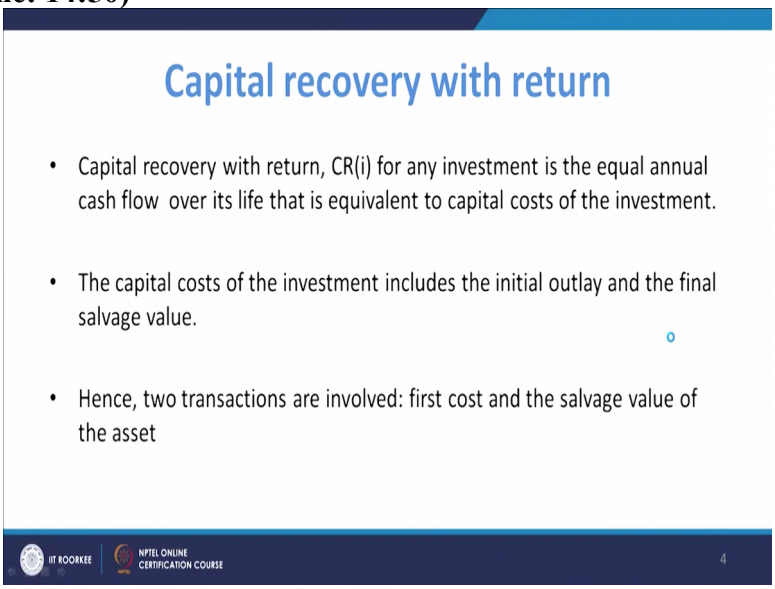

Now what is capital recovery with return this is for a foreign investment it is the equivalent well cash flow over its life that is equivalent to capital costs of the investment. So, basically capital cost means whenever we have an investment and we know the time period for which the investment is going to take you know it is being done in that case when the investment is over and you have put the capital then you have also some salvage value of all this capital.

So, you have that includes the initial outlay whatever you have invested and in the end basically you are going to get some you know amount after selling it so that is your salvage value of the plant. Suppose you are establishing a plant then once you are selling the land cost or the equipment cost or so, so that will be the salvage value. So, basically when we are taking that into account in that case whatever we because you have also the recovery of the capital so that is why that amount that you know that annual equivalent amount basically will be known as the capital recovery with return. So, here you have two transactions involved first cost and the salvage value of the asset.

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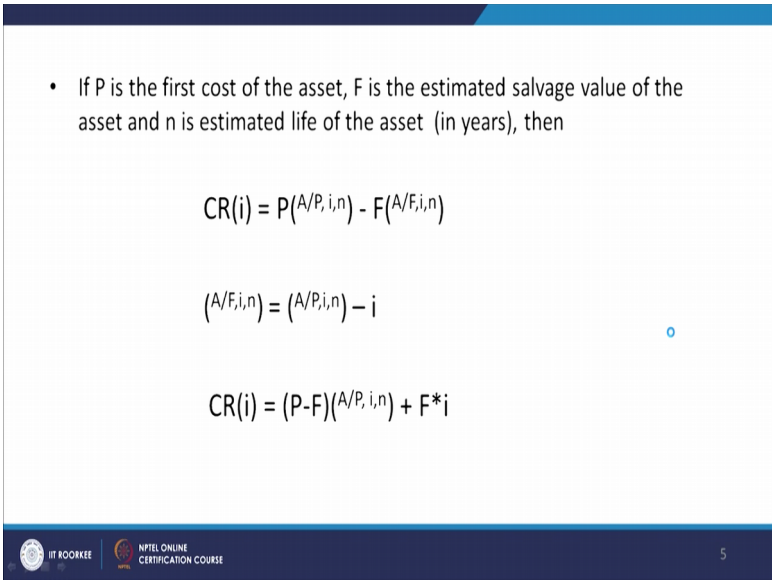

Now what we see that here we see that you have the P as the first cost of the asset and F is the estimated salvage value of the asset and n is basically the estimated life of the asset and in that case the capital recovery so the capital recovery with return if you try to you know define now what we see that you get some kind of you know cash flow pattern.

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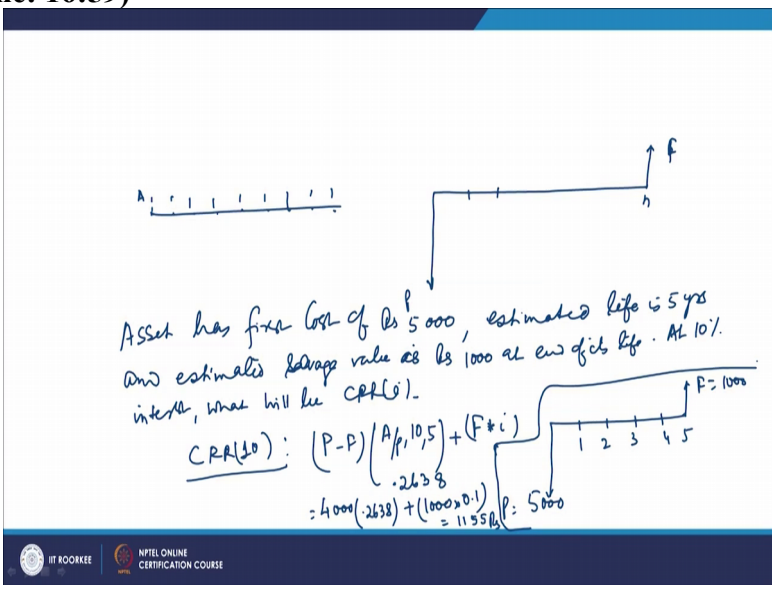

So, basically your cash flow pattern goes like this that you have invested some amount here you have invested here and at the end basically you are getting some salvage value so that is what the cases. So, for this basically you require to know one equivalent cash flow you know you know diagram or this A you are trying to have. So, this will be P and this will be F and P is your investment capital amount which you have invested at present.

And F is known as the future salvage value that is what it is telling this F is basically the estimated salvage value of the asset and n is the estimated life of the asset. So, this time will be you know you have n years. Now in this case you will have some P this P has some you

know a component ultimately you are going to have a you that amount of equal annual value which should represent this you know cash flow diagram so that is what is the capital recovery with return.

So for that you know what we do is that you are getting the P multiplied by the factor A/ P, i, n so this factor are being multiplied with P it will give the equivalent annual value for that P and then similarly the F when multiplied with A/ F, i, n that will give you the equivalent annual value for the salvage value what you are getting. Now one is positive and another is negative so this P is negative and F is positive.

So if you take from the you know perspective of you know the investor so he has put in P and we are taking that the negative direction downward direction as for the positive one so it will be P and then this upward direction will be F-. So, that way you can get the capital recovery with return value as P of A /P I, n - F *A /F, i, n. Now you can we have discussed that there is some correlation between A/ F, i, n and A by P, i, n. So, we know that A/ F, i, n will be i / 1 + i n, raised to the power you know n – 1.

Similarly A / P, i, n will be i * 1 + i raised to n upon 1 + I raised to power n - 1 so that way you see that once I will be subtracted from 1 you will get this. So, if A/ F, i, n will be subtracted now you know we will be represented by A/ F, i, n - i then it will be again P * you know A/ F, i, n F * A/ F, i, n and then again F i so this will be the final expression P - F A/ F, i, n + F * i. So, that basically is and the equal annual cash flow over its life that is equivalent to the capital cost of the investment which includes the you know first cost of the investment and also the salvage value of the investment.

Now this we can understand with a problem and this problem will be something like suppose you have a situation where your asset is you know has first cost of rupees 5000 and estimated life is five years and you know and estimated salvage value so estimated salvage value is suppose rupees1000 at the end of its life. So, salvage value is at the end of its life so now you have to know the capital recovery with return.

So now at 10% interest what will be the capital recovery with return so basically you are going to have CRR 10. So, CRR 10 is to be found out now as we know that you have P as you know 5000 and you have F as so this is P and this is F as 1000 and this is your 5 years 1 2 3 4 & 5 so we know that we can get this P * A / P i 5, A by P and then 10, 5 - and F of A / F and then you have i 5.

So, that also you can get or else you have this as P - F of A by P than 10 and 5 and then you have + F * i so this also factor can be used to get that. Now we know that this P so if you take then you know 10% of table you can go and see the 10% table and for the 10% table basically we need A / P, 10, 5 so it is coming as .2368 so this value is coming as .2368 P - F is basically 4000 thousand.

So, 4000 * 0.2638 + F is given as 1000 into L the interest rate is .1 so this is how you are getting the capital recovery with return and this amount is coming out to be you know so this will be something close to 1000 and then 300 so it will be 1155. So, now basically we are getting this 1155 rupees that is known as the capital recovery with return and this will be mostly used for such cases.

In many cases you have you know you need to represent those cases where you have the capital cost and then in the end you also have tea you know return in terms of selling of the; you know items and the salvage value of the plant. Now we can also deal with another situation for such problems of the capital recovery with return. Suppose you have one company which has been given the two mutually exclusive alternatives.

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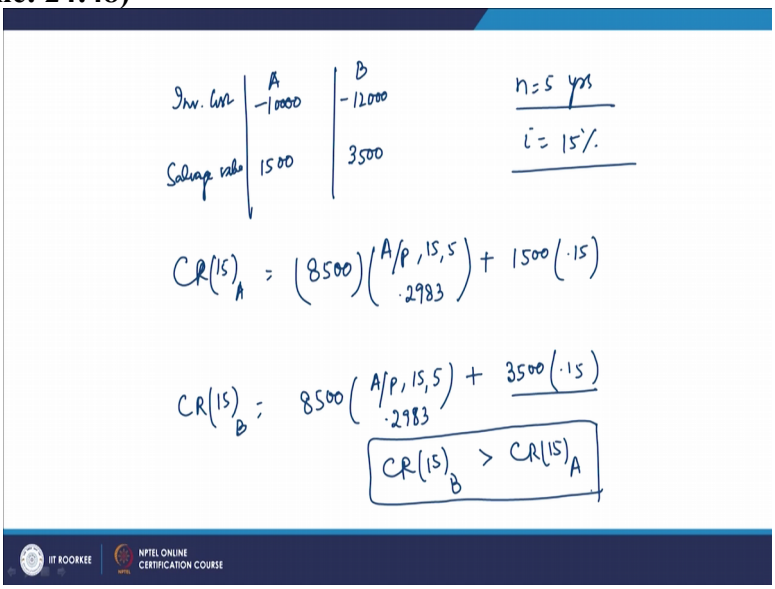

And suppose the you know for the in the first initial cost which is their the investment cost is you know for alternative A is given as -1000 and then it is salvage value, so salvage value is given as suppose 1500 similarly you have another alternative B and it has the initial investment is -12000 and then you know salvage value is given as say you know 3500. Now the time is supposed 5 years so if n is 5 years and you have to compare the capital recovery with return for the two cases.

So, for the two cases and interest rate is suppose the interest rate is given as 10% so or you may have interest rate is taken as say suppose 15% so you take i as 15%. Now you can basically calculate the values of the capital recovery. Now capital recovery if you take at 15% for A so it will be you know 10000 -1500 so it will be; so this is P and this is your F say 10000 -1500 it will be 8500 and then A / P, i, n so A / P i is 15 and n is say now 5 + salvage value is 1500 into you know interest rate is 0.15.

So, similarly A / P, 15, 5 if you have to find A/ P and 15 and 5 will be 0.2983 so it will be 0.2983 so that will be the one and then if you take so suppose capital recovery of you know B option then in that case you can calculate as 12000 - 3500 so again it is coming as 8500 and what you see is again you have the factor A/ P 15 and 5 so it will be 0.2983 and then + you know F i.

So, in this case the salvage value is 3500 and you have i as you know 0.15 so what you see that in such cases when your P - F is same in that case it is obviously you can come to this conclusion that since these two you know you know quantity is being same we see that this amount is larger because this is 3500 and this is 1500 so you can come to the conclusion that CR 15 B is more than CR 15 A.

So, you basically it will be the one however now if you are further told that what should be you know how much your salvage value of you know this amount should be less so that their capital recovery cost is same so in that case you can say that you know by what it should be you know increased or decreased so that they become equal. So, you know that this is 3500 and differences so if it is 1500 in that case both will have the same capital recovery.

So, now basically by looking at such situations you can come to the different conclusions. Now so what we discussed in this lecture that we have two types of you know finding these equivalence formulas we have discussed two types. Ne is your capitalized equivalent where we normally used this concept of finding you know the present amount and that present amount will be giving you some return you know in future in perpetuity.

So, in that case that will be used for you know many times with societies may think of setting certain funds which will be required for the you know a maintenance of the club suppose so in those cases when you know that what will be the expenses and how if you are even taking the you know increment in the expenses after maybe certain years. In those cases that concept of the capitalized equivalent amount will be utilized.

Similarly whenever you have to discuss about the capital recovery you know you have any plant where you have the initial investment putting in and then ultimately you have to the time period which is defined for which the investment is going to run. So, in those cases you will have also the; you know salvage value earning so that will be at the end of its time period. So, in those cases you are going to deal with such situations and basically how much return is coming that also will tell you that in what way the capital recovery cost will vary.

So, different types of problems basically you can deal with you can solve the problems you know that will be given in the assignments and you will be having the more and more understanding about this type of problems which will help you understanding these financial transactions of certain these type of organizations. So,, that is about the these two concepts of you know capitalized equivalent and capital recovery with return, thank you.