**Financial Mathematics**
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**Lecture – 21**
**Methods of Comparison of Alternatives**

Welcome to the lecture on, Methods of Comparison of Alternatives. So now we will go to this new chapter, where we are going to discuss about, what are the different methods of comparing the different alternatives. In the case of financial world, many a times we are have some different alternatives we have option one or option two and the cash flows or transactions at different times are basically provided and we need to you know come to a decision and for that we must find the, you know, the value of these, you know, transactions at a particular time.

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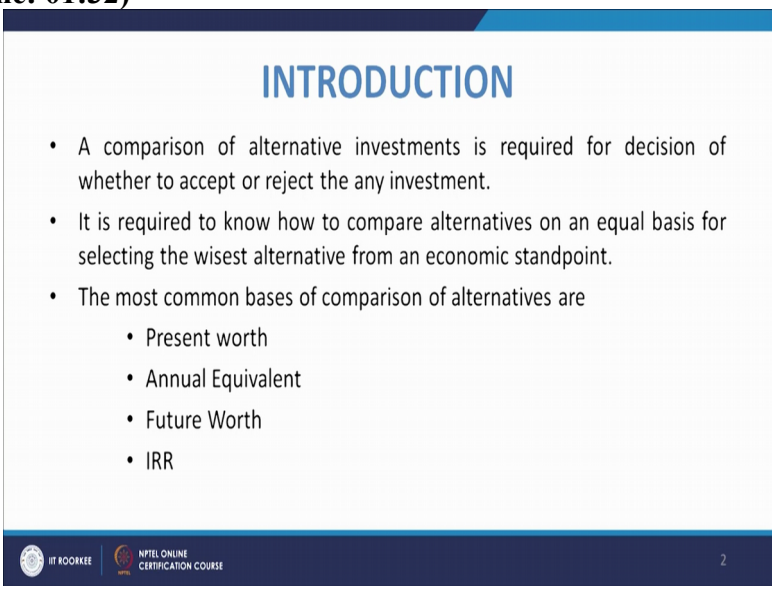


So how to, you know calculate what are those, you know basis of comparison by which you can talk about these alternatives. So we are going to have a discussion on that in this lecture. So a comparison of alternative investments is required for decision of whether to accept or reject any investment, so that is many times the need of the hour. Then it is required to also know how to compare the alternatives on an equal basis for selecting the wisest alternative from an economic standpoint.

And for that there are basically four ways by which you can compare these alternatives and these four ways are basically the Present Worth, Annual Equivalent, Future Worth and the Internal Rate of Return. So these are the four methods of comparison of the alternatives and we will talk about them, one by one.

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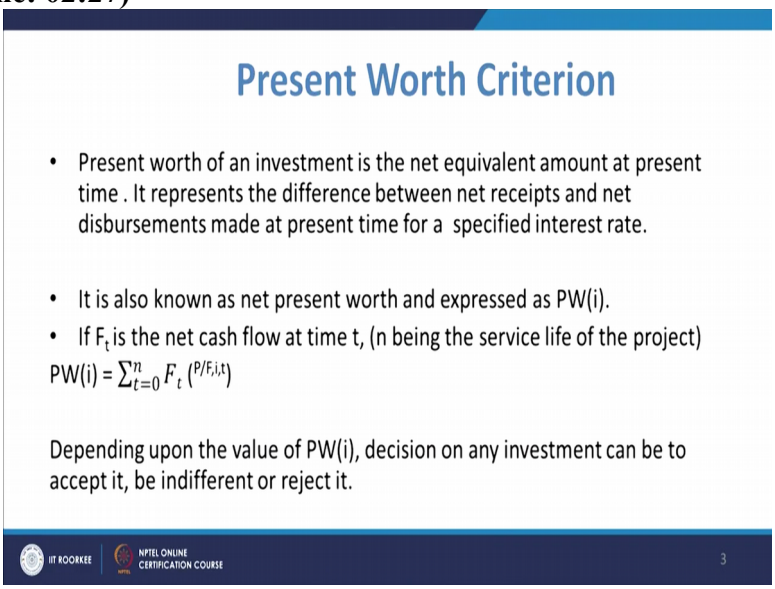


So, Present Worth criterion. Now Present worth of investment is the net equivalent amount at present time. So basically what we see many a times that you have you know investment or the transaction of cash transactions at different times. So you can just get these present worth values of all those transactions and it represents the difference between net receipts and net disbursements made, made at present time for a specified interest rate. So what it tells that in a particular you know cash flow diagram you will be having you know either you know receipts or disbursements.

So we all the receipts together, it will have a net you know receipt at the present time. Similarly all the disbursements taken together you will have a net disbursement and the arithmetic form of these two, because one is positive one is negative, so that will give you the net present worth values. So basically based on that you can compare many a times the situation you know comes like you are going to get something today and maybe in another case you are going to get the other some maybe a five years later.

Now you have to compare these two, you know, alternatives like Bank one may tell you that I will give you fifty thousand rupees, now and Bank you know B will tell Bank two will tell that it will give you fifty thousand after maybe three years, and you know and once you know the rate of interest, you know in that case you can just, or there are many payments in between at different times.

In those cases you can compare by finding the present worth. So all, what is the worth or the value of all these transactions at the present time. When we take that into comparison, then that criterion is known as the Present worth Criterion. It is also known as the net present

worth and expressed as PWi, i is the interest rates, at a particular interest rate, what will be the present worth.

So that can be calculated and if Ft is the net cash flow at time t and n being the service life of the project, and the project, in that case if you look at how can you find this, you know, PWi will be t equal to 0 to and you have Ft into P by F i,t. So all these Fts in f1 f2 or f3 or so all that, all that will be individually multiplied with with this factor Ft by F i,t. So if suppose there is a payment on, at the end of third period, so it will be multiplied with factor P by Fi3, so that will give you the equivalent value at the present time.

Then so one by one individually for all the you know cash flows F1 to FM, you have to individually, you know, change it into the present worth and then altogether you have to add or subtract them and then you have to find the net present worth value and depending upon the value of PWi, decision on any investment can be to accept it be indifferent or reject it.

So when you have two options and you have to come to a conclusion then you find the present worth and ultimately you have to see that in whatever case in which every case PWi is more, you can go for that investment, where when you are to you to get it or if you have to if you have to pay it then that case and you know contrarily you can think of taking that you know investment alternative which has, which is giving you PWi and lesser. So that way you can use these PWi methods for the different cases.

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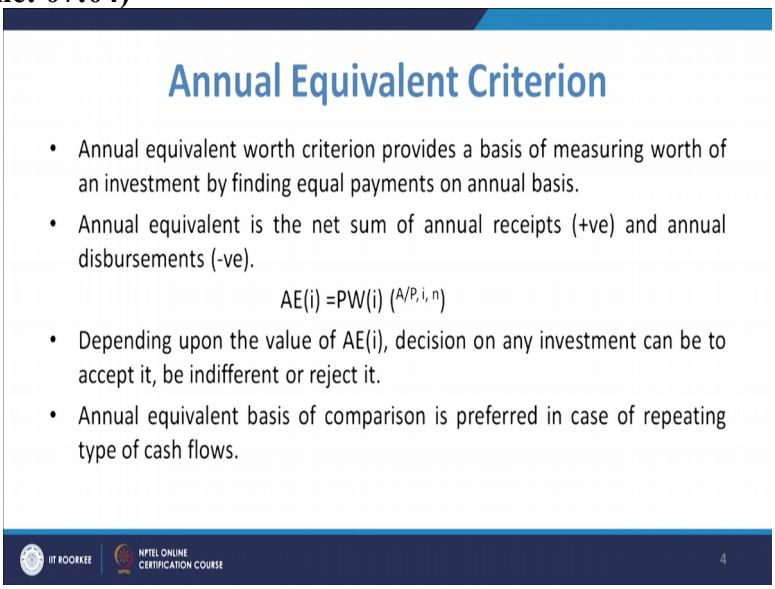


Now the next method which is you know present to us is about the; you know Annual Equivalent Criterion. So similar to the, you know, present worth criterion, you have another criteria that is, annual equivalent criteria. Now annual equivalent criteria you are going to get the equal annual amount for the different you know alternatives, so you have the cash flow

diagram, and for the two you are going to have the annual equivalent value. So it is this annual equivalent worth criterion, so that is providing basis for measuring worth of investment by finding equal payments on annual basis.

So we know that you know by multiplying any payment with respective factors like if you have P you multiply with A by P i, n , so it will give you the equivalent worth payment series for any transaction, you have to multiply with the respective A by P i,n factor and accordingly you will get the equal payment series value and for the different alternatives you can have the value of AEi, so annual equivalent is the net sum of annual receipts and annual disbursements.

So if you have suppose two you know financial alternatives, in that case, AE i1 and AE i2 will be you know calculated, now that can be calculated by many ways. One of the way will be that, you get the present worth as we discussed that in the last point was the calculation of PWi, so once you calculate PWi, then that will be multiplied with the factory by P i, n. So that will give you the annual equivalent value. So or even if you can get the future value and then also, you can multiply with the P by F i, n so that way you can get the; you know annual equivalent value, and that will be basically, the parameter to compare the two types of cash flows.

So depending upon the value of AEi, this is a non investment, can be to accept it be indifferent or rejected, similarly, like if you are you know if you have paid in that case you have to select, see that, you are getting the lesser value or if you are on the game side you can go for the larger value of AEi and annual equivalent basis of comparison is preferred in case of repeating type of cash flows, many a times you prefer this way, it is very easy, to think that the present, you know what type of, you know comparison, the present what value calculation has you know explicit meaning you know that if it is larger you are you will say that yes we should go for it.

But then, many a times for some repeating type of cash flows and then that too it is going on for very large time, in that case, it will involve large and large you know calculation, it will take more computers in time, and also it will you know, in that case, this annual equivalent basis, is having more meaning that because it will tell you where it is repeating. So the same thing will be repeated, you know in future also.

So the annual equivalent value if you take that that circle or that period in which the cash flow is there, which is further going to be repeated in the next periods, in that case, you just

take the annual equivalent value and that will be representing the nature of the investment and you can come to a conclusion for such cases.

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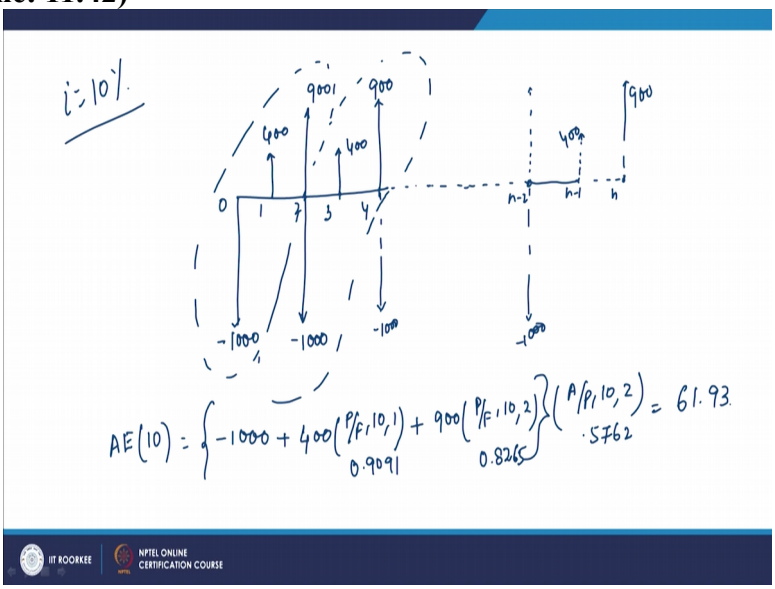


So we can see we can also find that this method is very much useful for the repeating type of cases, like for example, if you see that you have one cash flow diagram where the cash flow diagram goes like this, so at zero time, you have minus of 1000 rupees, and then at one, you have 400, and at two you know you go for 900 then at 2 again, you have disbursement of minus 1000 and at 3 again you have the receipt of 400 and at 4 you have receipt of 900.

So further you have the disbursement of minus 1000 and then this goes on continuing. Now if you look at this if it is a repeating type of cash flow, now for them it is advantageous to look into the aspect that you must calculate the annual equivalent value that is having more meaning. Now what you see is that you see that this cycle is basically getting repeated, so further you know you are going to this cycle.

So, same thing is going to be repeated. So in that case, it is better that you break down this type of cash transactions into such a manner that you get the annual equivalent value and they will be more appropriately representing the, you know, case. So what you will see that and also this will because this cash flow is going on, and what you see that you have you know n minus 2 period, then you have n minus 1 period and then you have n period.

So, you know it is all given that, it is minus 1000 here, and then you have further a 10 plus, so you will have all these values of 900, then you have also this as 400 and then you have here as 900. So this is given, so what you see that this is ultimately, the same thing is repeating so that, such a repeating type of cash flows you know we can solve by looking at

only this. So whatever is the annual equivalent value for such case it will be remaining the same. So basically this will represent the annual amount and that will be same.

What would ever be in that time position, so annual worth basis of comparison annually to valid basis of comparison holds very much good for such kind of, you know, you know, cash flow transactions, which we see many a times. Now if you have to solve for such case, what we see that, if that if there is a, you know, rate of interest i is defined as 10%, now in that case, what we see is,, that we will write we will calculate this annual equivalent at 10%. Now what we see that for this you have, you know, minus 1000 is the, you know, disbursement, so you will take it minus 1000.

Now 4000, this 4000 is to be multiplied with, you know, P by F so use this 4000 also to be you know mal you see its equivalent value is to be found at zero time, so this 400 it will be multiplied with P by F 10,1, then, you know, you have further you have, this 900 value. So this 900 value will be multiplied with P by F 10, 2. So this value, is basically telling you the present worth value, for this repeating type of flow. Now, this whole amount is must so this is giving you the present worth value.

Now for this time domain of two years, you have to find the annual equivalent, what will be the equivalent annual payment, receipt and disbursement, for that you have to multiply with the factor A by P, so because you know P and you have to find A, so you will be multiplying with A by P, 10, 2. So you can calculate this by knowing all these factors from here now P by F 10, 1 will be point 9091, then you have P by F 10, 2 is point 8265, and then A by P 10, 2 is coming out as point 5762.

So, you can refer to the 10% interest table and you can get these values for the different factors and once you see that you will get the value just 61.93. So 61.93 basically is the value of the annual equivalent. Now this 61.93 will be for this, so it will be same as, so it will be like at 0 so then 1 and 2 will be 61.93. So similarly if you go further then 3 and 4 will be also be 61.93, so if this is 61.93 is not going to be changed, so it is going to be defined so

For such kind of repeating flows, this kind of, you know, analysis of the comparison or method of the comparison like using the annual equivalent value comma basis, that has more meaning, as compared to other you know comparison methods like present worth, and even in future, we will discuss about the future what matters and so. So this is about the annual equivalent method of comparison.

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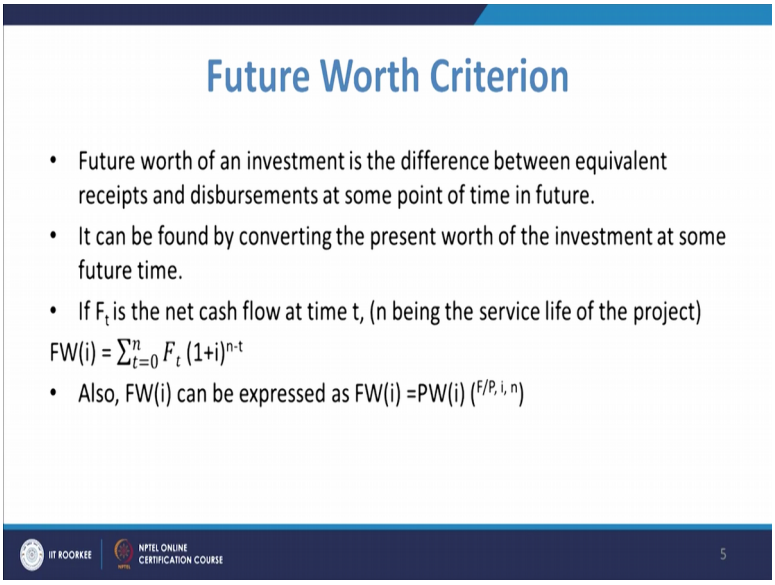


Next is the Future worth Criterion. Now Future worth of an investment is the difference between equivalent receipts and disbursements at some point of time in future. So in this case you are going to have the you know, future worth values, and then you are going to compare how, which will want to be decided, and if Ft is the net cash flow at time t, n being the service life of the project, so we know that we are going to have this formula, future worth i for any particular interest rate will be Ft into 1 plus i raise to the power n minus t.

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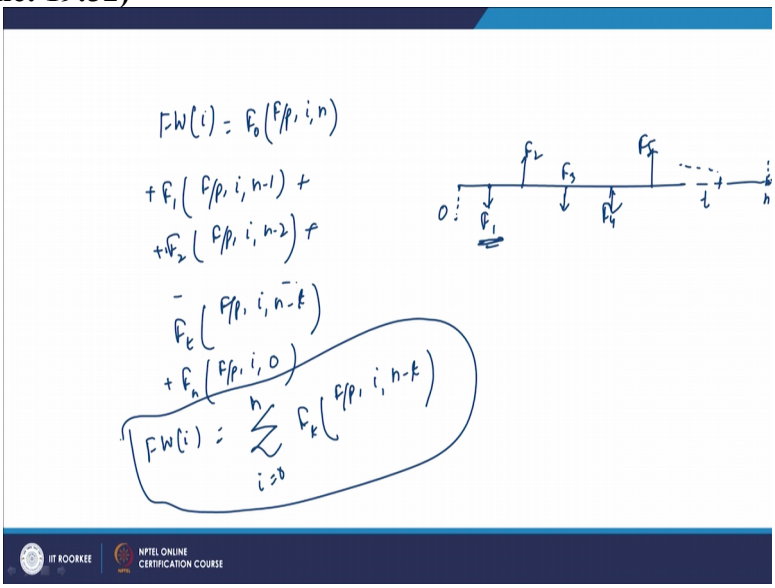


Because as we move in the direction of time, you will have certainly, lesser and lesser time available, for it to earn interest, so that is why, the power becomes n minus t, and this is how you get the future worth i, you can even see that what happens that your future worth will be basically when you have many transactions at all the different, you know, times. So you will have F1, F2, F3, F4, F5, whatever it be, so F1, F2, F3 like you know F4, F5, so this way, you know t and it goes to n.

So suppose if you trying to find the value at future, now this F1 will have more time. So it is for basically it is, its future value is to be calculated, you will be having you know n minus one time of, you know compounding period. So whatever Fn is there, if suppose you have any Fn is there, it has no time which is 0, n minus n is 0, so that is why, what you do is, that you know for you are multiplying with that power 1 plus i to the power n minus t,

So that basically, we are, we have if there is F0 also, so you have F0 and that will be multiplied with by P you know i, n, then if you have F1, so it will be F by P i and minus 1 then if you have F2, then it is F by P i and minus 2. So this way, this factor will go Ft will be there. So, F by P i and minus t so this way it will go and up will go up to FN will be F by P i 0. So that way it goes and so that is why Fwi, when we write, it will be summation of this i equal to 0 to n and it will be Ft into F by P i and minus t, it comes out.

As, so so that way, we are taking the summation and we are getting the future value, future worth and this is known as the Future worth criterion, for finding the, you know, these equivalent future values, and then deciding as to which one is better alternative. Now, what we know so far that we have three methods of comparison, one is, you know, that present worth method of comparison PWi, then you have the annual equivalent method of comparison either, AEi and you have future worth comparison FWi.

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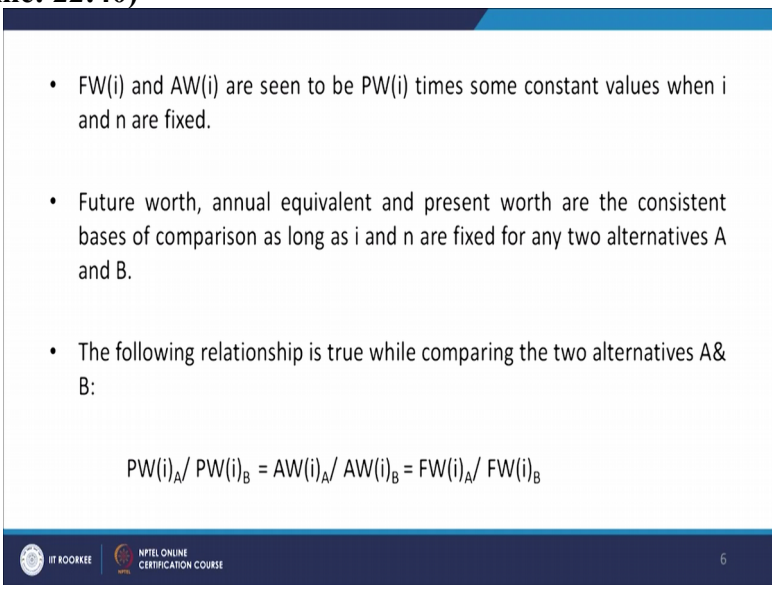


Now what we see that FWi and AWi, you know or you can say AEi annual equivalent, that is, are seen to be, PWi times, some constant values when i and n are fixed, you know, that we have to multiply with certain factor and this factor will be depending upon i and n, so once we fix the i and n values, then basically, they are basically some constant values, you know, multiplied by one or other factor.

So, future worth, annual equivalent and present worth are the consistent basis of comparison as long as i and n are fixed for any two alternatives A and B. And what we see that, like what we get is that when we have to get the future worth, and you know the present worth, then you multiply with the F by P i, n. So similarly if you have to get other things you know AWi, AEi, so you have to multiply with some other factor, like that.

So, what we see that normally this relationship holds quite good. PWi, A upon B will be same as AWi, A upon B or it will be same as if FWi A upon FWi B. So basically all these you know, comparison, they are, their ratio becomes same so they are basically the measures of, all they are the measures of equivalence and the difference only is that, at what point you are basically comparing them. In one case, you are comparing, at present time in another case you are present comparing, at the future time and in one case you are comparing, by taking the annual values together so that's the only difference between them.

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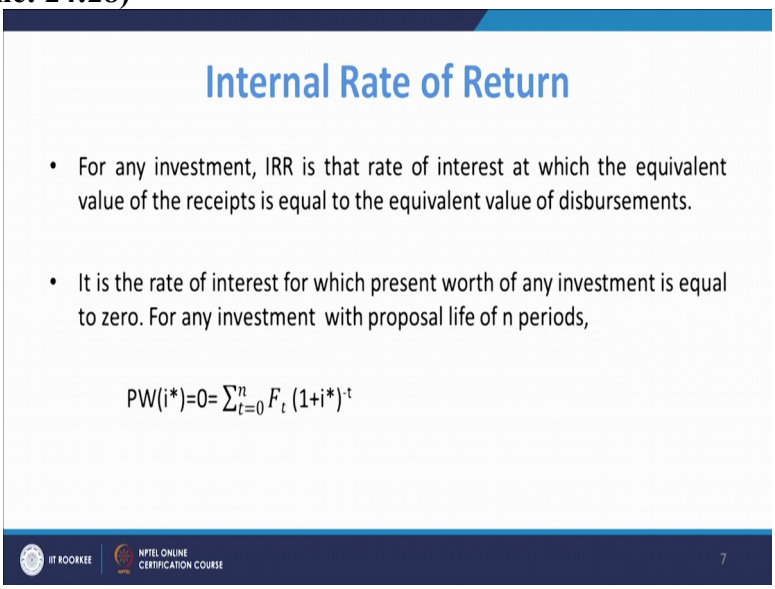


Next what the base of comparison is an Internal Rate of Return. So for any investment the internal rate of return is that rate of interest at which the equivalent value of the receipts is equal to the equivalent value of the disbursements. So that is how your internal rate of return is defined. So we know that you have in any investment, you have the receipts and also you have the disbursements. So this, internal rate of return you have all the receipts and also all the receipt, disbursements,

Now when you are going to calculate the equivalent value of all these receipts be it the present worth or be the annual equivalent or be it the future worth, you know, they are, you know, present worth value of all the receipts and the present worth value of all the

disbursements if you take the net value of that, then it should be 0. So, for that, that for whatever rate of interest, this value becomes 0. That is known as the, you know, internal rate of return.

So it is a rate of interest for which present worth of any investment is equal to zero, that is summation of t equal to 0 to n of Ft 1 + i star and raised to the power minus t, we know that when you have any transactions going on in the future then its present worth will be calculated by using the factor P by F i, n, and P by F i, n is 1 plus i rate to the power minus n.

So, now this 1 + i raised to the power -t, all that will be taken out for individual transactions and its summations must be 0, because Ft may be positive or negative. It will be positive for receipts and it will be negative for the disbursements. So, that how you calculate these internal rate of return so the equation or the value of i, for which this condition is satisfied, that will be only the value of the internal rate of return.

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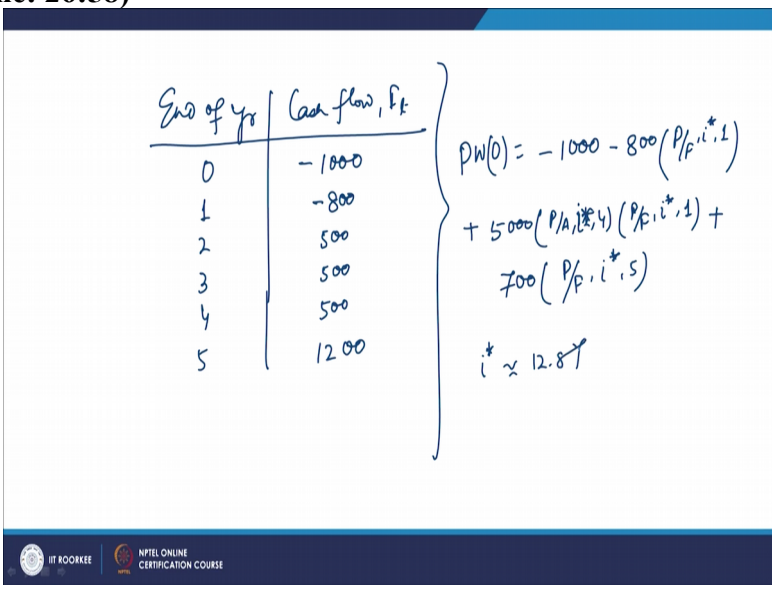


For example, suppose you have been given, you know, cash flow pattern, which talks about this following values, like you have end-of-year and you have cash flow. So that is Ft. Now at the end of year 0 you have minus 1000 rupees and then at the end of you have next five years your cash flow transaction talks about minus 800 in the first year end and then 500 for the next three years and in the end you have 1200 rupees.

Now for such transactions if you have to calculate the what is the rate of return for our internal rate of return, for such investments, then as per definition, you have to calculate the present worth values and of all the receipts and that must be equated to the present worth values of all the disbursements and then or else, the net present worth values must be equated

to zero. So, that time you have to take plus, and for disbursements and minus, plus for the receipts, and minus for the disbursements.

So if you know try to define the internal rate of return so the internal rate of return will be i star, and for that i star, your present worth ultimately will be 0 and for that if you take the present value you know -1000 is anyway, it is the time at 0 time. So you have this is your present worth itself. Then you have for minus 800, so this will be multiplied with you know P by F i1, P by F star 1. So now certainly it will be by 1 + i, 1 + i star is the rate of return.

Then you have you know then you have again you have three 500s, so you have to convert them to the three, you know the present value, so you can go for the 500 and multiplied by you know P by A, and what we do is, you can get this 500 from here and then just seven, you know 100 will be remaining. So you can have, you can have, P by A 12 4. So this 500 will also be taken from here, so that you have your 700 is still, you know, remaining, and P by A 12, 4 will define the value at here.

So you have to further multiply with P by Fi1, so it will be P by F. so this is i star, this is i star and P by Fi star then you have 1. So and then you have 700 remaining here. So this 700 will be multiplied with P by F and this is 5 periods so you have i star and 5. So this is how you will have an equation and the value of i star which will satisfy this, you know, equation, that will be your the internal rate of return and that is how you calculate this rate of return in such cases.

So you can solve and you can by hit and trial and by trial and error maybe we have to use it because it will be all raised to the power minus 4 or so. So by using that you can come to certain interest rate values and it may be used, because you have to go, take some i star values by, you know, by check, by hit and trial and then maybe you can do the interpolation for finding the i star and you can solve such problem and you will see that it will be coming close to 12.8%.

So that is how you get. So for 12.8% basically, this altogether value, quantity will be 0. So these are the three you know, four methods for the comparison of the alternative, they are the basis of comparison of the different alternatives in financial world. Thank you very much.