Financial Mathematics
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Lecture-20
Problem Solving on Mathematical Functions and Statistical Measures

Welcome to the lecture on problem solving on compounding frequency and equivalence, so we will deal with few questions and we will try to see that they how it can be solved. So, in this lecture let us start with the question.

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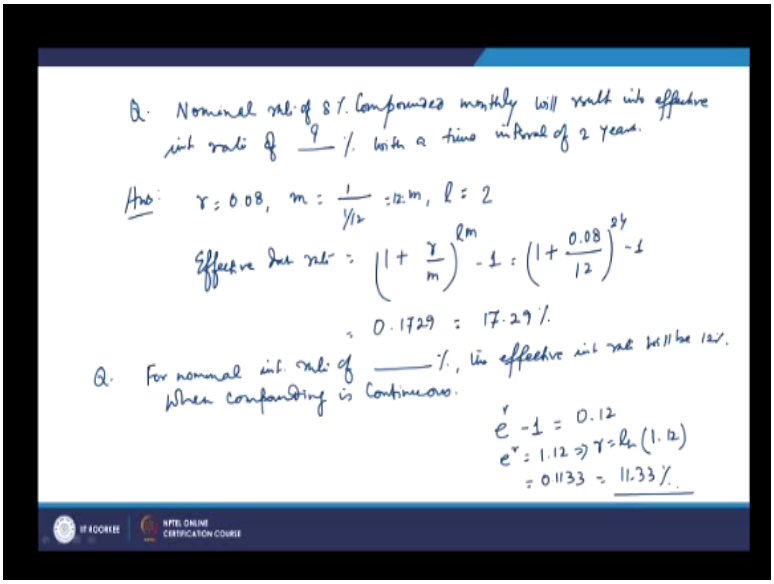

Now let us say the question is like you have nominal rate of 8% compounded monthly will result into effective interest rate of what % with a time interval of 2 years. So, we have discussed about these type of problems and we know that the question is effective interest will be equal to 1+r/m and raise to the power l*m then -1. So, that way we have to use that formula and what are the values given what are you know data given that we have to write first.

So, what is available to us is that we have nominal rate is 8% compounded monthly, so r will be you know 0.08. Then you have you know this is effective rate, so it is compounded monthly and we know that the m is reciprocal of these compounded period in years. So, it will be 1/1/12, so it will be m and L we know that L is the period for which we have to find the effective interest. So,

it is 2 years so L will be 2, so now we can have the you know effective interest rates formula being applied.

So, effective interest rate will be 1+r/m * which is power lm-1, so it will be 1+0.08/m, so m is 12. Then raise to the power l*m l is you know, so m basically it is coming as 12 and then here you have 2*12 is 24-1. So, if you look at it is value if you calculate it, it will be coming as 0.1729 it means it will be 17.29% per 2 years ok. So, for the interval of 2 years if you calculate the effective interest rate than that value comes out to be you know 17.29%.

Then we can have another type of question, now in this case we have to find for suppose for nominal interest rate of what percentage the effective interest rate will be 12% when compounding is continuous. So, basically you need to know what will be the nominal interest rate very effective interest rate is 12% when there is a case of continuous compounding.

And as we know that the effect this continuous compounding formula is e raise to the power r-1. So, what we do is that your e raise to the power r-1 and you know. So, it is given as 12%, so it will be 0.12. So, you have to find this value e raise to the power r will be 1.12. So, you have to have solve this, so we can take the you know ln of on both these sides, so, r will be ln of 1.12 and it is coming as 0.1133.

So, it means you know r is 11.33%, so basically this value will be coming as 11.33% and that will be the answer in such cases. So, that is how we solve the problems related to the you know compounding frequencies and we can have even several type of question that you can practice.

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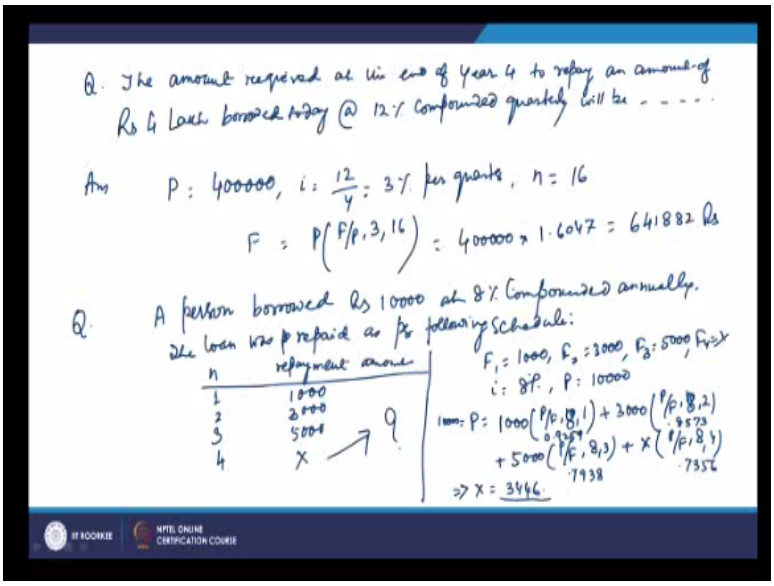

Next we may have a question like many a times you may face such kind of problems like the amount required at the end of year 4 to repay and amount of rupees 4 lakhs. So, that is borrowed today and the interest rate is 12% compounded quarterly will be. So, you have to find this you know amount which is required at the end of year 4 and the amount borrowed today is rupees 4 lakh and you have 12% compounded quarterly.

So, we know that compounded quarterly means it will be percentage effectively the effect interest will be 3% every 3 you know month every you know quarter of the year. So, in fact what we have here is we have been given P as you know 4 lakhs and then the interest rate here we have to take that as you know 12/4 that is 3% per quarter. Now since we are going to have the money you know it is value calculated after year 4.

So, in this case if you take the you know time of compounding as 1 quarter or 3 months. In that case in 4 years you have the compounding 16 times. So, that is why n will be 16, now you are you can use the normal you know compounding factor formulas and you can find in the value of f. So, f will be calculated as p*then f/pi, i is 3 and n is 16, so that is how you are going to calculate and it is nothing but 1+0.03 raise to the power 16.

So, this factor basically will be coming you know you have to you can get it from the table and it will be becoming as 1.6047. So, you have to have this value 4 lakhs *1.6047, so it will be

coming something close to 641882 rupees. So, that is how you calculate this kind of you know problems because this you solve such kind of problems when you are encountering such problems.

And you can even find the you know effective interest rate also in such cases and you can if you take the effective interest rate then suppose you are calculating the effective interest rate for year. So, it will be 1+ you know and then you have to calculate this effective interest rate for 4 years. So, it will be again we have to you know 1+0.12/4 raise to the power 4-1.

So, 1.30 raise to the power 4-1, so it will be something like 12.you know 55%, so that also again multiplied by 4 times. And so that will give you this same thing then you may further have a question of some other type like a person borrowed rupees 10000 at 8% compounded annually. Now the loan was paid according to following schedule repaid, the loan was repaid as per following schedule.

So, you have a schedule given how this loan is being paid and that schedule is that if your this is your n and this is your repayment amount. In that case your in the first year you are paying 1000, second year you are paying 3000 and then you have the third year. So, third year you are giving 5000 and in the fourth year you are giving x. Now this you have to find this value that is your x in this case this x is what.

Now you have to calculate what will be this x, so that this loan is repaid fully this 10000 rupees amount is repaid fully at the interest rate of 8% compounded annually. So, in that case what you have to do is that you can have the you know mapping it is value at a particular time and then you can basically get that equation equated on both the sides and then you get the you know amount x.

So, what you see that you are getting f1 as 1000 and f2 as 3000, f3 as you have 5000 and f4 as x and i is 8% and p is basically 10000. So, if you take and equate the p2 it is sides, so p basically will be equal to, now this 1000 is there, so it has to be you know driven back 1 you know period

towards left . So, it will be multiplied with the p/f factor, so it will be p/f 10, 1 basically it is nothing but 1+i raise to the power -1.

So, then you have a 3000*p/fi2, so this will be 8, this is 8, 1, so 8 2 and then 5000 p/f 8,3+x P/f 8, 4. Now it is nothing but if you take p/f 8,1 it will be 1/ 1.08 similarly if you have this is 1/1.08 raise to the power 2, so 1/1.08 raise to the power 3. And this will be 1/1.08 raise to the power 4, so basically you that will be used, so all these values can be calculated and as we know that this will be coming as 0.9259 then it will be 0.8573.

And this is 0.7938 and this is 0.7350, so if you solve this so certainly all these values are known only this is unknown and p also we know that p is 10000 you know here. So, this 10000 is equated to it and if you solve this you will be getting x as 3446 so something close to that you will get use, you can just solve this equation and you will get solve basically when you are given a table and you have to solve such problems.

You can just solve this equation and get this value of x which is to be paid in any particular year made be third year of fourth year or first year. You have to just find the equivalent of the values on both the sides, next we will have another question.

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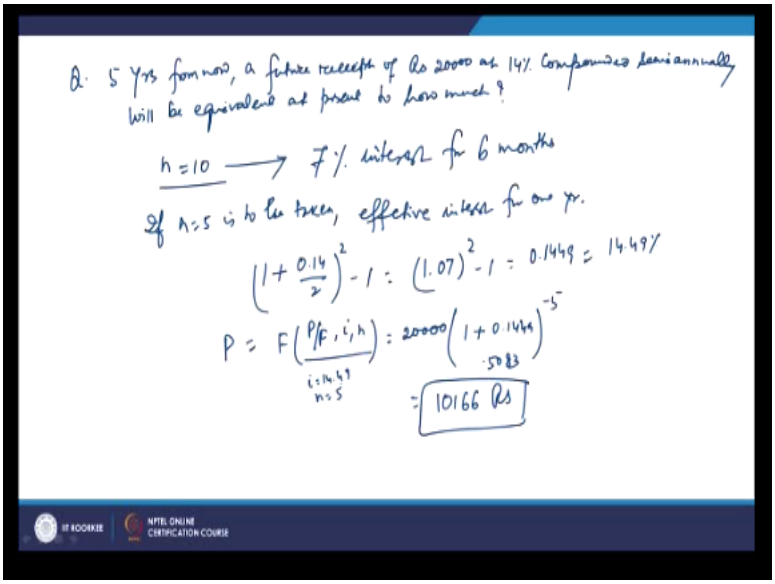

So, we have question like 5 years from now we have a future receipts, so a future receipt of rupees 20,000 at 14% compounded semi annually. So, will be equivalent at present to how much. So, suppose you may face with certain type of questions that you have to have this from 5 years I mean from now you have a special receipt of 20000 and the interest rate is 14% compounded semi annually.

So, again you have to use this concept of the frequency compounding of interest, so 14% compounded 7 only means you are using 7% effective interest for the 6 months. So, and then once you have 7% interest for the 6 month then the n becomes you know 5*2 that is 10. So, you have you can have the values, so you have n will be coming as 10 you know and either you know or you can have you can calculate the effective interest value.

Because once you are using this you know compounding semi annually, so then you have to calculate the effective interest for the year, that will be 1.07 raise to the power 2-1. So, it will be 14.49%, so that way you can use it, you can have the you know effective interest also or so there are 2 methods 1 is n/10 n=10. So, in that case your 7% will be there so 1.07 and you know raise to the power you know so 10 and it will be reciprocal of that.

So, -1, so that will be the you know 1+so that will be divided basically, so that way when you take n=10 you have to use 7% interest for 6 months. So, your interest rate will be i=7 and that can be used or if you use n=5 itself in that case you have to find if n=5 is to be taken then in that case you have to find effective interest for the year for 1 year. Now in that case effective interest for the year will be 1+you know 0.14/2 and raise to the power 2-1.

So, it will be 1.07 raise to the power 2-1, so it will be 0.1449, so it will be 14.49%, so in that case you have to use n=5 or else you can use this when you go for the you know interest rate for the 6 month period, in that case you can take 0.07 as the n. Now once you get so any of the you know method can be adopted and then you have to find the value of the p.

So, p will be basically, so in that case you have to multiply the f with the factor p/f I, n so you can use any of these. So, f is given as 20000 and then you know if you take the effective interest

rate itself, so it will be 1+ you know 0.1449 and then raise to the power -5. So, that is how you get it and it is value will be coming close to like 0.5083, so in that case your value comes out to be 10166 rupees.

So, this can be seem and also you can see from here for i=you know 14.49 and n= you know 5 or you can you could have taken i as 7 and as 10. In that case so you will be getting the you should have similar value of this on that factor value is 0.5083, so that will come out to be 10166, so this way you can solve such kind of a problems.

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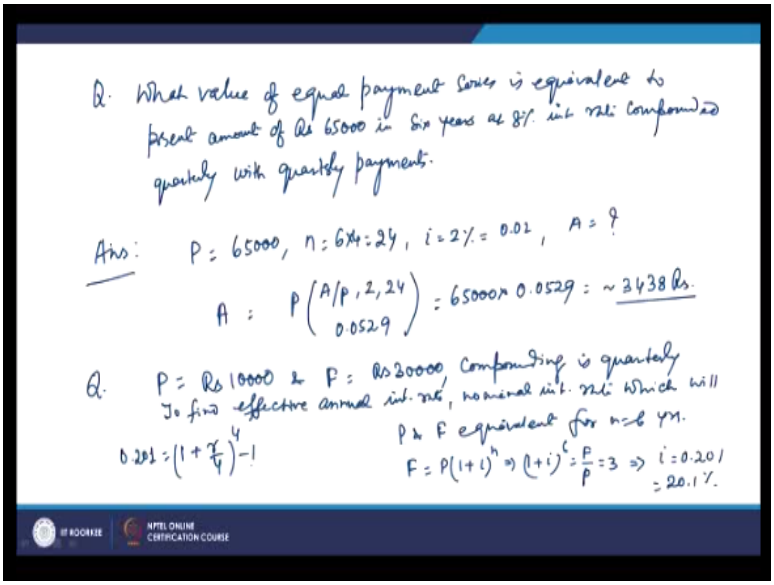

Next type of question will deal with in the sense that what value of equal payment series is equivalent to present amount of rupees 65000 in 6 years at 8% interest rate compounded quarterly with quarterly payments. Now in this question what we see that your interest rate is 8% interest you know compounded quarterly it may is it is 2% interest rate every quarter and also the payment is quarterly.

So, you know you have 6 years, so all together you are going to have 24 payments and for that you have to have the value of a required that is equal payment series. So, the question tells that you will proceed in this fashion that given amount is p as 65000 and you know in this case you have n as 6*4 because it is quarterly payments it will be 24 and i will be certainly 8%, so 8%/4 that is 2% it will be 0.02 and then you have to find a.

So, you know it is obvious that p will be, so you have a will be p*and then you have the factor a/p i, n so i is 2 and n is 24. Now a/p we know the factors you can look at the you know table else you can you know the formula a/p i, n=i*1+i raise to the power n whole the valid by 1+i raise to the power n-1 that is the formula for a/p i, n. So, you can use either this formula or it is better to refer to the interest table.

And this interest table if you look at it is coming as 0.0529, so this 65000 will be multiplied with 0.0529 and this comes out to be you know close to 3438 rupees. So, basically you know if you are depositing 3438 rupees every quarter you know every quarter and the interest rate is 8% compounded quarterly and every quarter basically interest rate is 2%.

In that case that value will be equivalent to the present value of rupees 65000, so if 6years otherwise if you look at since 34 or 3500 close to the *4, so it will be 14000 and *6 years it will be 84000. So, certainly since we are talking about the present time so because of the interest rate it is coming to 65000 of present time. That is all you can solve these problems.

Then we may have another you know example of solving some different problems like you have you know present amount is given as rupees 10000 and future amount is given as rupees 30000. And you know compounding is quarterly, so you know you have to find what will be the effective annual interest rate. So, to find you know you have to find effective annual interest rate.

Then you have to also find the nominal interest rate now you have to find all these interest rates which will make p and m, f, e equivalent for n=6 years. So, now the thing is that we know that when they are to be made equivalent in that case there are 2 things which are further required to be known 1 is interest rate and another is your time, so time is again given, so you can have the value of the effective interest rate.

And also the nominal interest rate but compounding is set to be quite early, so how to solve such problem. Now in this case what we see is that we know that f is basically p*1+i raise to the power m because f is found by multiplying p with the factor f/p, i, n and f/p i, n is 1+i raise to the

power n. so, f will be you know p*1+i raise to the power n. So, this way you know we know that 1+i raise to the power 6 will be f/p and f/p is basically 3.

So, we can get it is value we can take the log you know on both the sides and we can get the value of i and if you solve the i for i, i is coming as you know 0.201 that is 20.1%. Now this 20.1% is basically the effective you know rate which is there, now what will be the r basically. Now for r you have to use again the formula because the compounding is done quarterly.

So, you have to again use that is i=1+r/4 raise to the power 4-1, so if you do that you will be getting the value of the r. So, now let us say you can solve it, so you will have you will be getting the expression for the r as so 0.201 that is your effective interest it will be 1+r/4 and raise to the power 4-1. So, this is basically the effective annual interest, now as we are talking about the quarterly compounding.

So, you have you know m is 1 upon ¼ that is 4 so m is coming as 4 here and then since we are talking about the effective annual interest, so l is 1. So, 4*1 is 4-1, so **so** this way you can have the value of r and r is coming as 18.7%. So, basically this is your you know 18.7% is coming out to be the nominal interest rate in such cases. So, this is how you are you have to use these concepts of interest and equivalence to find the value there maybe we can solve furthermore 1 question like you have.

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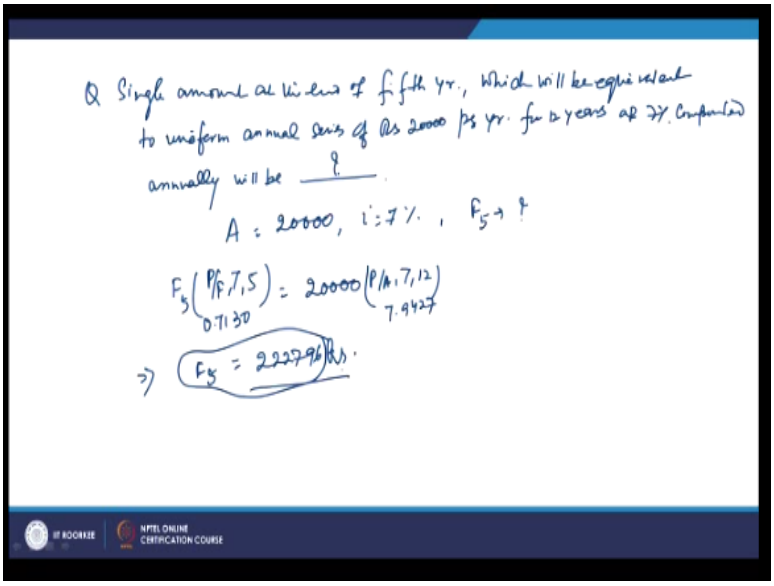

Suppose you have similar amount at the end of 5th year which will be equivalent to uniform annual series of rupees 20000/year for 12 years at 7% compounded annually will be how much. So, you have to calculate this single amount at the end of 5th year which will be equivalent to the uniform annual series of rupees 20000/year for 12 years. So, such problems for that we know that the we are given A is 20000 and i is given as 7%.

And we have to have the value f5 that is your you know equivalent future value at n=5 f5 equal to be known. Now for that you know we will be calculating first the p and then ultimately we are going to use it use the formula for finding this future value at the time n=5. So, f5 and then you have p/f and 7, 5 this will give you the present of this f5, so f5 will be multiplied by p/f 7 and 5, so that is coming as p.

Now again you have to convert this 20000 to the rest through the equivalent p, so for that this 20000 will be multiplied. So, this is a, so you are going to multiply with factor p/a i, n so this will be multiplied with p/a 7 and this is going for b12 years. So, if p/a 7, 12 is coming out to be 7.9427 and this is 0.7130, so if you solve other things are known, so f5 if you calculate it is coming as you know this is 20000.

So, that will be coming as 22279, so this is how you know 222796 so that is how you are going to get it value. So, because you are going to some deposit this 20000 for you know 12 years and

then you are getting it is p and then further you are coming 5 years down the line, so you are getting this value you can solve this and you can get these answers and get your confidence boosted by solving the different type of questions from the books, thank you very much.