Financial Mathematics
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Lecture-18
Introduction to Economic Equivalence

Welcome to the lecture on Introduction to Economic Equivalence, so we discussed about you know many types of interest factors then we also discussed about the compounding frequency of interest. Now many a times you need to know you have a cash flow where many transactions are taking place and then you have transactions taking place at different times in different quantity.

And also it may so happen that at some point of time you know between certain periods the interest rate also may vary. So, basically at that time how to tackle these transactions which are taking place what will be it is you know effect cumulatively if you try to compare with certain other type of you know cash flow or alternatives because many a times you need to have an understanding of the situation you know what way you will be benefited by the offers.

Suppose there are 2 banks and they are giving you 2 types of offers or a bank itself is giving you a 2 offer you know. In that case how to compare these situations for that you will have you know to understand things you have to see the transactions, you have to see the cash flows you have to add or subtract take into account the interests which are you know taking place, interest rates which are there in certain period or so.

So, for studying these things you need to have the understanding of the equivalence economic equivalence. And we will have some introduction about this equivalence and also some concept about it in this lecture.

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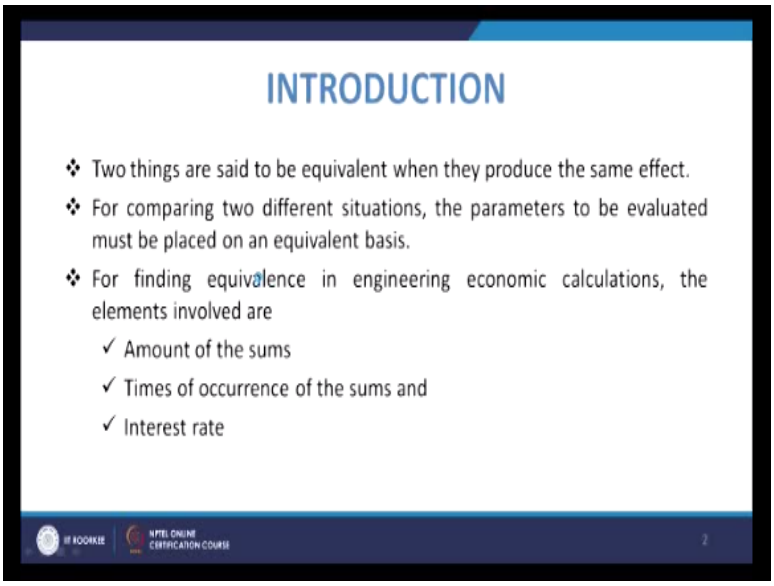

So, 2 things are said to be equivalent when they produce the same effect, so when they are producing the same effect you tell that they are equivalent. In the sense that if you have 2 cash flows if both the cash flows are giving you same return you know or in fact both are yielding same future value or same present worth something like that or same manual equivalent value all that.

So, in that case you can say that they are basically equivalent now for comparing 2 different situations the parameters to be evaluated must be placed on an equivalent basis. So, basically you will have to you know place or you know you have to evaluate the parameters on an equivalent basis like if you are you know you have 2 cash flows and then if you have to you know compare them.

Then there are certain parameters like what is the you know how much is the transaction going on or when the you know occurrence is there what time these transaction takes place where the interest rate. All these things you know they must be placed on equivalent basis many a times what happens that if 1 will affect we will be changing other will change.

So, suppose what way we can understand this that for finding equivalence in engineering economic calculations. The elements involved are amount of sums times of occurrence of the sums and interest rates. So, basically when we talk about you know analysis of the you know

system financial transactions or so. In those cases basically you have 3 elements involved 1 is the amount of sum, so how much is the transaction, what is the amount of transaction which is taking place .

Then what is the times of occurrence when it is taking place some of the receipts maybe at the second, fourth or fifth you know time and some may occur even at 4th, 8th or 10th time. So, that time of occurrence is also important and then the interest rates, so that interest rate also is to be seen that what is the interest rate many a times you need to find the interest rate which will make something equivalent.

So, what will be the interest rate suppose which will make today's 1000 rupees equivalent to 1500 after 10 years. So, that way that time you need to have a interest rate, see the interest rate will be varying for different interest rate they will have different value of you know f you know r, e or so. So, what we need to say that out of these 3 few are basically fixing you know 2, so third will be fixed basically.

And if you are fixing 1, so 1 by changing 1 you can even change the second, so that you will get the equivalent value or the cash flows will be equivalent. So, such as you know such cases we have to see that how you will see that how this equivalent concept or equivalence concept you will be tackling with. Now what you can say that.

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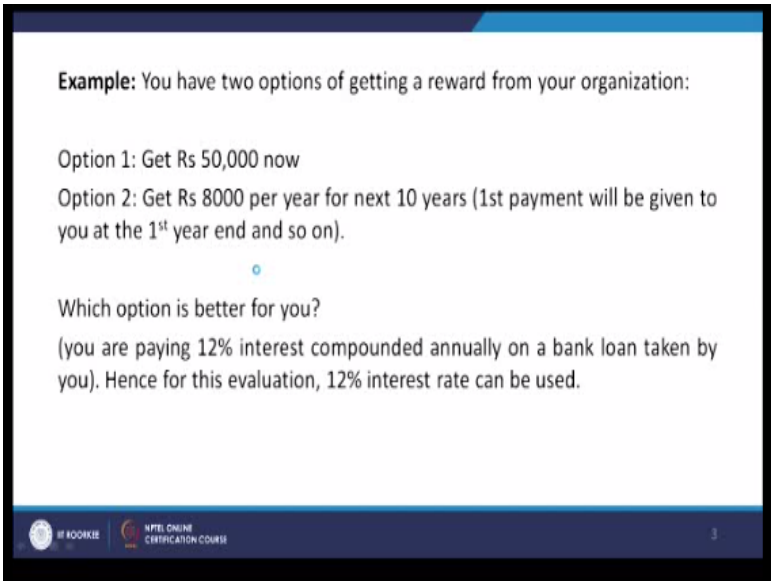

Suppose you are given 2 options of getting a reward from your organization, now let us say the option 1 is that the organization is telling you that you will get rupees 50000 now. And in the second option you are told that you will be getting a rupees 8000/year for next 10 years. And you know that 8000 rupees will be given to you at the first year end and then further 8000 second year and so.

So, it will be given 8000 rupees for the next 10 years and a rate of interest you can have it equivalent to 12%. Because you have taken a bank loan and for that it is 12%, so you can take that as the criteria for the interest rate. Now if you are told to decide that which one should be prefer, so if by looking at you know cursory if you look at 1 point what you see that you are getting 8000 for 10 years it will be 18000 something like.

But you are getting only 50000 now, so the thing is that you know what to decide which one will be better deal. So, for that you must understand that what will be the equivalent values for this you need to calculate the equivalent value, now how to calculate that equivalent value.

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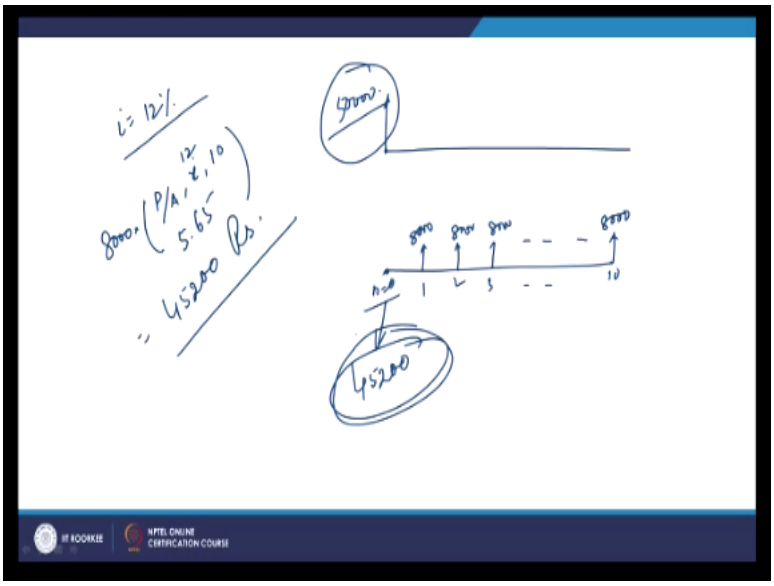

So, if you try to see that you have 2 situations, one is that you are getting you know 50000 now, another is that so your p value is coming as you know 50000. In this case you know in the another case what is the coming is you are getting 8000 rupees every year, so your 8000 is coming every year. And you are going up to 10 years. So, 1, 2, 3 and then you got to 10, now if you have to compare these 2 you will have to have the equivalent value of this at a at the time you know tn=0.

So, what you do is that you get the equivalent value and for that what you do and your i is 12% compounded annually. So, for finding that equivalent value what you will do is that you will get the present worth and for that and this is a is given. So, you will multiply 8000 with P/A and then r and you know or i you can say and n is 10 and this i is 12, so it will be 12. So, P/A 12, 10 now P/A 12, 10 factor is to be you know found out and if you try to see the factor value of this P/A 12, 8.

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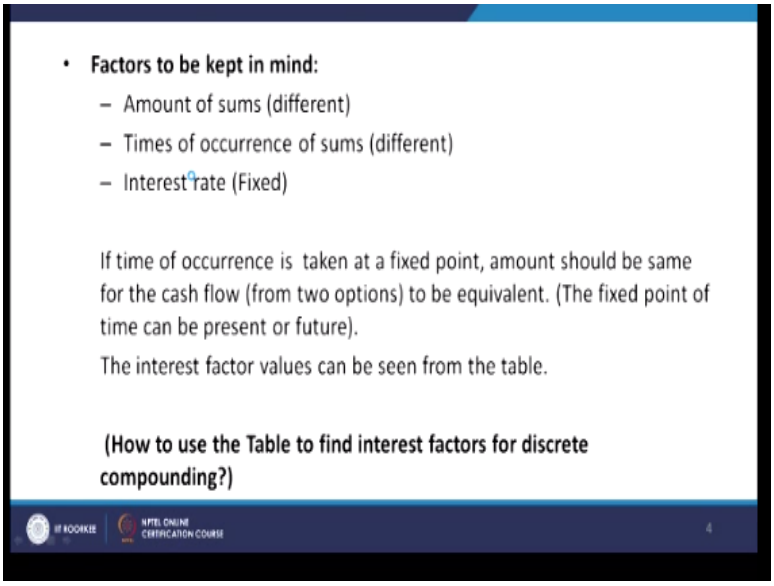

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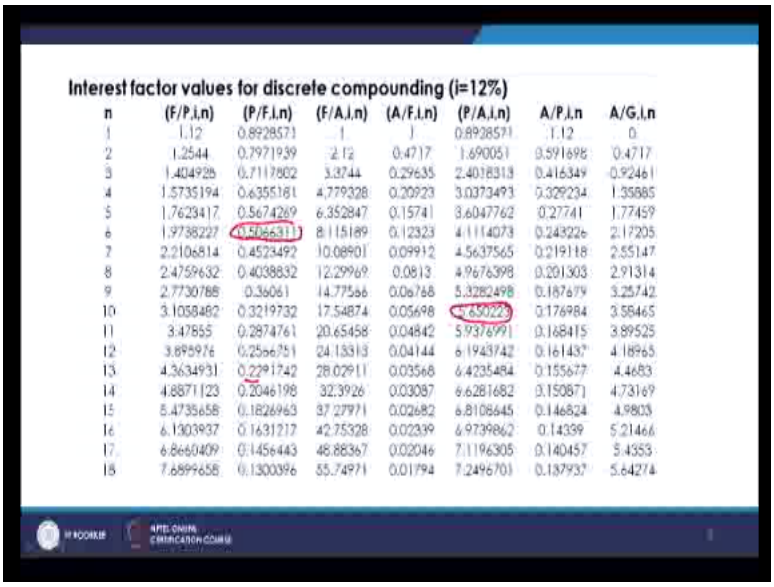

n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	A/P,i,n	A/G,i,n
1	1.12	0.8928571	1	1	0.8928571	1.12	0
2	1.2544	0.7971939	2.12	0.4717	1.690051	0.591698	0.4717
3	1.404928	0.7117802	3.3744	0.29635	2.4018313	0.416349	0.92461
4	1.5735194	0.6355181	4.779328	0.20923	3.0373493	0.329234	1.35885
5	1.7623417	0.5674269	6.352847	0.1574	3.6047762	0.27741	1.77459
6	1.9738227	0.5066311	8.115189	0.12323	4.1114073	0.243226	2.17205
7	2.2106814	0.4523492	10.08901	0.09912	4.5637565	0.219118	2.55147
8	2.4759632	0.4038832	12.29969	0.0813	4.9676398	0.201303	2.91314
9	2.7730788	0.36061	14.77566	0.06768	5.3282498	0.187679	3.25742
10	3.1058482	0.3219732	17.54874	0.05698	5.650223	0.176984	3.58465
11	3.47855	0.2874761	20.65458	0.04842	5.9376991	0.168415	3.89525
12	3.895976	0.2566751	24.13313	0.04144	6.1943742	0.161437	4.18965
13	4.3634931	0.2291742	28.02911	0.03568	6.4235484	0.155677	4.4683
14	4.8871123	0.2046198	32.3926	0.03087	6.6281682	0.150871	4.73169
15	5.4735658	0.1826963	37.27971	0.02682	6.8108645	0.146824	4.9803
16	6.1303937	0.1631217	42.75328	0.02339	6.9739862	0.14339	5.21466
17	6.8660409	0.1456443	48.88367	0.02046	7.1196305	0.140457	5.4353
18	7.6899658	0.1300396	55.74971	0.01794	7.2496701	0.137937	5.64274

So, this P/A 12 this is 12% and 8th you know this time, so I think it is for 10 years not 8 years. It is for 10 years P/A 12, 10, so P/A 12 and 10 and in the 10 it comes here, P/A 12, 10 is 5.65. So, this 8 12 10 is 5.65, so if you multiply that it will be 45200 rupees. Now you can compare because if you while comparing you must have so the equivalent basis and you have to compare the you know these 2 amounts at a particular time.

And present time you are comparing and it is coming as the present value is coming as 45200. Now it will be easy for you to compare these 2 situations and you can very well say that the offer which you are getting the option 1 50000 you are offered now. That is more you know

advantageous or it will give you more you know profit I mean as compare to the option 2 where you are getting 8000 rupees for the next 10 years.

So, this way you know these principle of equivalence is to be utilized and you have to find these you know equivalent values for all these cases and then you have to decide what should be done. Now when you know finding these equivalence you may have the single factor which is involved and at that time you have to understand how to proceed.

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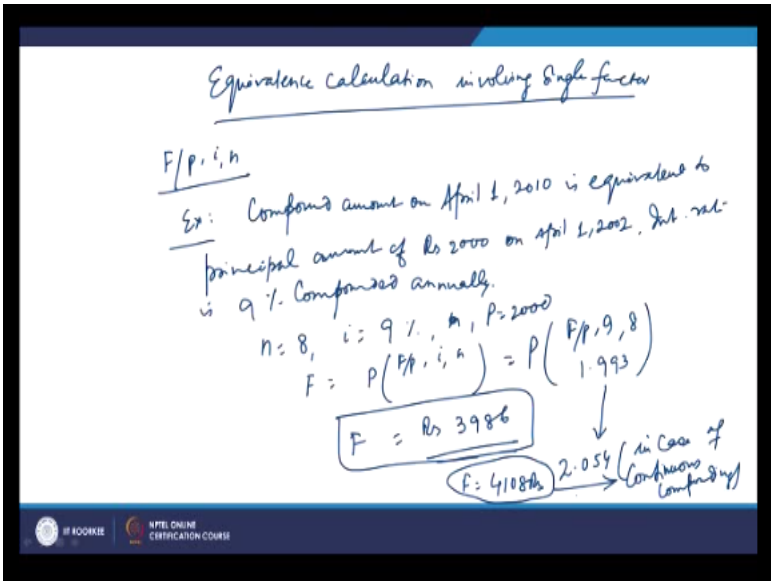

So, you may have the equivalence calculation involving single factor, so you know what happens that you know you must have we must we have all have discussed about the different you know factors like P/F i, nor f/p i, n or you know F/A i, n a/f i, n all that we have studied and here what we will see that you have a single factor which is required to be known to which will tell us that whether they are what will be that value which will make this equivalent.

So, in those cases that single factor is required to be known and in that case you know for example suppose you have the f/p i, n factor calculation is there. So, you know that this is the factor which is known as single payment compound amount factor. Now what may happen that you are given some future amount and present amount and you maybe told or you are given you know time period interest rate and present amount.

And you may be told to find the future amount, so in that case you know you have to calculate these values accordingly. For example suppose let us say that you know compound amount on April 1 2010 is equivalent to principle amount of rupees 2000 on April 1 2002. Now an interest rate is 9% you know compounded annually, so in that case if you are required to calculate you know the values that what will be the compound amount you know equivalent compound amount.

Now in this case as you see that your n becomes equal to, so you have 8 years of you know time span in between, so you have n as 8, i is 9%. So, ultimately you are going to have the f as p and then you will have the factor f/p i, n. So, this present amount what 2000 and if for the 9% you are we will refer the 9% table and if you refer the 9% table f/p you know and 9, 8.

If you see this value that value will be something close to 1.993 and p is given as 2000, so if you multiply that it will be rupees 3986. So, now this is you know this is the amount which will be equivalent to the present amount of u principle sum of 2000 on April 1 2002. Now this may change this amount of f, so this f amount will change if the one of these factors are changed like i or n.

So, suppose your interest rate is changed and you are told to find that the interest is compounded continuously in that case. Now if the interest is compounded continuously in that case the same factor, so this factor will become 2.054 in case of continuous compounding. So, if you do that it will be something like 4108, so this amount will be 4108 rupees for the case of continuous compounding.

So, the thing is that if these things are changed I that case the value will change and then we can say that under the condition of continuous compounding of interest the rupees 2000 on April 1 2002 is equivalent to rupees 4108 on April 1 2010, so that way you will have the 1 factor calculation you know in the case of these equivalence value calculations.

Similarly you maybe you know given the future value and the present value or principle sum . And also you are given the time but you are told to calculate i, so in those cases you have to

calculate the you know rate of interest and you can have the 1 example suppose you have you know number of years is known. And you know and then so p and f is known.

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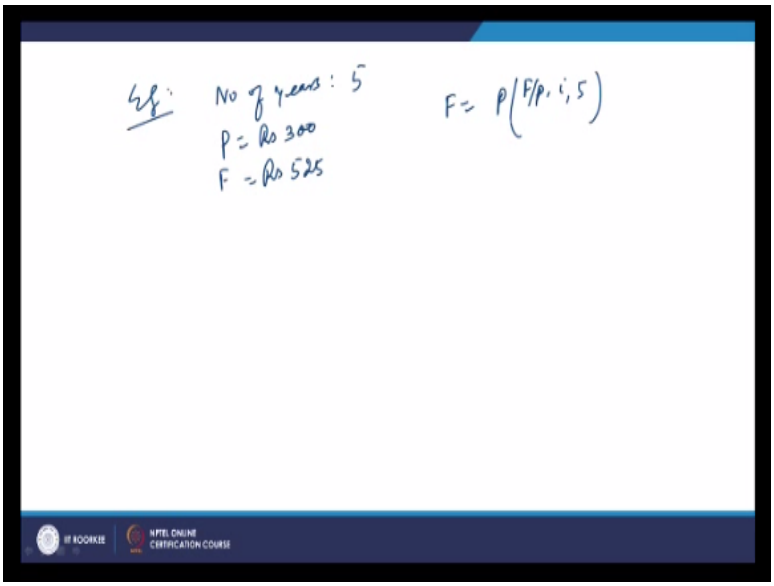

So, p is rupees 300 and f is rupees 525 and number of years is 5. Suppose now what rate of interest will make this principle sum of rupees 300 present value of 300 rupees equivalent to now rupees 525 in 5 years. So, here the concept of equivalent equivalence will be used and what you know that f=p* you know f/p and i and n is 5. So, now you have you know f/p i, 5 basically we know that is 1+i raise to the power 5.

So, based on that you can have these you know calculation and you can find the value accordingly. So, this value can be calculated and also from by looking at the interest table also you can find it. Now so similarly you know as we told that sometimes you may also be required to find the time. So, the time again further can be calculated.

And whatever parameter is required to be understood or calculated that can be found out you know using this principle of equivalence. So, even for other factors also you can use the same thing the equations will only be you know going more and more complex in case f/p or P/F it is somewhat you know more simple. But the at equation suppose we are taking for P/A or a/p i, n.

In those cases it will be more you know complex in that case many a times you may need to go for write and trail method or you have to go for interpolation. So, by using the interpolation you can find these values the now what we further try to understand is that when in most of the financial transactions when we talk about we deal with the transactions which are taking place you know with time.

And then ultimately you will have to find, so the equivalent value is at a particular time. So, in fact we are trying to see that how in the case of you know cash flows how these principle of equivalence works. So, we will be dealing with equivalence involving cash flows.

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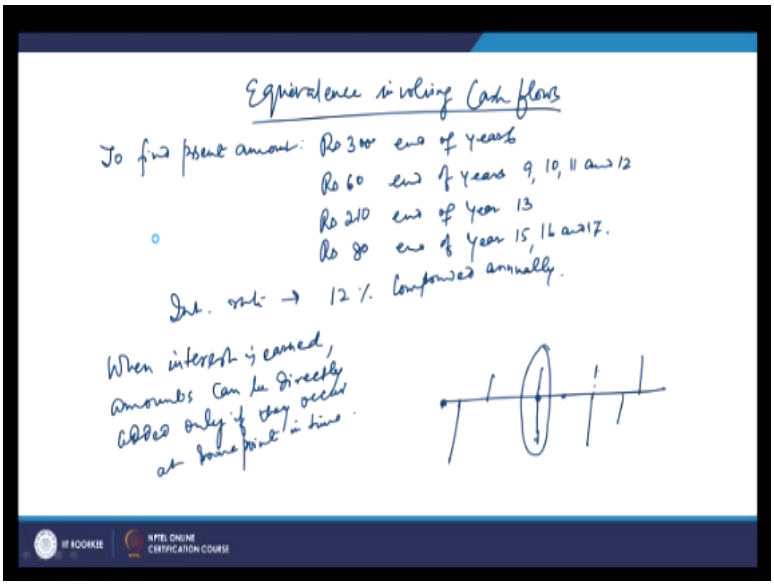

Now this we can understand by taking an example suppose you have to find the present you know to find the present amount equivalent amount basically. And the cash flow is going like this that you have you know 300 rupees 300 you know end of year 6 and rupees 60 end of year 1910, 11 and 12. And suppose rupees 210 end of year 13 and rupees 80 is end of year 15, 16 and 17 and the interest rate is basically shown as 12% compounded annually.

Now what we see that these are the transactions you have 17 years of time domain. In that you have only rupees 300 is transacted at the end of year 6 and from rupees 60 you have you know years 9, 10, 11 and 12. Then 210, 13 and 80 is 15, 16 and 17. So, what will be the equivalent

present amount for such cash flows. So, we have you know to analyze such you know cash flows and then finding the equivalence.

So, the n is that for all these you know transactions you have to have it is equivalent to be defined at time 0. So, if you try to you know further see before that we must understand certain principles certain you know rules what we deal with. So, when you are earning the interest basically then any transaction which you which is occurring at 1 point of time there maybe you know you are getting the equivalent at certain value.

Suppose you have many transactions going on at different points of time, now the thing is that if you are finding you know suppose this is also 1 transaction. So, if you have to add this and this in that case either it is equivalent is to be found here or it is equivalent is to be found here. Then only if they are occurring if you are finding it is equivalent at a particular time then only you can you know add or subtract them.

So, basically when interest is earned amounts can be directly added only if they occur at same point in time. So, at same point in time, suppose in that case they could have been 1 transaction here, in that case you can write that there will be net either receipt or disbursement. But otherwise if you have to add them or subtract them you will have to bring them at a common point of time. Now what we see that here these things are occurring at different times how to you know see you know solve such cases.

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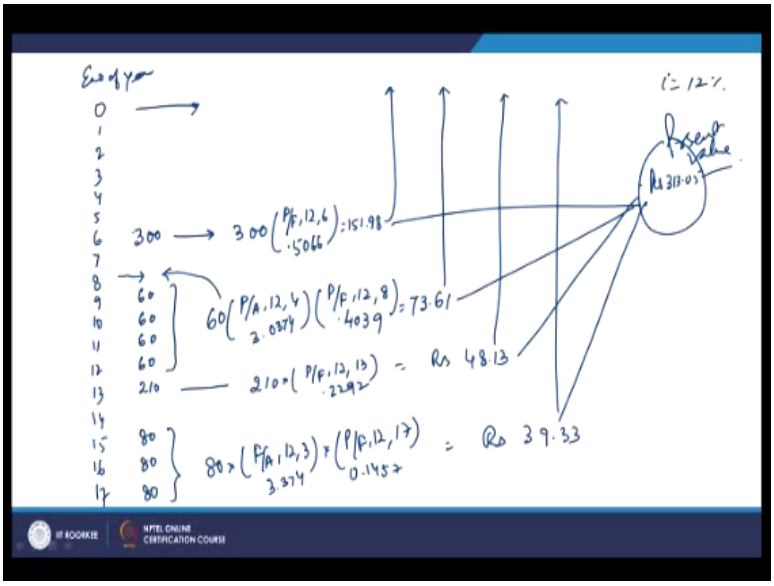

So, what you can do is you can have the you know time, so this will be your end of year and you can have 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 and 17 that is what you have 17 years in which the transactions are taking place. You know that in the 6th year and you have rupees 300 taking place. So, you this is coming as 300 here then in the you know in the 9, 10, 11 and 12 is rupees 60 is going on.

So, in the 9, 10, 11 and 12 rupees 60 is you know transacted, then you have rupees 210 at end of the year 13. So, at end of year 13 it is rupees 210 and then you have the at 15, 16 and 17 you have 80 rupees. So, this is 80, 80 and 80, so the thing is that when you want to find you know the present equivalent value you will have to have the equivalent values of all these at time 0.

So, how to have that, so you will have the partially you will calculate it is present value calculation. Now if you try to find the you know partial value of the present value calculation of this amount. Now this amount will be multiplied with a factor because this is a future you know value at the end of 6th you know time period. So, you will multiply this with the factor you know P/F i, n and i is you know that 12%.

So, you will multiply with P/F 12, 6, so you will are going to multiply this it is value will be 300*P/F and 12 and 6 and if you look at the factor table. Then P/F 12, 6 it will be 0.5 0 66, so in that case you will multiplied this with 0.5066 and you know this value will come out to the

151.98. So, this will be 151.98, now you will go to the next set of transactions, now these 4 transactions.

Now we know that now in this case what happens that you are if you try to get now what you can do is that for all this they individually can be you know mapped for the year 0. So, 60 will be multiplied with you know P/F factor 12, 9 again 12, 10, 12, 11, 12, 12 but that will be you know that is not the proper way. Basically you know that these are all the equal values, so basically you can use the is that annuity concept formula.

Now what you can say that for these 4 if you apply the P/A formula, P/A is the find 1 point here ahead. So, if you take P/A 12, 4 and multiply with that, so that will be defined at time 8 and then again you have to multiply with P/F 12, 4. So, what you do is that you multiply this rupees 60 with P/A and 12, 4, so this will be defined at this point and further to define it at this point you will again multiply is with P/F and 12 and this is at 8, so P/F 12, 8.

So, if you see that P/A 12, 4 if you see P/A and 12 and 4 this factor is 3.037 3 or 74. So, you multiply that with you know 3.0374 and then you have P/A 12, 8 so P/F 12, 8 again you can see P/F 12, 8 will be 0.40388. So, it will be 0.4039, so if you now this amount basically will be, so basically this amount is defined here. Similarly this amount which is coming out to be 73.61 and this is basically defined at this point.

So, you are getting the equivalent value of these 4 values at this point. Now again this 210 it is occurring discretely at time 13, so for these 210 is to be multiplied with you know P/F and this will be 12, 13. So, P/F 12, 13 is you can further see P/F and 12 and 13 it will be 0.2291, so you can have this factor as and then you have 0.22917, so 0.2292 you can 8, so it will 0.2292.

So, this value will be coming out to be rupees 48.13, so this value will be again here this is again you know it is equivalent is found at time 0. Then for these 3 values again you have to calculate and not in this basically can we can get P/A and then they can calculate by P/F or you can get F/A, so F/A so 80 will be multiplied with F/A and you these on 3 times.

So, it will be F/A 12, 3, so it is defined at time 17 then you can P/F 12, 17, so this refer to the table it will be 3.374 and this will be 0.1457, so it will be coming as rupees 39.33 and this is again at this point. So, basically you are going to add these 4 values and it will give you the value which is coming out you know you can add them and it was coming as your own rupees 313.05.

So, this is the present value equivalent value of the particular cash flow, so that is how we try to you know deal with the you know cases of cash flow in the case of the finding the equivalent values. And this will be useful for many cases when we deal with problems of such type in the long run, thank you very much.