Financial Mathematics
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Lecture-02
Important Mathematical Functions and its Characteristics

Welcome to the lecture on important mathematical functions and its characteristics, so in the earlier lecture we had introduction to the financial mathematics. And we had we also had the concept about the numbers. Now we will we try to interact and make you aware and basically refresh your knowledge about the different type of mechanical functions which are typically used for the financial you know mathematics.

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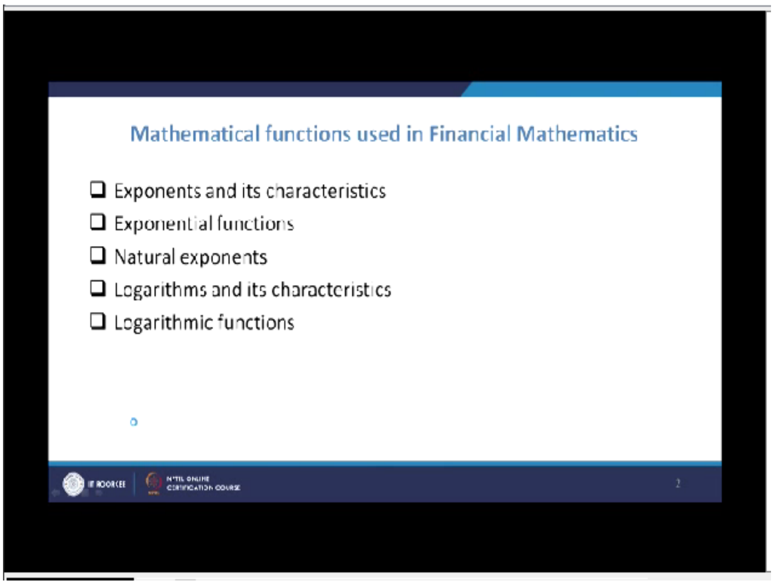

And in that basically what we see that normally the exponents or exponential functions and the logarithms and logarithmic functions. They are mostly used when we talk about the financial you know mathematics, so we should know something about it. And will also discuss in brief about the laws which or its characteristics how they are in used how what are the different laws of exponents what are the different laws of logarithms that we will see here.

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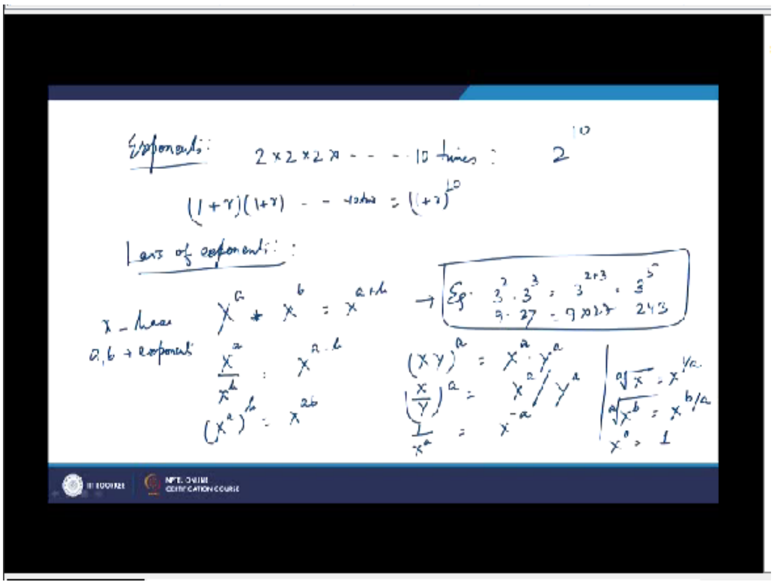

Now when we talk about exponents nothing but so the number multiplied by itself that is exponent. So suppose 2 is multiplied with itself and to 10 times, so we tell that 2x2 is 10 and 10 is the exponents. So, that is how the exponent is defined it can be done for any term like you have 1+r and 1+r is multiplied 10 times so that will be so again it is also 10 times if it is done. So, it will be 1plusr to the power 10.

So, that the this 10 is the exponent that is all the exponent is defined and this is very much used in the case of financial transactions or economics. Because many a times you need to so suppose you are talking about the interest or you are talking about the depreciation or you are talking about the investment cases. In those cases the exponents are many a times coming now there are many laws for the exponents and common laws of exponents .

We must be knowing but you just brush up that what are these laws of exponents. Now in this case when we have the you know if you have a base. So, this base and this is exponent basically similarly you have the base and you have the a and b are the exponent and x is the base. So, x is base and a and b they are known as exponents. Now for that there are many rules like when we define multiply these 2 you know quantity.

In that case this **ex** exponent is added, so that we know this is one of the law of exponent similarly when we are dividing these 2 members Xa/Xb. So, in that case the exponent will be subtracted similarly when you are having Xa and then that is raise to this b exponent. Then we telling it is to be X a*b, so you can have any number like 3 raise to power 2 and 3 raise to the power 3.

So, that will be 3 raise to the power 2+3 that is 5. So, the example is very clear 3 raise to power 2 and 3 raise to power 3, so it will be 3 raise to 2plus3. So, 3 5, so it will be 9 and it will be 27 it will be 243, so same thing it is. So, this way the rules are there and they are used for making this numbers. So, then if you have, so if this same thing can be applied for all this formulas all the laws like 3 raise to the power 3/3 raise to the power 5.

So it will be 3 raise to the power 3-5 that is 3 raise to the power-2, so that way 1/3 square. So, all this things can be you know tested other laws which are coming like when you have 2 different bases. And if you have one exponent then it can be taken as X exponent will be Xa and Y rise to the power a. So, this way there is another rule, so that is defined then from here you have X/Y rise to the power a means Xa/Y raise to the power a.

So, this is another law which is obvious then you have 1/Xa this we defined as X rise to the power –a. This is also a law of exponent then if you are finding the ath root of X. So, if you have aX, so it will be nothing but X raise to the power 1/a. So, that is another law of exponent and you have if you have X raise to the power b. And if you want to find the ath root, in that case it becomes X raise to the power b/a.

So, this is another rule the so these are basically the laws of the exponents and one more this that one you have 0 as exponent when it becomes equal to 1. So, these you know typically these exponent laws are used. So if 0 is the exponent in that case whatever with the base, so with 0 as the exponent it will be equal to 1. So these are the normal laws of the exponent, now coming to the term known as exponential function.

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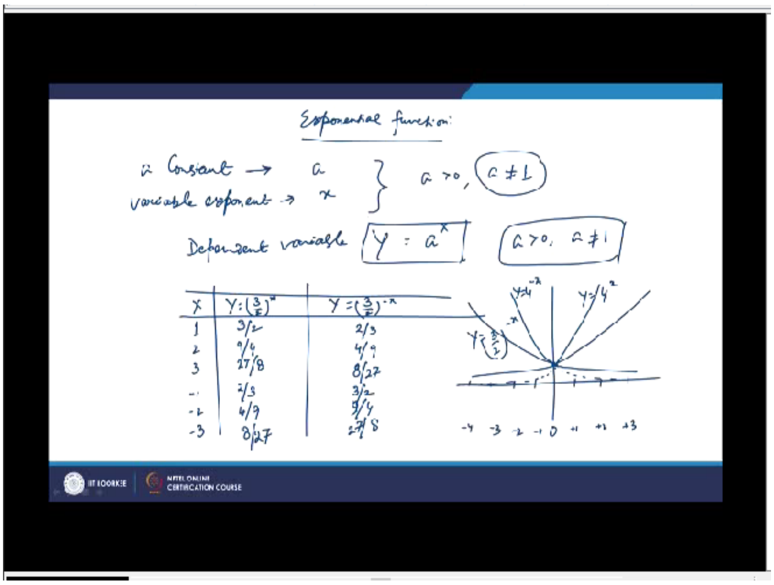

So, you have a function we were this exponential term will be used. So, you have exponential function and here you have a constant a and it has variable exponent X. So, that is you have a constant that can have any number says suppose a and you will have a variable exponent that is x, so now this x. So, a has to be the larger than you know 0 and it is should not be equal to 1.

So, if that is 1 in that case its value will be always 1. So, you know in such a case you have these are the conditions that you have you know a has to be larger than 0 and you know and it should not be equal to 1 and also you have the so basically when we talk about many things in the financial terms like growth rate or so in those case these functions are used.

And in that case you have basically dependent variable, so that basically is expressed. So suppose if you have a dependent variable Y. So, this y they can be expressed, so suppose something is growing at certain rate or so. So, if you have one independent variable x, so that can be expressed like a raise to the power x. So, you have one independent variable and this will be depending upon the x.

So that is why Y becomes the independent variable and in that case you have a has to be more than 0. And then is has not to be equal to 1, now this is for growth rate basically normally we try to you know see that such functions normally you will be coming across and that is how this exponential functions become important in equivalent decisions. Now if you look at the values suppose you take this example of the you know if you have X value as 1, 2 or 3 or -1 or -2 or -3.

Now in that case if suppose you are taking Y as I mean a as 3/2. So Y will be now you can have the value of Y, so this X is basically independent variable. And if suppose Y is 3/2 raise to the power x. Now in that case, so a is basically 3/2 and in that case you will have you know values coming up. So this is 1 it will be 3/2 if it is 2 then it will be 3/2 raise to the power 2, so it will be 9/4, it will be 3 in that case it will be 27/8 if it is -1 then it will be reversed it will become 2/3 similar it will be 4/9 and similarly you will have you know 8/27.

Now what you see that when you have this X value is positive you have growth of the values like 3/2. So, it is value is 1.5 you become to x raise to the power 2, so in that become its 9/4 that is 2.25, so it has increase from 1.5 to 2.25. Similarly you have 27/8 for X=3, so it will be .375. So, this way this is there is growth, so that growth functions basically will be dealing with that similarly you will have the in another case the X value maybe negative.

Now if you that becomes the case, so that will be decay value, so there will be decay as you know X is increased or so in that case if you have Y=3/2 raise to the power-X (()) (11:50). So, if you look at 1 it will be 2/3 similarly you have 4/9 and you have 27/, so it will be 8/27. But if you are this value becomes negative, in that case as you see it will be becoming like 3/2 then you have 9/4 and then 27/8.

So, that is how you know this values will be changing and you can see that this you know exponential functions will behave in the different way. If you try to take the you know its graph what you see is that if you have, so this is your suppose you take the example of you know this Y=3/2 raise to the power X. Now this value if you look at, so when this X will be 0.

So, its value will be 1, so at this 1 now, so if it is 0 then in this side you have -1, -2 or so. So, you will have -1, -2, -3, -4 here you have +1, +2, +3 and +4, so when this X exponent value 0 then you have 1. And now if you have a growth value now if you look at so you are as the X is increasing you will have the increase in line like this. And it will be decreasing like that, so this is a type of curve which you get in the case of the X as the positive value.

And once you go towards the -, so this will be for 3/2X when you go for –X. So, in that case you will have this way and then it will go like this. So, it will be 3/2 raise to the power –X, so this way if you look at and all this exponential you know functions in such cases they will be passing from here. If you have 4 raise to the power X it will go like this it will be go lying like this.

And then it will be coming like this way it will be a moving. So, it will be just going it will be touching like this and this will be 4 raise to the power X and this will be for Y 4 rise to the power –X. So, what we see is that when the X is increasing it this a decay curve, so it is decreasing and when in this case it is increasing. So, that is growth curve and decay curve they are normally you know represented by the use of this exponential functions.

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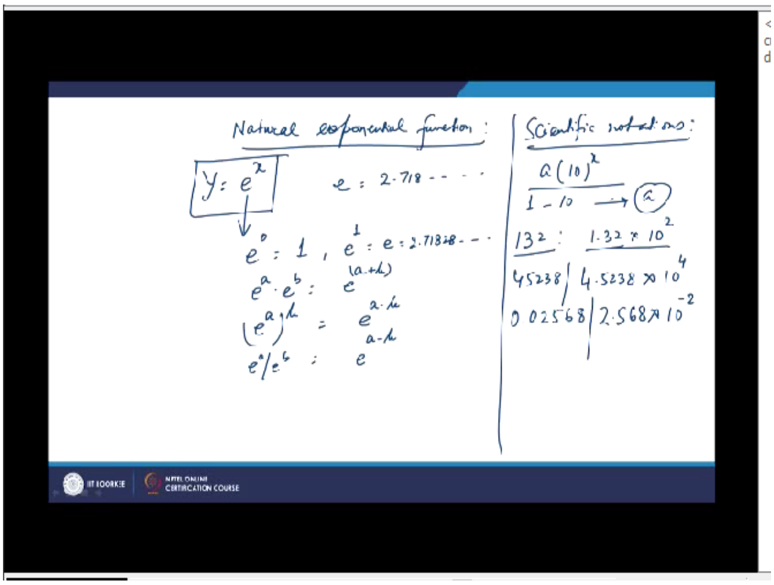

Now there is another function that is natural exponential function which is many a times used, now when you are basically base that value is set to one you know natural base. Then it is known as the natural exponential function and in that case they the base, so it will be e raise to the power x. So, this is basically the natural base and its value is basically fixed and all we know that its value has 2.718 like this.

So, we know that it is the e value is defined as limit of you know extending to infinity and 1+1/x raise to the power X. So, that way you define this e value of e and when you have you know this, so that is known as the natural exponential function and for that natural exponent also there are many laws. And basically you have a constant rate of growth this so this function will be describing the constant you know continuous growth at some constant rate.

And it will be used in many cases like growth of population then you have also negative rate of capital growth all these things. I have been seen that they are going in following this exponential function natural exponential function and they are used. So, they are also used in the case of depreciation of assets or so. Now for the natural exponents also there are many laws and this laws are like if you.

In this case also if you have the exponent its exponent as 0 then it becomes as 1 and if you take its 1 exponent then it becomes as e, so it will be as we know that, so 1828 and it will go like this. Then same law of exponent will apply here, so e raise to the power a and e raise to the power b it will be e a+b. So, this is exponential will be you know summed up similarly ea and raise to the power b.

So, ea*b ea/eb, so it will be ea-b, so these are the you know typical you know laws of the exponents which are used in the case of this natural exponential functions or in fact the exponential functions. Now we will deal with certain other you know mathematical terms and one of the term is basically the scientific notations. So, when we talk about the scientific notations.

So, as we know that when we talk about the notation of any number what we see is normally in a most convenient form. We write it as a 1o raise to the power x, so the value a is taken something from you know 1 to 10. And, so it will be varying from 1 to 10, so this value of a will be varying from 1 to 10. And then you are trying to you know express that number for 10 and this will be the exponent.

So that way we try to suppose you have 132 now this can be represented as you have 1.32. And then since we are taking the decimal towards the left, so you will have the exponent 2 to be used 10 raise to the power 2. So, that way we use these exponents, so depending upon how you are moving the decimal one which side you are moving the decimal if you are moving decimal towards left.

Then you will have the use of positive exponent and if you have the movement towards the right. In that case you will be using the exponent as negative one, so suppose you have 45238. Similarly, so now you are having the decimal point here, so this decimal will be now this number is 45238.0 basically. Now decimal is move to 1234 point, so it will be 4.5238 and then it will be since you are moving to 4 you know places towards the left.

So, you are going to this exponent as 4 as the positive value, so that way we represent now on the other hand if you have number like 0.02568. Now in this case to represent in scientifically to in the scientific notation terms the decimal this a for when we try to define a. It has to be between 1 and 10, so you have to move the decimal towards the right, so you are moving the right to 2 places.

So, the then it becomes 2.5, so in that case, so you will have 2.568 and since you are moving towards the right this decimal place to 2 you know places towards the right. So, you will have the rise to the power -2, so this way what we do is that we normally represent these you know decimal you know this scientific notations by moving the decimal points towards the left or towards the right.

Now next as we discuss, so we are discussed about the length about the exponents, now we will discuss about the logarithmic functions and concept of log.

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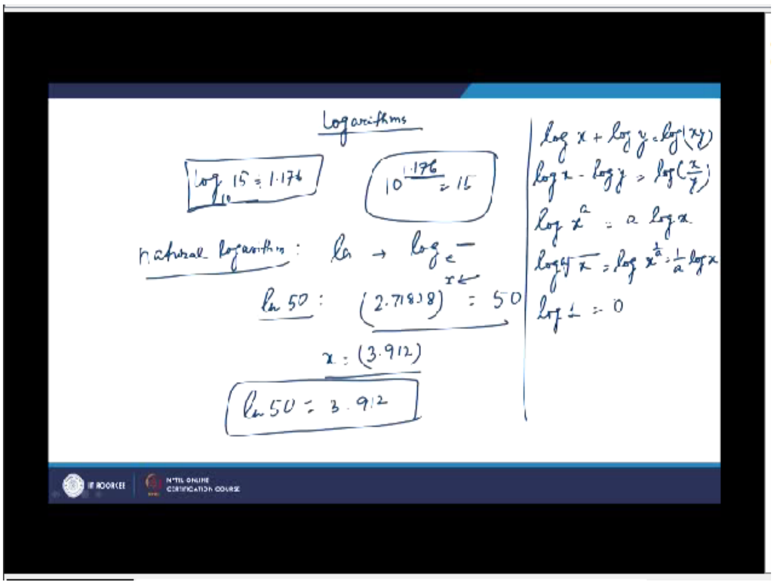

Now, so you know we must have the idea of these logarithm now when we are discussing this logarithms. So, as you know that many a times when we get a number by raising the exponent to something suppose you are getting 8 by raising to 2 2 raise to the power 3 that is 8. So, basically if you try to have the log of 8 and base is 2, so that will be equal to 3. So how many times you have multiplied the same number.

So, to get certain number, s o if you are taking the log of that number with this base that the exponent which are used that will be the logarithm value. So, that is how you so basically it is nothing but the special form of the exponent itself. So, normal practice is to have either 10 as a base or the natural log taking e as the base and suppose you want to have the log of 100 as you know the 10 has to be put in.

So, 10 raise to the power suppose exponent if it is 2 then only it is 100. So, if you take log of 100 and base is 10 in that case the value is 2. So, that is how the log is used and the value this exponent value it not be always whole number. So it may be something like a fraction or a decimal points, so that value the log value will be also a decimal value. So if you take the log of suppose 15, so what you how can you find it.

Now if you see that if you take the 10 if you have the exponent as 1.176 then it becomes 15. So, since this exponent is 1.176 and that relates, so if the base is 10 in that case it will be 1.176. So, this is how this is the concept of the you know log, so it is basically special kind of exponent formula only being utilized to have the you know value of this logarithms. Then you have also a natural logarithm, so in that base is you know e.

So, in that case we will be call it as ln, so in this case you have the base as e and also called it has log e. So, you will have any number now what we see is that this number you know that 2.718 28 and all that. Now that number has to be raised to any you know, so that you get a particular number suppose you want to have the value of ln15 it mains the 2.718 28. Now this is to be some exponent should there such that this be becomes equal to 50.

So, this x is the value of this ln50, so if you try to see you will see that it will be something like 3.912. So, this x will be this, so you can write ln50 is 3.912, so this is how the concept of log is normally used and in financial you know statistics many a times you need to know that what should be the exponent by which you know it has to be the exponent value it will be used.

So, that it becomes equal to the desired value which you want, in those cases this log will be used. Now the contrary to this the positive will be nTlog, so nTlog if you have similar to log you have the use of anti-log also and other are rules of logarithm also. So, similar to what we have seen the laws for the exponents and in the log also if you have logX+logy it will be logxy.

Similarly if you have logx-logy it will be logxy logx/y, then if you have log x raise to the power a, in that case this a comes further here, so it will be a*logx. So, this is the law of the log similarly if you have log and you know ath root of x. In that case x raise to the power1/a, so the 1/a will come to this side. So, you can write this log x rise to the power 1/a, so it will be 1/alogx then also if you find log1 it will be always 0.

Because any numbers take 10 as a base or e as a base is power is a exponent 0 when it is taken it is equal to 1. So, log of 1 will be always 0, now you have also the logarithmic functions and this logarithmic functions are also very much used in the case of these financial you know analysis and if you try to find the function of the logarithm the how it looks like.

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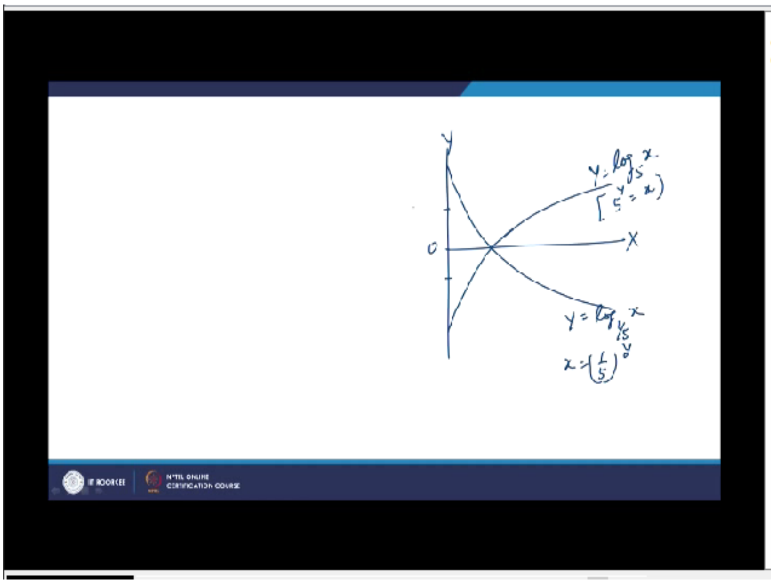

So, if you try to draw then a function something like you know this function is there or this function is there. So, this will be a logarithmic function, so you have this x and y and this is the y= log5x or you so basically what is there? How you define this one?. Basically 5 rise to the power y is nothing but it is x basically from here the function is 5 rise to the power y is basically x.

Similarly you know log this function will be y=log 1/5x, so basically here x will be 1/5 raise to the power y. So, this way this logarithmic functions you can see that it will be used at when you place as you will have -1, -2 this is like – and this is you have plus. So, these are the you know different type of you know logarithmic functions which are used and this a kind of characteristics which are you know very much clear that they will be used in many cases as we study this course.

I hope you have understood this you can have the practice of this you know mathematical functions, so that we can discuss more about the mathematical terms in our next lecture thank you very much.