Financial Mathematics
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Lecture – 22
Payback Period

Welcome to the lecture on, Payback Period. So in the financial terms or in general terms also, Payback period means, something a period in which the investment is supposed to pay back. So many a times we use the terms like this investment will pay in itself and in less than suppose three years or four years it means it will pay back whatever you have invested if, if it goes for four years and depending upon what you know is the cash flow at different times.

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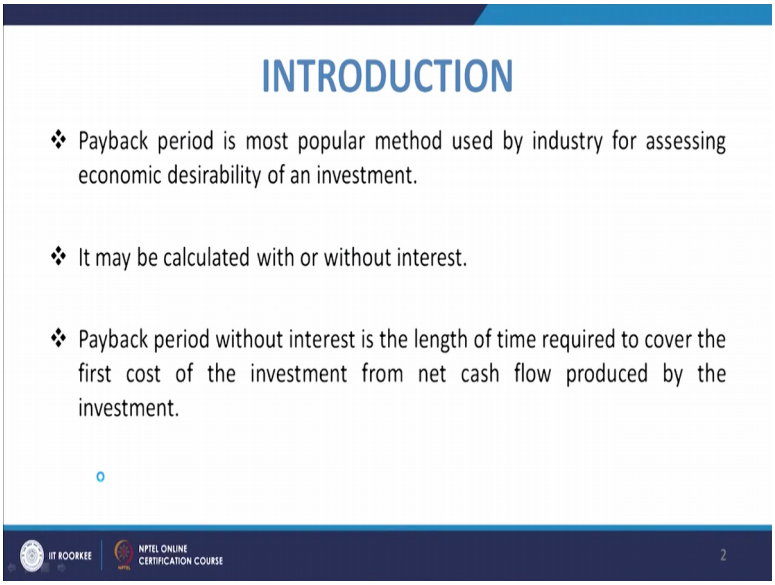

So there is a period in which, it is supposed that whatever you have invested, it has come back. So no so that is what the payback period is, and we should know because it is the most popular method used by the industry for assessing economic desirability of an investment. So whenever you know, any investor invests any amount then he is interested, you know to know that, for how long he is, you know, once he starts the business once he starts gaining something from it, so how long you know he has to continue with certain type of flow pattern, cash flow pattern that he you know there will be payback, so for the investment and that period is normally used, known as, the payback period.

It may be calculated with or without interest, so certainly when you have the without interest cases, then, in that case, you know, your calculation method will be different, you do not have to see the time value of the money and when certainly there is, you know, interest factor to be

taken into account, in that case, the payback period will be certainly different than that being found using the case without interest.

So when we talk about the payback period without interest it is the length of time which is required to cover the first cost of the investment from net cash flow produced by the, you know, investments. So that is what the definition says, that it will be that time when your first cost of the investment will be you know, covered like you have invested 1 lakh rupees and then you are getting for the subsequent years maybe 20 thousand rupees each.

So when there is no interest to be taken into account, in that case, you know, in that case, whenever this 1 lakh rupees is covered so if it is 20 thousand in that case it will be 5 years. So that we can see even by example, but before that so we should have some other more concepts. So what's there that if suppose F naught or F0, is the first cost of investment, so whatever you have invested, so that is your F0.

And Ft is, the you know net cash flow net cash flow at in period t. So, net cash flow in period t. So you know in that case, what you do is, that your payback period will be payback period is smallest value of n that satisfies this following equation. So that equation is summation t equal to 0 to n, and then this Ft, that should come greater than or equal to 0. So that you will be finding that and whenever it is approach, coming to 0 or in whatever time period in which our time you know time this ft becomes you know summation of ft becomes equal to or more than 0.

In that you know minimum value of n will be known as the payback period. So certainly we have to, we are interested to find this n, and let us say for the case of you know for case with, without interest, so when suppose you have case without interest, in that case, it is very easy to see, that how you to calculate this payback period.

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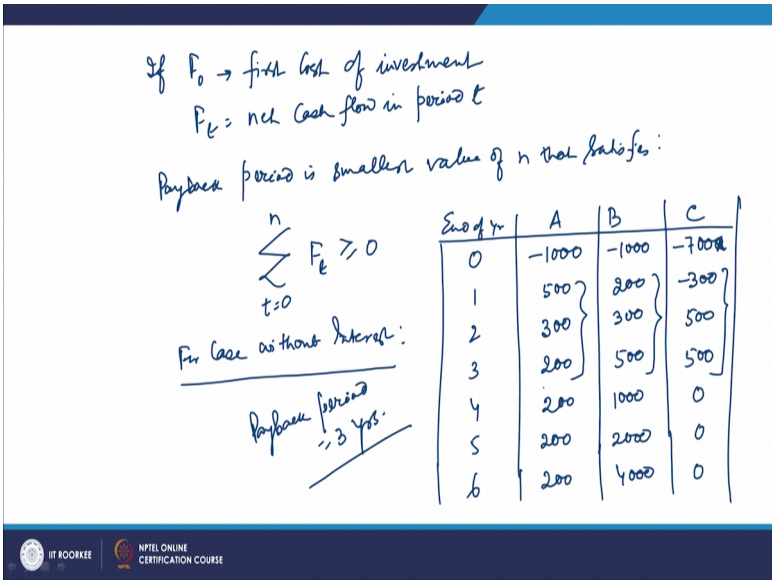

Suppose we can have one example and suppose you have an example like end of year is given, and then you have different alternatives A, B and C. Now suppose end of year will be 0 1 2 3 4 5 6, now let us say, we know that, in the K in the year 0, your first investment is being put and put in by the investor, so we have put in, you know, 1000 rupiah, investor has put in 1000 rupees.

Now, now here this so this thousand rupees is the, investment amount, by the investor and you have three, you know options, three alternatives. Now payback period payback which you are getting or cash flow which is supposed to be you know to get from these three alternatives are like 500 in the first year, 300 in a second year and then remaining 200.

So suppose this is the type of investment which you know this is a type of return which you are getting or count cash flow which you are getting from alternative A. Similarly suppose for B it will be 200 300 and 500 and then you have 1000, 2000 and 4000. And for C suppose, it is minus 300 and then 500 and 500 and then you have 0 0 and 0. So suppose this is a case without interest, now in this case, if you see, you, you would see that, here you go to three periods and when the interest is not taken then simply you have to take the arithmetic sum of the Fts and you see that at three it comes a 1000.

Similarly here also at three it comes 1000. And if you go to this, you know, for this particular, you know, investment and you know this then in that case, suppose no you know in this case in this case of C it is 700 basically not one thousand, because 1000 will will never be you know get we cannot obtain it. So it is 700 in that case, it will also be three years. So in this case the payback period is payback period for all these three cases is three years.

We cannot, so with that, there was a mistake in writing this 1000, 700, so this way now if says that that will be the case, whenever you have, you do not have any, you know, interest factor into, no question, in that case, you have to simply add these things because there is there is no change in the value of the money and that's why you have equal payback period for these three, you know alternatives.

Now the thing is that, this is the case, when you do not, you have not considered, the effect of interest you know, now, now when you have, now the thing is that although they are taking same time, but the thing is that, the only difference you know what maybe that, what rate you are recovering the amount. So suppose in this case, in the first year you are recovering half and in the second year you are recovering sick value, you know 30% and then you 50% then 30% then 20%.

So you have recovered you know 80% after 2nd year and in this case you have recovered only 50% of the second year and in this case you have recovered only 20% after the second year. So basically the only difference is that at what rate you are recovering but otherwise the payback period, in these two cases are basically same, and you can go for you know for this you know decision that how long the project should go, so that your investment amount is you are getting back to it to yourself.

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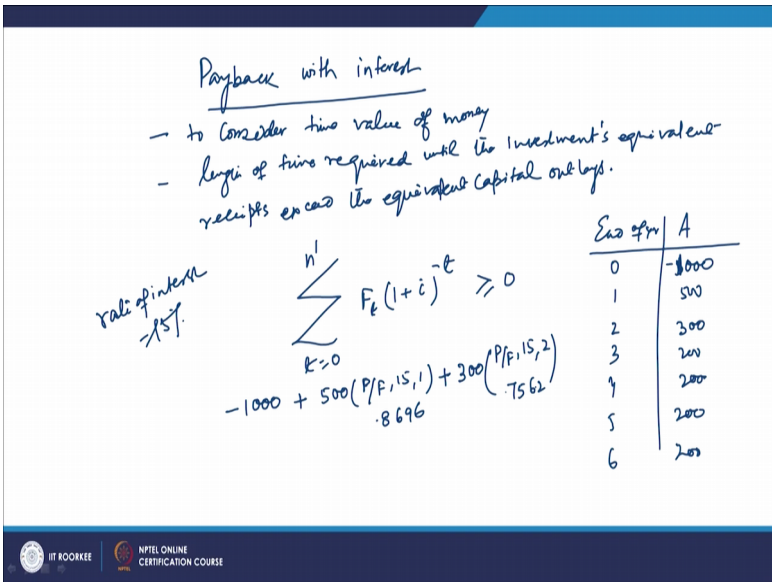

Now we will deal with the situation, when you have the pay back with interest, so that is what the normal cases, when you have, or you are investing, whenever some person is investing we will try to you know and certainly the interest amount is into picture.

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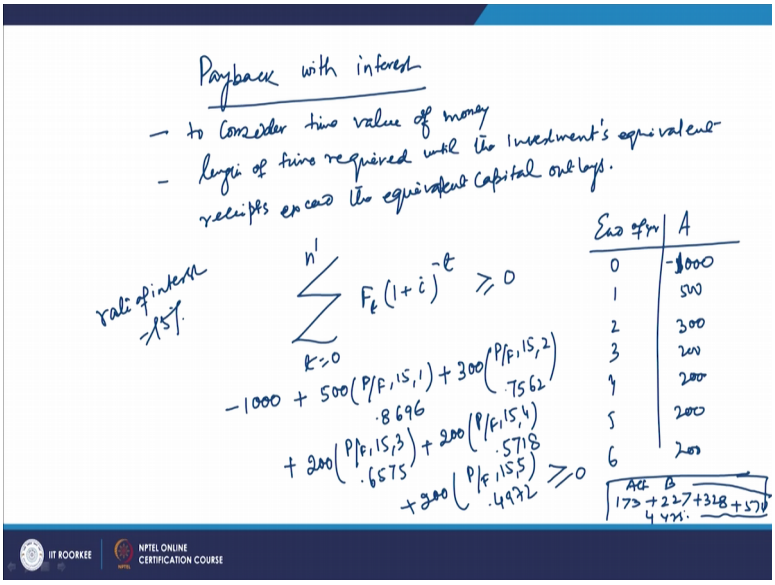

So in that case what you do is, that you take that into account, so pay back with interest, now in this case, you are as we discussed, that you have to take the time value of money into account so, to consider time value of money. Now this is, here also you have, you know it will be same as the length of time, you know so it is length of time required until the investment equivalent investments equivalent receipts, that will be exceeding the equivalent capital outlays.

So the meaning is that, when there is you know there is advancement of time and you are getting different transactions, you are getting different receipts then you have to see that the; what is the equivalent receipt and at and at what amount of time it is basically exceeding the equivalent capital outlay. So that is what you have to see and in that case, what you go for the the condition is that summation t equal to 0 to n Prime and then Ft and 1 + i raise to the power minus t, so that must be greater than 0, so what it tells that, whatever is, you know the transaction, now this so F is already defined. Now this amount must be greater than equal to 0.

So you are going to take all that from 0 time itself. So the zero time, whatever you have invested that will be a negative quantity and later on the f1 f2 or f3 that will be positive, that may be negative, it is negative, you have to take it as negative and the, the sum of all that whenever it exceeds 0, or it is equal to zero that time, we call it as the payback period. So let us say we if you calculate for the same problem which we have done.

So suppose we had, we saw the investment A, where basically we are getting 500 in year 1 300 in year 2 and 200 in year 3 and then further we had 200 in 4, 5 and 6, so what way, you know, your payback period is going to differ so that can be calculated, and then in that case,

what you do is, that you are calculating that value. So if you have the end of year, as we discussed, that end of year, you have 0, 1, 2, 3, 4, 5, 6 and for alternative A, it will be minus of 1000 and then 500, 300 and 200, further for the 4 years.

Now in this case if you see the values, what happens, that if you find the value it will be going like -1000. So this is the present, so it will be Ft into 1 + i - t, so it is zero times, it will be zero. Then you have plus 500 into and if the rate of interest suppose is 15%, so we are taking the rate of interest as 15%, compounded annually so if the 15 the internal rate of return is 15%, in that case it will be you know F by you know so it will be P by F and 15 and 1.

So it will be basically 1.15 raised to the power -1 and this factor is coming out to be 0.8696. So that is what we are getting, 0.8696, then we are going to 300 and this will be multiplied with P by F 15 2 and this factor will be basically 1 + 0.15 raised to the power -2 and that will be 0.7562, you we can refer this,

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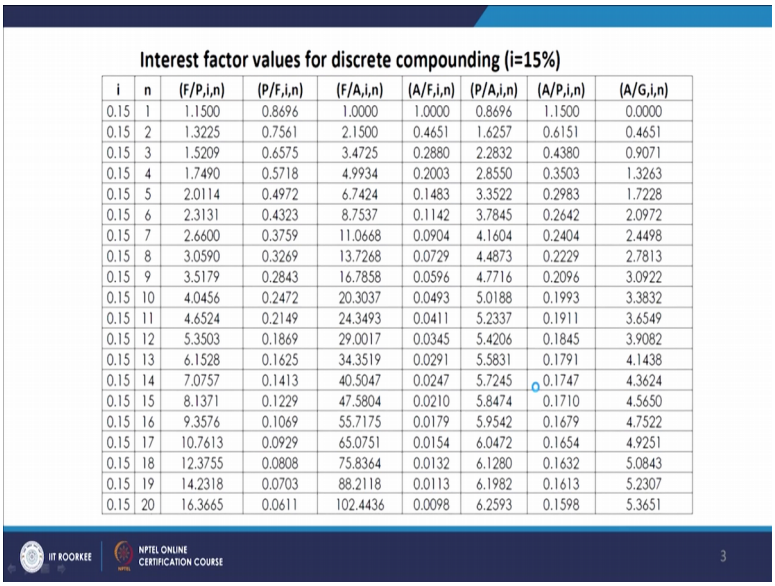

i	n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	(A/P,i,n)	(A/G,i,n)
0.15	1	1.1500	0.8696	1.0000	1.0000	0.8696	1.1500	0.0000
0.15	2	1.3225	0.7561	2.1500	0.4651	1.6257	0.6151	0.4651
0.15	3	1.5209	0.6575	3.4725	0.2880	2.2832	0.4380	0.9071
0.15	4	1.7490	0.5718	4.9934	0.2003	2.8550	0.3503	1.3263
0.15	5	2.0114	0.4972	6.7424	0.1483	3.3522	0.2983	1.7228
0.15	6	2.3131	0.4323	8.7537	0.1142	3.7845	0.2642	2.0972
0.15	7	2.6600	0.3759	11.0668	0.0904	4.1604	0.2404	2.4498
0.15	8	3.0590	0.3269	13.7268	0.0729	4.4873	0.2229	2.7813
0.15	9	3.5179	0.2843	16.7858	0.0596	4.7716	0.2096	3.0922
0.15	10	4.0456	0.2472	20.3037	0.0493	5.0188	0.1993	3.3832
0.15	11	4.6524	0.2149	24.3493	0.0411	5.2337	0.1911	3.6549
0.15	12	5.3503	0.1869	29.0017	0.0345	5.4206	0.1845	3.9082
0.15	13	6.1528	0.1625	34.3519	0.0291	5.5831	0.1791	4.1438
0.15	14	7.0757	0.1413	40.5047	0.0247	5.7245	0.1747	4.3624
0.15	15	8.1371	0.1229	47.5804	0.0210	5.8474	0.1710	4.5650
0.15	16	9.3576	0.1069	55.7175	0.0179	5.9542	0.1679	4.7522
0.15	17	10.7613	0.0929	65.0751	0.0154	6.0472	0.1654	4.9251
0.15	18	12.3755	0.0808	75.8364	0.0132	6.1280	0.1632	5.0843
0.15	19	14.2318	0.0703	88.2118	0.0113	6.1982	0.1613	5.2307
0.15	20	16.3665	0.0611	102.4436	0.0098	6.2593	0.1598	5.3651

From our table that you have 15 percent table and P by F we are looking at 8696 7561 or 2, then 6575 5781, all these factors we are going to use. So 0.7562, then for the next 200 you have again P by F you know and then you have this 15 and 3. So 15 3 will be 0.6575. So further you have 200 P by F 15 4, it will be 0.5718 and then 200 P by F 15 5 and this is 0.4972, these tables, from the table, we can get these values.

Now what we see that if we calculate these values you have to take all that 5 to get it, you know this value, more than or equal to zero. So in the earlier case when we are dealing with without the interest case, in that case, we are only confined up to this time t = 3. However in

this case, now let us say if we try to see the approximate value. So it will be something like 434 and it will be something like maybe 226 or 227.

So, 434 + 227 it will be 6 and you know 61 then further it is around, 130 so it will be 790 only. Then again it is only 115. So, again it is only something close to 905 and then it comes maybe about 80 98 or 99. So that is how it all together, all these five factors, is to be taken into account, then this value basically is becoming more than zero. So it's coming close to 7 rupees which is coming, before that it is not coming near zero.

So it is not going you know more than zero so basically what we see that when we take these, you know interest amount, into picture in that case, you know the payback period will be changing and under this you know case of financial transactions we can say that the payback period is five years. So if before payback period you are stopping, in that case you are not able to completely recover the investment amount.

So that is what the you know meaning of the pay back with interest is and if you calculate for you know alternative B and C both, now in those cases, if you try to calculate it will be for B it will be coming as maybe four years and for C it will be you will never be able to get it because for B in the fourth year you are getting quite a good amount. So, what happens that you have 200 here.

So, 200 will be something 170, so far if you take the case of ,you know B so if you take alternative, you know B, so 2,3 and 5, so two hundred will be here, so it will be 173, then 300 will be here, so it will be something like 227 and 500 will be here, so it will be something like 328 and so what you see that in three years it is coming only like it will be 300 400 plus three 728. So in the next case, you have 1000 rupees here, so then you are getting something like 570 so here itself in the 4 years, you are getting, the 1000 rupees.

That is what the 1000 is coming here, so they are going to cancel. So that is why in the case of B it will be four years whereas in the case of A it was 5 years, in the case of C if you look at in C you see, you slow that it is minus 300. So it will be going something close to minus of 260 and then you have you know minus 300 and 500 and 500. So for C, if you look at C for C it will be looking 3 and minus 300.

So, it will be minus of 260 plus it will be 500, so 500 will be you know 378 and then it will be again 500 so it will be 328. So it is not going to get you know it is not it is less than you know, 1000. So anyway this C is, we are not give able to have the payback situation with

alternative C, and that is how, we can understand you know the situations of pay back with interest.

Now we will see that, how this payback of with interest, can be you know further presented with the help of, you know, so suppose you are paying back certain amount you know you do take one example or you take one case, where basically you have given one investment capital you have invested capital P and you are taking amount A. So every year so you are getting that. Now how that you know amount is going to how you are going to have the payback period calculated.

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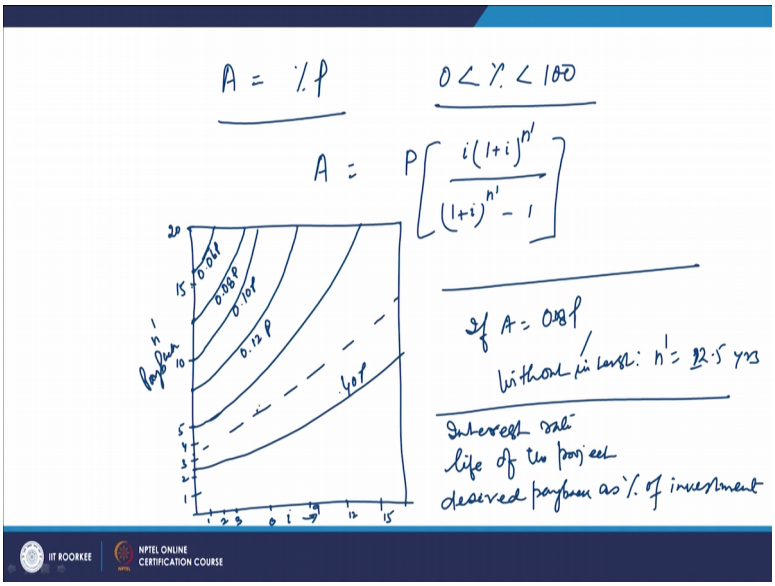

So you can have that period payback period calculated by you know by getting that using that function. So what you can do is that suppose you have A which you are getting; which is some percentage of P, so in that case where zero is so percentage is zero to 100. So what you do is normally, in that case, you will get A, as so we know that A by P, so this will be A by P factor is there. So P is to be multiplied with A by P i, n and A by P i, n, we know that it will be i * 1 + i raised to the power and so n prime it will be the payback period divided by 1 + i raised to the power n prime - 1.

So that way we can have the value of the n prime being calculated for a particular rate of interest and when a interest rate and n prime is changing. So for different interest rate that n prime with changing n prime, how this you know, you can have the calculation of this n prime for different interest rates that can be calculated, and that can be calculated with a particular type of curve, which will tell you that, how this payback will be achieved. So a curve will be generated and that curve will show like this.

So you will have a curve like that and in that curve, you have basically this is your interest rate i, and this is your payback. So that is your n prime. Now what happens that, suppose you are dealing with a case and the A is basically 6.25% or maybe 6% of P, so, and there is no interest case, in that case we know that payback inter you know payback period will be hundred by 6. It will be something like 16.66.

Then when the interest rate comes into picture, then that time will be changing, it will be going on increasing, because the A amount is basically being fixed. So what we do is, that we take this graph so if you take suppose this is 1 2 3 4 5 so you have 5, this is 10 and this is your 15 and this is your 20. So, suppose this way it goes now we know that when n and i value also, will be, it will be, 1 then 2 so it will be 3, you know have it has 6 then you have 9 12 and 15 and so it is going like that.

Now what we see is that when you, you do not take any interest and if A is, 0.06 P, in that case, what you see is, it will be something here, payback period will be here, it will be 16.66, and that basically goes like this. So this way, so this is this line is known as, 0.06 P. So this way you know the payback period will increase if your interest rate basically is increasing, so interest rate becoming to the pay back in time will be going like increasing so that way your payback period will be changing.

Suppose you are taking 0.08 P, so for 0.08 P, you know if A is 0.08 P, in that case your without pay back, without interest, your we know that n prime will be 100 by eight that is your 12.5 years. So it will be 12.5 years and it will becoming something close to here so it will be 12.5 and then it will also move like this. So now so what we see that if you go for so it will be 0.08 P.

Then you have if you take 0.10 P it will start from 10 and it will move like this, then you have, so this will be 0.10 P, then you have 0.12 P, so it will be if it is 0.12 it will be 8.33. It will be something like here, and then it will also move like this, so it will be 0.12 P like that. So basically there is you know for the annual recovery you know, amount there will be, this is, this, this curve may be generated, which can tell us that with different interest rate, how these payback periods are going to vary.

And you see that if you have interest as 20 percent so no interest is 0.2 P. So it will be taking only 5 years. So 0.2 P graph will start from here and then it will go like this so it will be going like that, and this these graphs basically talk about the you know different points,

where, you know how with the changing interest rate how the payback periods are basically changing.

So this graph basically will be useful to find because you have three you know variables, you have variables like you have one is variable is interest rate, then you have variable as the life of the project and then you have desired payback as percentage of investment. Now in this case what from this gird curve, what is there that if two of these you know variables if they are, you know, fixed, in that case the third can be found out from you know, these curves where which is I guess for the i versus n Prime.

And that is how you calculate, this you know no payback period and it will go from till bottom basically it will may go if suppose it is 0.4 t P. So it will be true and half so to start from here and then later on it goes like this so it is going to as 0.4 t P. Similarly if it is you know 30 P then it will be 3.33, so it will start from here and then it will move like this. So it will be 0.3 P you know so that way if you know two of the things, of these three things in that case so you have 1 2 and 3.

So that way you can find the third one and you can say that this will be basically the payback period and you can estimate about it and accordingly you can plan about the investment you know you know investment things. So that is what about the payback, payback period so in this lecture. Thank you very much.