Financial Mathematics
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Lecture-19
Principles of Equivalence

Welcome to the lecture on principles of equivalence, so we in the last lecture discussed about the equivalence concept and we here seen that how what are the factor is to be calculated. The 3 elements which are required you know for the equivalence calculation is the amount of sum then the time of occurrence of this sum and the interest rate. So, basically when you have a cash flow also we saw that in the cash flow type of situations.

You have to you know find the equivalent value at a different time and for that you need to have the value of these factors you know the compounding factors which we have seen many type of factors are there. So, these factors are to be used and then accordingly you have to find the equivalent value at a particular time. So, if you are finding the values are the particular time you can add or subtract these numerical values at that particular time.

The transactions which are occurring at different times cannot be directly added or subtracted, that is what is there in the case of equivalence. Now when we talk about the 2 type of cash flows and we are to a certain whether the 2 different cash flows are equivalent or not. Again you have to put the equivalent principle like you know that if you have a cash flow which talks about the present value of something like 300 rupees.

And then we have a cash flow which talks about the future value you know of maybe 798 rupees. Now whether they are equivalent or not and if the time period is given as 7 years, so for a particular you know rate of interest they will be equivalent, so that you know if the this present value of 300 at that particular rate of interest gives you the future value of 798 maybe at the end of 7 years then they are said to be the equivalent cash flows.

Like that what we have understood by looking at the cash flows, now we will talk about other principles of finding the equivalence and now one of that you know principle is that you know.

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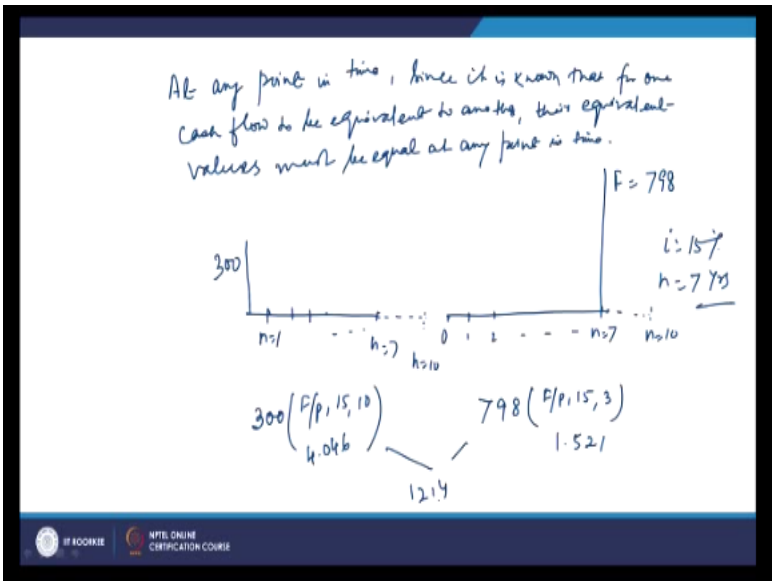

At any point of time point in time since it is known that for 1 cash flow to be equivalent to another they are equivalent values must be equal at any point in time. So, basically this is one of the equivalence principle that if your one of the cash flow is set to be equivalent to another. In that case at any point of time if you find the equivalent values they have to be equal and for that basically we had seen that case of the 2 cash flows.

Suppose one of the cash flow as we discussed that it is giving you the present value of 300 for 7 years of time. And in another case you have the future value and this future value is coming as 798 rupees and it is again giving you the time of n=7. Now if they are equivalent then if you take at any point at any point of time if you find the values they must be equal.

Now the rate of interest is 15% and time is you know time is 7 years. So, this principle tells that if you try to find the equivalent value at any point using for these 2 you know cash flows. They must be equal at any point of you know time, so if you suppose extend this 2 any particular time of suppose say n=10. In that case you know if you take go to this n=10 in that case and this also is suppose taken at n=10.

Now for this 300 has to be multiplied with p/f, so this will be f/p and 15 and 10 and similarly this 798 has to be multiplied with it is only going to 3 time you know periods. So, it will be again f/p

15, 3, now it will be basically 4.046 and this will be 1.521. So, both the you know both these result into the value of 1214, so something like that you are going to get if the 2 of the cash flows are equivalent.

At any point of time if you have find the equivalent they are suppose to be equal. Now so we can say that 2 or more cash flows the if they are equivalent if they are you know and if you have 2 cash flows and if one of the cash flow you know is equivalent to one of the cash this cash flow. Then certainly that cash flow will also be equivalent to this particular cash flow.

So, the 2 are cash flow, cash flows is being equivalent and one of them is equal to another cash flow then they also will be equal to the other one that is also again you know are no need to be discussing further about it. Because that is very clear that if you know 1=2 and 2 and 3 is equal so 1 will be same as 3. Now very you know interesting situation occurs in the case of these financial transactions.

That many a times during the you know different time intervals you are faced with different interest rates. So, maybe that if you have a time of 10 years maybe for this year 2 years you may have a case of 10% compounded annually. Then initial then second to fourth year it may go to 9% then from 4th to 6th it may go to 8% like that. So, you have many a times during these you know time periods you have different rates of interest.

And that needs to be taken into account while finding these equivalence cases, so in those cases you will have to take into account the time during which a particular interest rate is applied. Now that cannot be you know taken as this same as the next you know time also together because they have different you know interest rates in those times. So, you cannot take all together full whole the I mean.

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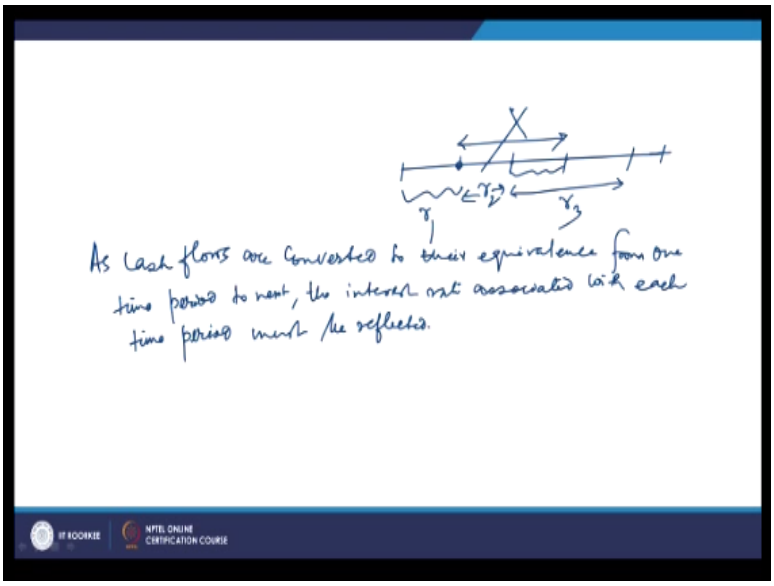

Suppose you have 4 or 5 zones and you have some rate r1 here and some rate r2 and some rate r3 here. So, basically you cannot combine these 2 this cannot be practiced because in this you know time domain half is you know with the rate of interest r2 and it is and here is r3. So, you cannot combine them you cannot take equivalent value something which is defined here you cannot directly jump into this point.

You will have to come to this point and then further you have to go to this point, so this principle of the differential you know interest rates different interest rates in different time that must be taken into account. And for that the tax is that as cash flows are converted to their equivalence from 1 time period to next the interest rate associated with each time period must be reflected. So, while calculating this equivalent value you must take into account the different you know interest rates which are there during the you know different times.

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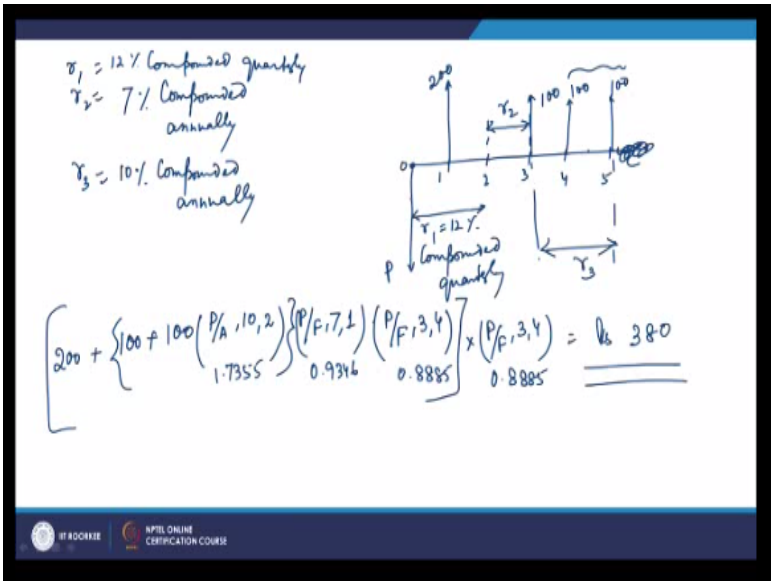

Now we can have 1 example of the you know such kind of case. Now we can take an example of a cash flow where we see that you have suppose you have to find the p value and you have different times 1, 2, 3, 4 and 5, 5 here now this is not the case. So, here we are of taken considering this case, now in this case what we see is that from 0 to 2 you have a rate that is defined in of interest r1 and this is defined as 12% compounded quarterly.

And simply then you have between 2 and 3 you have r2 defined and r2 is basically you have 7% compounded annually. Then between 3 to 5 you have defined this r3 and this r3 is 10% compounded annually. Now for such cases if you have to find the equivalent u present amount now what we see that from 0 to 2 you have 12% that is compounded quarterly, so that rate is different.

Now r2 is 7% compounded annually, so r1 12% compounded quarterly, now what you have to do is you have to find see the present amount or the future amount. Now the transaction which are taking place is at 1 you have receipt of 200 then at 3 you have receipt of 100 and 3, 4 and 5 you know you have receipt of 100. So, if you try to see that what will be the value of p in such cases.

Now in this case as you know that from 3 to 5 you have the same rate of interest being applied. So, from 3 to 5 you can have you know the value of you know the equivalent value can be calculated but then you cannot apply. So, as you know that this is equal amount which is applied,

so your annuity concept is utilized. Now you cannot use the p/f formula because once you define p/a formula that will be defined at 1 time ahead.

So, it will be defined at time 2 and this cannot be possible because you have different interest rate which is occurring here same interest rate is occurring here. So, basically you have to use f/a and i and 3. So, you have 3 transactions, so 100 will be multiplied with f/a and then rate of interest is also defined and rate of interest r3 is 10, 4, so if you look at this will be you know 100*f/a i3.

And that will be defined at this point you know at 5th time period and then it has to be mapped you know now it will be coming to here then further you have to convert here then it will coming further to 1 and then it will go you know you can directly come from there. Because you have a 1 typically you know interest rate in between, so what we do now.

Now the thing is that you can have the values like this that you have f 100*f/a you know 10 2 now this another way what we can understand is if you take this 200 transactions and use p/a formula it will be defined here. So, you cannot take these 300 transactions and define use p/a formula because that will go to this time step and you have different interest rate.

So, you can use 100 and you can use these 2 to define you know present value at this point. So, you will be using p/a and the interest rate r3 is 10 and time is 2, so this will be telling you the present value at this point. Now this point you have already 100 rupees, so this is 100 is added here, so this will be defined at the point 3. Now this point 3 now from here you have to come to point 2.

And for point 2 it is you know compounded this is 7% compounded annually. So, this is basically the future value and you have to go to the present value, so you have use the p/f factor, so this will be multiplied with p/f and that will p/f and interest rate is 7 and this is for 1 year. So, this will be multiplied with this, so this amount now you are coming to at this point.

Now you have to further bring this amount which is defined at 2 to 1 and this you know interest rate is different that is 12% compounded quarterly. It means this is 1 year, so there will be 4 times 3% of interest is added. So, you have to use the interest rate of 3% and time period will be 4, so you will be multiplying with you know p/f interest rate is 3 and time period is 4.

So, this is what you are now you are coming to this point now further you are you know bringing it to this point. So, now you are p/a 3, 4 you are coming to this point, now here you will add 200 because now you are defined you are coming to this point already you have 200 rupees of transaction. So, this is how you are now getting the equivalent amount at this point.

Now for finding the present amount this is to be further you know you have to find a equivalent amount at 0 time. So, you will again multiply them with p/f 3, 4, so this is how you are going to take into account the different interest rates which are you know effective in the different time domain. So, as we see that you may have the different way to solve it you could have calculated have using f/a formula.

And then you know coming to but again you have to come to this point, so that is why we got using these 2 transactions you got p here by adding 100 also +to it and then further finding here**.** And then finally coming to this different interest rates and if you are finding these value p/a 10 to you have to look into the different tables that of the interest and this values will be coming as 1.7355.

Then you will have may having the value and 0.9346 then you have 0.8885 and finally you have this also as 0.8885. So, ultimately if you do all this calculations you are going to get rupees 380, so basically this is how you are going to get these values of the present value for such you know cash flows where in the different times you have different you know interest rates being applied.

Now if you have to calculate the future value for suppose this transaction then your present value is known this. So, you can go to first of all you will go to this time, so that will be multiplied you know with p/ you know f/p factor with you know time will be 8 because it is compounded quarterly. So, I will be 3 and n will be 8 then you go to here further using again f/p formula and

then further here. So, that way you can have the future values also calculated, so this is one of the rule of finding the equivalent values. Next we have further inline the another equivalence principle.

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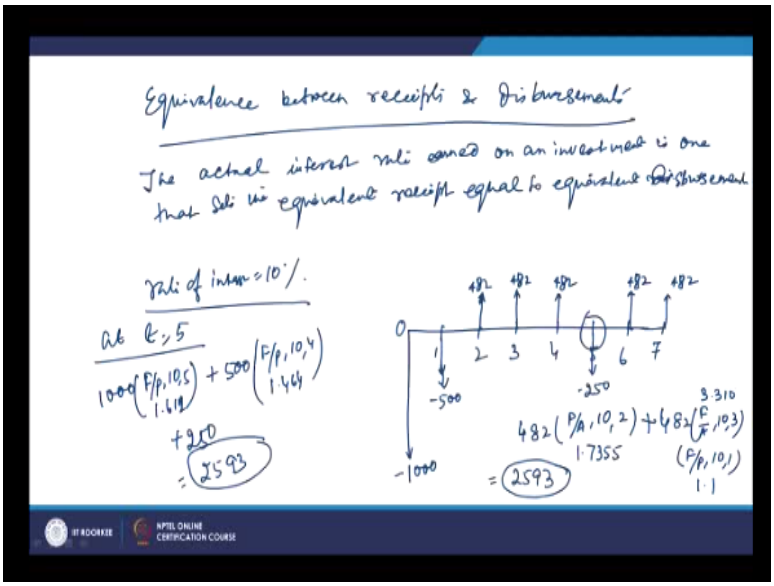

And that is equivalence between receipts and disbursements, now what happens that when you are dealing with you know a cash flow then you have receipts and disbursements you know going on a different times. At sometime you have receipts and at another time you have disbursement, now the you know the equivalent receipt and equivalent disbursement they must be you know equal that is what the principle tells.

So, it tells that the actual interest rate earned on an investment is one that sets the equivalent receipts=equivalent disbursement. So, the principle is that if you have you know you have some cash flow and you know the actual interest rate earned, now that will be the same rate of interest that will be the that particular interest rate earned. Because of which whatever you know you have the receipts at different times.

So, receipts we normally represent in terms of positive quantities and disbursements are represented by the negative quantities. So, whatever receipts you have it is equivalent value will be same as the equivalent value of all the disbursements. So disbursement we take as a negative

but then there are absolute values will be equal. So, that if that is equal then only we can say that is actual interest rate which is specified or the you know.

We call it as the (()) (22:51) rate of return or minimum attractive rate of return all that there are terms or rate of return or so. All these term indicates about the rate of interest which is applied and you know for that you know the for the actual you know interest rate earned the equivalent of all the receipt will be equal to the equivalent of all the disbursements. For example if you take 1 example suppose you have a cash flow diagram which talks about you know different disbursements which occur.

And receipts also you have you know all together 7 time periods 4, 5, 6 and 7. Now in that at some point of time you have receipts going on. So, at 2, 3 and 4 the there is receipt and receipt of 142 rupees each is you know going on and then you have h6 and 7 also there is receipt of 142 going on and at the 5th time you have disbursement of 250 you know going on this is basically.

Because this is disbursement, so you are taking it as a negative value and here at first time you have disbursement of you know -500 and then you here -1000. Now suppose for any particular cash flow such is the transaction you have you know the disbursement of 1000 at 0 time 500 at first you know at the end of first time period and 250 at the end of 5^{th} time period and receipts are at 2nd, 3rd, 4th and 6th and 7th time period.

So, if you have to say that how to see that whether you know you have actual rate of interest tells that this is this cash flow diagram is the actual representation of you know what is happening in actual case. So, that can be checked by setting these values of the receipts and disbursements, equivalent values of the receipts and disbursements.

Now in this case your rate of interest is shown to be 10% so now you can check now what we do is we are going to calculate you know the equivalent value of all these you know receipts 142 which is there at 4 points. And then we are also going to get the equivalent value of all these disbursements, so the purpose is we can have the values you know equivalent values at any time.

And the equivalent of all the receipts should be equal to equivalent of all the receipts at any particular time. Now let us say that we try to find the equivalent value of all the receipts and equivalent value of all the disbursements at suppose t=5. Now at t=5 at this point we try to find the net equivalent value of the receipt and disbursement. Now for this 1000 rupees it is value will be you know future value at you know 5 you know time period away.

So, it will be multiplied with f/p 10, 5, so this will be 1000*f/p 10, 5 10% is the rate of interest. Then similarly you will have 500, so 500 again multiplied by so this is 4 you know you have 1, 2, 3, 4 interest periods. So, it will be again f/p 10, 4 and then you have you know 250, so that is you know taken as the value of all the equivalent value of all the you know disbursements which is there.

Now if you take all these factor values it will be 1.611 and this will be so 611 then this factor value by looking at the table you will get 1.464. And this will be 250, so this is coming out to be 2593, now you have to find the equivalent value of all the disbursements. So, what we see is that this is basically not 142 this is 482, so this is not 482 and now what we see here that this 482 you can have the value at this point.

So, this 482 it is p/a and 2 time it will be defined directly here, so this 482 will be multiplied with p/a and rate of interest is 10 and 2. So, this will be directly defined at this point because once we use the p/a factor, p/a factor defines the present value at 1 you know time period you know previous. So, if you take these 2 p/a values it will be giving you the value here.

Now this 482 will be further taken into account, now this 482 will be multiplied with suppose f/a 10, 3 so it will be defined here. And then further you have to use for 1 time step away, so that will be 4828*f/a 10, 3 and it will be also multiplied with. So, you have to get further it is future values f/p 10, 1. So, it will be f/p 10, 1 so if you take these factor values p/a 10, 2 is coming as 1.7355 and f/a 10, 3 is 3.310.

This you can refer it from the interest table and f/p 10, 1 is coming out to be 1.1, so if you do this calculation also. It will be coming as 2593, so this is what you see that in the case of you know

receipts and disbursements which is occurring at different times if you are trying to find the equivalent values at any particular time the equivalent value of all the receipts will be same as the equivalent value of all the disbursements.

And their absolute values are equal otherwise when we talk about finding the net values we are going to have you know the consideration of it is sign also and this is how it can be shown. So, that is what the principle of equivalence tells apart from there are also certain equivalence rules like for any you know cash flow if you try to find at a particular time you know of the 2 you know regions the equivalent values.

Then they will also be equal but with opposite sign, so that is also another equivalence rule and that also can be checked and found to be correct. So, these are the equivalent equivalence you know principles which must be taken into account and when we will talk about you know the transactions at different time domains that time is you know equivalent formulas will be for the you know calculation of different you know p, f or other you know financial parameters, thank you very much.