Financial Mathematics
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Lecture-17
Interest Factors for Continuous Compounding

Welcome to the lecture on interest factors for continuous compounding. So we will discuss first how the (()) (00:34) compounding defined and then we will also derive the expressions for the continuous compounding cases. Before that let us see that how the you know compounding is changed when we have seen that when the number of periods per year is changing. In that case in the earlier you know lecture we have seen that when we are changing this compounding periods per year you know number of periods per year.

In that case the effective interest rate you know per period will be changing you know not particular year for any particular you know time. So but we suddenly it will be similar so we can have the you know example from here basically we can have the table which tells us about the comparison of interest rates for 12% nominal rate of interest.

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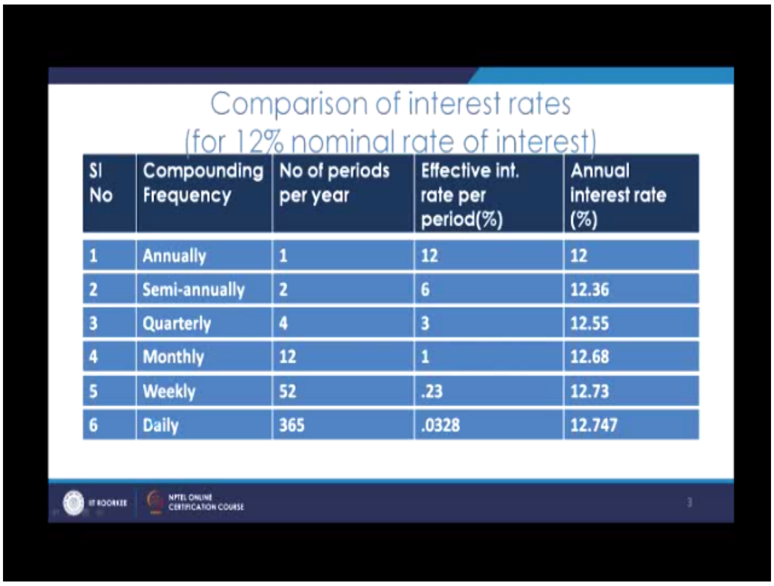

Sl No	Compounding Frequency	No of periods per year	Effective int. rate per period(%)	Annual interest rate (%)
1	Annually	1	12	12
2	Semi-annually	2	6	12.36
3	Quarterly	4	3	12.55
4	Monthly	12	1	12.68
5	Weekly	52	.23	12.73
6	Daily	365	.0328	12.747

And how we when we change the compounding frequency then how the annual interest rate will be changing. So if you look at the first case of compounding frequency annually it means the number of period for compounding is done per year it will be 1 and effective interest rate per period will be 12. So certainly it is annual interest becomes 12%, so we call it at 12% compounded annually.

When we talk about the 12% compounded semi annually we discussed it that then your period becomes number of periods you know for which the compounding is carried out is 2 and per period you know effective interest rate because your period for which the effect interest rate is compounded is 6 month now and for 6 month interest rate will be 6% but if you calculate the effective interest rate for the whole year it will be difference.

So it will be 1.06 multiplied by you know 106 and then that will be subtracted you know so 100 that number -100 so that will be the interest, so basically that becomes 12.36, then if you do it quarterly so you are doing the you know compounding 4 times an effective interest rate per period will be you know 3% because you know in that case with 3*4 is in 12 and in that case annual interest rate if you compute take 1+r/n which is call ln-1.

So ln is normally taken as C also where that was shown, so that -1 becomes 12.55, then go to monthly in monthly basis your number of periods per year becomes 12, effective interest rate for 1 month will be 12/12 that is 1 and in that case your annual interest rate effective value will be 12.68%, if you compute further weekly then you have 52 times compounding. So compounding number of periods for which the compounding is done is 50 you know 1 week. So it will be 52 times and you know for every week you have 0.23% so 12/52.

So that way you get this 0.23 and annual interest rate if you compute it will be 12.73%. Now if you do the daily basis the same thing comes out so you are doing 365 times and your effect interest everyday will be 0.0328% and your annual interest rate computed will be 12.74 7% or so. So that is how you see that your annual interest rates will be very, now another case is which is very much practice by the financial institutions and banks is the continuous compounding and for that basically we are going to define that what is continuous compounding.

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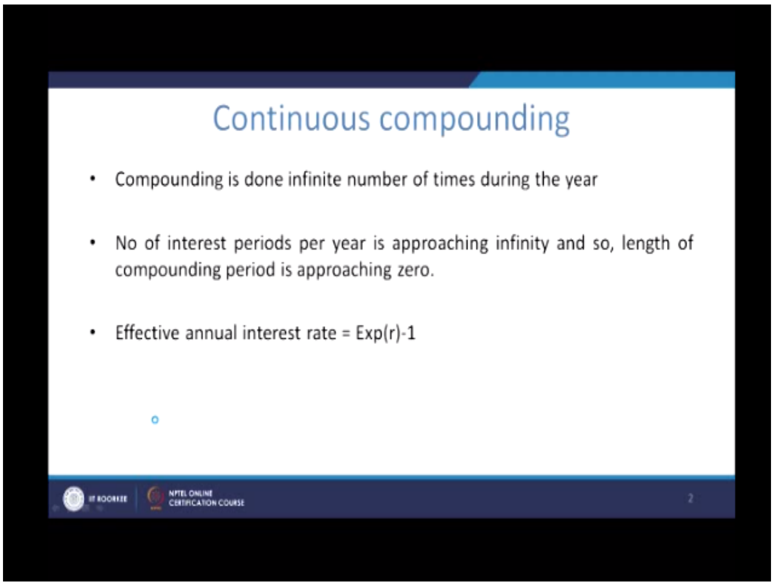

Continuous compounding is you know that case when compounding is done in finite times you know infinite number of times during the year. So basically the number of interest periods per year. So that is approaching infinity and that is why the length of this compounding period will be you know approaching zero. Now so in these cases your value of these effective interest rate per annum will be to be computed.

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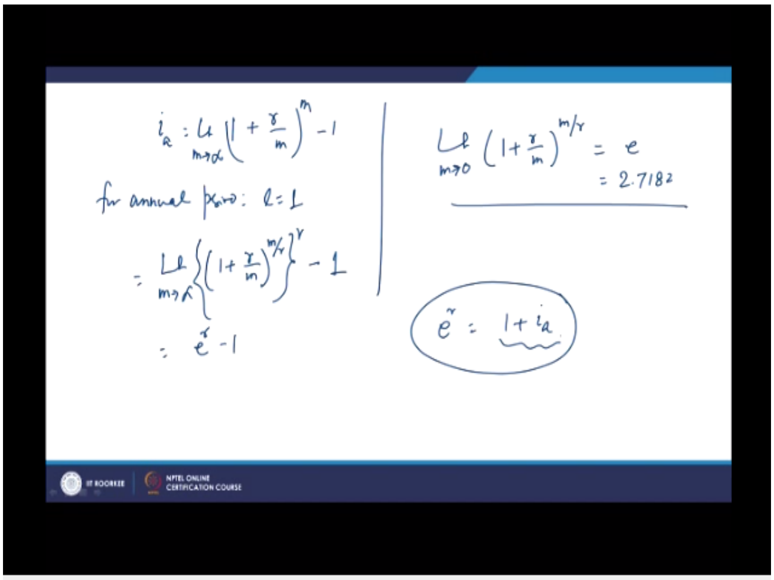

Now that can be computed by using that formula again, so what we have seen that effective annual interest rate ia you know that this case you have 1+r/n raise to the power lm and -1 and l is 1 year so since you are doing annually, so for annual case or annual you know period you know l will be 1. Now in this case 1+r/m raised to the power lm, so it will be 1-1. Now in this case the thing is your period that is you know this m.

M is going to be infinity so you are taking m tending towards you know infinity you know that way, so in that case if you try to define now what we see you can recall that expression that limit m tends to 0 and 1+r/m and it is raised to the power m/r it is defined as the exponential function that is e and this w is nothing but 2.7182 and then you are further the number.

So this can further be written as limit m turns to infinity and then you will be writing 1+r/m then m/r and then raise to the power r, so this can be written as you know a 1+r/m raise to the power m. So and then -1 so we know that this come you know this quantity becomes e so it will be e raised to the power r-1 and this is becoming e, so you will have e raise to the power r-1.

So the effective interest h which is computed in such cases will be e raise to the power r-1. So when you are you know the nominal interest rate is defined then you can have these value of you know this effective interest rate for the continuous compounding case e raise to the power r-1. Now what happens that when we are dealing with such situations where are you have to compute these interest factors for all these cases.

In that case you have to use this e raise to the r-1 as the effective interest rate per year and if you look er so basically er becomes 1+ia, so most of the times you are using this 1+ia and that will be replaced with now er in all the situations when we are dealing with you know these continuous compounding cases. Now if you are dealing with that you know continuous compounding formulas we are I think the conversant with all these terminologies which we have used earlier.

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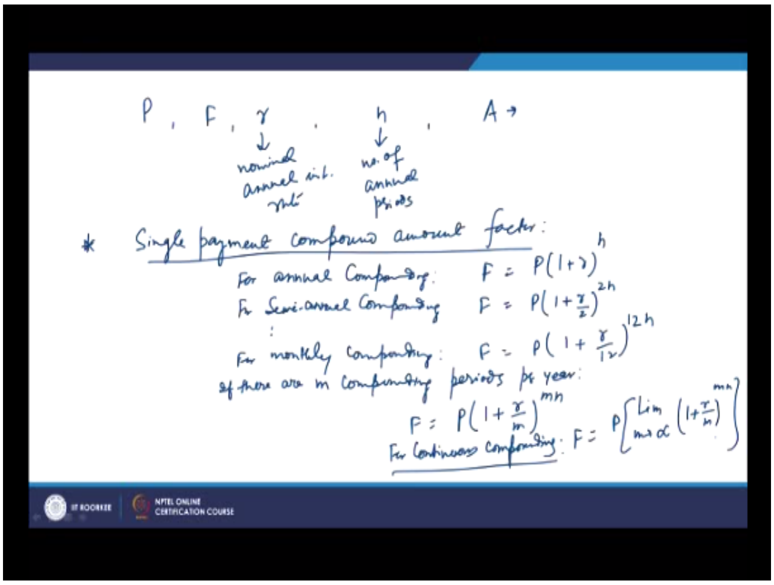

That is e will be your present amount, so that is the present principal, so now you will have p then that is your present principal, then you have f, f as the you know future sum, so future value or FV sometimes you also call it as then you have you know you have call as r here you will have returned an as r that is your nominal interest rate. So r will be your nominal interest rate, nominal annual interest rate.

Then you will have n now n is the number of annual periods then you will have n, now n is the the number of annual periods then you may also come across the terminal as a what we have discussed earlier, so you will have all these factors like F you know rn and then F/prn or a/prn or pyrn or different you know based on all the different terminologies different factors will have different names.

And we can have the expression for that so a basically is the you know single payment which is in a series of unequal payments and may that be end of each of the you know annual periods that is what we call it as a that we also call it as amenities also. So now we will go one by one to the different you know factors. First of the factor is the single payment compound amount factor.

Now this we are discussing for the case of continuous compounding, now in the case of you know for annual compounding if you look at what we see is for annual compounding when we do we are telling that f=p*1+r raised to the power n, that is what the formula will be applied and factor will be you know 1+r raise to the power n, if there is annual compounding.

If it is for semi annual compounding then in that case we have seen that interest becomes r/2 that is 1+r/m and then l*m, so if you are doing you know are 2 times and in that case f will be p*1+r/2 and then you will have 2n because what happens that your time period is 1 and your this is becoming 2. So 1*2 and then raise to the power m. So that is how your r/2 and raise to the power 2n.

So similarly if you go for suppose you go for monthly compounding then you know that it will be f=p*1+r/12 raise to the power 12n. So that is how you go in those cases. So if you have so it means in general you can write that if there are m compounding periods per year in that case your equation becomes f=p*1+r/m and then mn. Now when we talk about the you know infinite times of you know compounding.

In that case your m becomes you know m is standing towards infinity, so that is the case of continuous compounding. So when we talk about the continuous compounding case in that case so for continuous compounding what will happen now for continuous compounding what will happen that the same thing you will have f=P and then you will have limit and then this m will be turning towards infinity and you have 1+r/m raise to the power mn.

So that is what you are you now this expression for the single payment compound amount factor will look like now what we can have is that we can write again that this factor as one limit you know m tending towards infinity 1+r/m raise to the power m/r that is e andm then you raise to the power rn, so you can see that this f will be basically we can further write you know p*you know.

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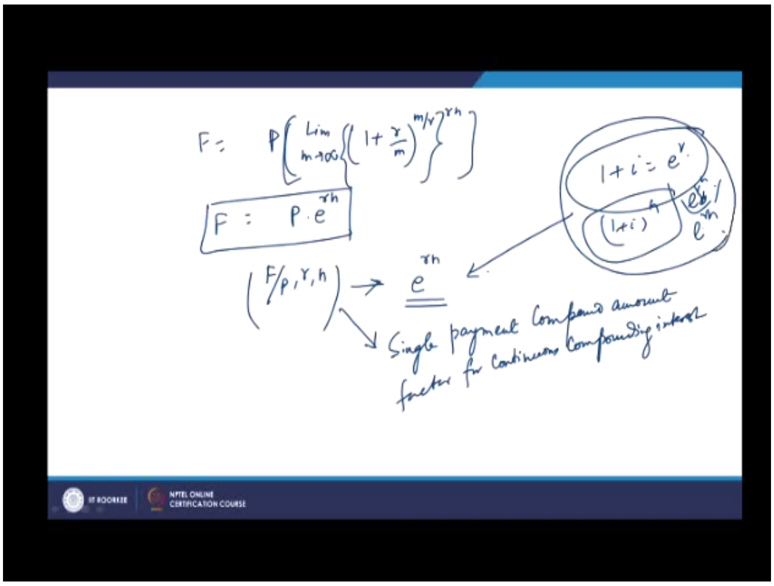

Then you have limit m tending towards infinity and it will be 1+r/m and you have mn so you write down m\r and this whole raise to the power it will be rn, so that this rn r cancels and your it becomes mn, so you had mn earlier, this mn was there, so this mn is still intact. Sothat is how you call it as for this case. Now in this case this will be basically this quantity will become e. So it will be p* then this will be e raise to the power n.

So what we see that this value will be p*rn so f will be equal to be p raised to the power rn. Now this factor so the factor single payment compound amount factor is normal f/p rn and this basically is nothing but it is e raise to the power rm. So so this is you know in the earlier case you had this one as 1+I raise to the power n-1 you know n basic is not a 1-1 and in this case as we discussed that 1+i basically it is power r.

And then since it is raise to the power n so it will e raise to the power rn and this is this factor is nothing but single payment compound amount factor for continuous compounding case, for continuous compounding interest. So that is how we try to you know define the case of the median of factor of the single payment compound amount factor in the case of continuous compounding.

Now we can see this calculation of the factor for the single payment present what factor now as we discussed that this how this came out. So we have already I told you that we have seen that 1+I will be e raise to r, so that is how 1+I raise to the power n is there in a normal case. So this raise to the power so er to power n, so e raise to r so that ern so that is how it becomes.

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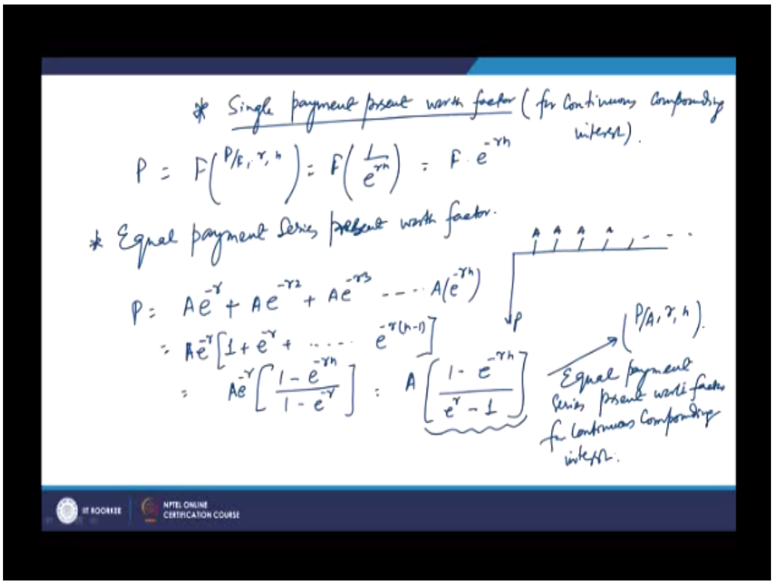

Now we will see in the next factor that is single payment present worth factor, now this single payment present worth factor as we know that this is p/f and rn it will be and it will be the reciprocal of the single payment compound amount factor and if you look at that **so** so basically it should be P=f*that factory p/frn, so may it will be P, so it will be reciprocal of the single payment compound amount factor.

And that is why it will be worn by e raise to the power m, so you can write f*e raise to power –rn. So basically 1/ern or it is –rn is the single payment present worth factor for continuous compounding interest. Now we will further moves, we will move to the equal payment series factor and then we will calculate for the equal payment series present worth factor. So we are interested to know the factory by p/arn.

Now in this case what is there is that you are you are trying to have a factor which should be multiplied with you know a, so that it gives you the value of p, now the thing is that in this case if you see so you have you will have p and then that value will be a will be there. So you will have a all that going on. Now in this case for such cases you can see that the p now this a now for you know.

And you have done this payment of a if you look at now if you try to find the expression for p, now if you find this total present worth value for these and that will be some of the individual present what values for these deposits. Now this will be for the a which is occurring at 1, so it will be e raise to the power –r. So it will be basically you know it is a present worth will be in the normal case it should have been 1/1+i. So this will be 1/1+I n 2times.

So that way in this case 1+i is nothing but it is e reciprocal of r, now it is reciprocal e –r, so this a will have e-r similarly you know then further for the next a it will be e then –r2, then e-r3. So it will be going and you have n payments to e and e raise to the power irn, so that is how it goes. So this series basically is geometric series you can have the A e raise to power –r as the common one.

So you will get 1+r raise to the power –r so this way it will go up to e raise to the power –r*m-1. So this becomes A geometric series and if you take its you know sum so it will be you know that for geometric series so it has the first term as 1 and the common ratio is e raise to power –r, so 1/e raise to power r, so that is basically going to be less than 1, so it will 1-rn upon 1-r and first term is 1 itself.

So it will be 1-r raise to the power n, so will have n terms, so e raise to the power –r/1-r and r is nothing but e raise to power –r common ratio. So this term will be again seen, so it is so you can further see it will be A*1-e raise to power –rn and then this e –r will be going at the denominator as er and er will be multiplied with this term. So this will be er and – this will be 1. So what you see that when there is a continuous compound in that case you have this e raise to the power r-1 will be at the bottom.

And this fact, now this factor is so this factor is used to find the present worth provided A is given and this factor multiplied with A will be the present worth, so that is why it is known as equal payment series present worth factor that is you know this is p/a rn and this is equal payment series present worth factor you know and in that case do you have for the continuous compounding interest.

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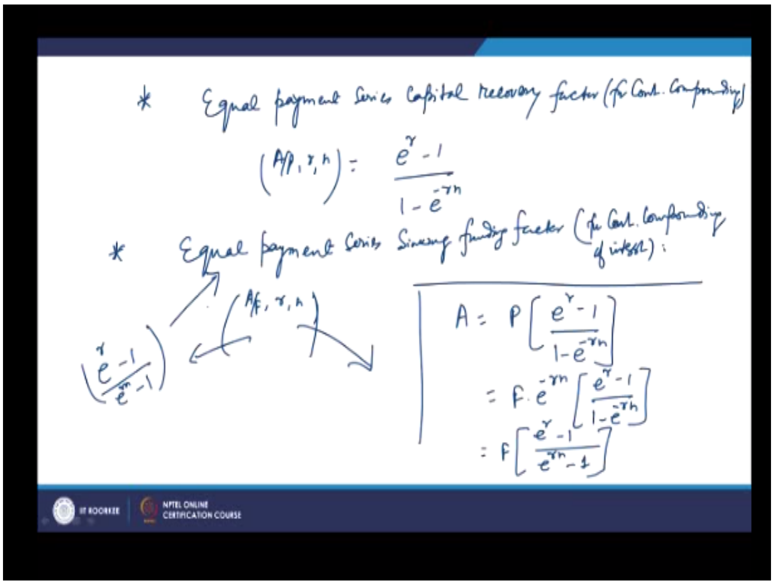

So that is how this factor will be denoted as you know p/A rn so you may have other you know equal payment series factors and if you have equal payment series capital recovery factor, so equal payment series capital recovery factor will be the reciprocal of the equal payment series present worth factor and this is for you know for continuous compounding and for this you know it will be reciprocal of the equal payment series or present worth factor.

And it will be the gain A/prn and you can have it, so earlier you had you know e raise to the power 1-e raise to the power rn and –rn/er-1, now it will be you know e raise to the power r-1/1-e raise to the power –rn. So this factor when multiplied with p that will give the value of the you know A that is equal payment series value, then you may have another factor that is equal payment series sinking fund factor.

So you know this is again we have to do for continuous compounding of interest. Now in this case what we do is that we know that this is A/frn so this factor when multiplied with F will give you the a values so it is nothing but A/frn, so this is the you know equal payment series sinking fund factor and we have equal payment series compound amount factor that is F/Arn.

Now if you look at A and p what we have seen is that is nothing but we know so far how to get calculate this for this you can have we know that A will be p*er-1 and divided by 1-e raise to –rm and then we also know that p is nothing but you know F*e raise to the power –rm, so that we have already got this factor. So it will be multiplied with er-1 and divided by 1-e raise to power –rm.

So you will get it this value as F* this e raise to the r n will go at the bottom so you will have e raise to the power r-1/e raise to power rn and then -1, so this factory e raise to the r-1/e raise to power rn-1, this factor is basically you know known as equal payment series sinking fund factor and for continuous compounding. So that is how we define for the continuous compounding cases.

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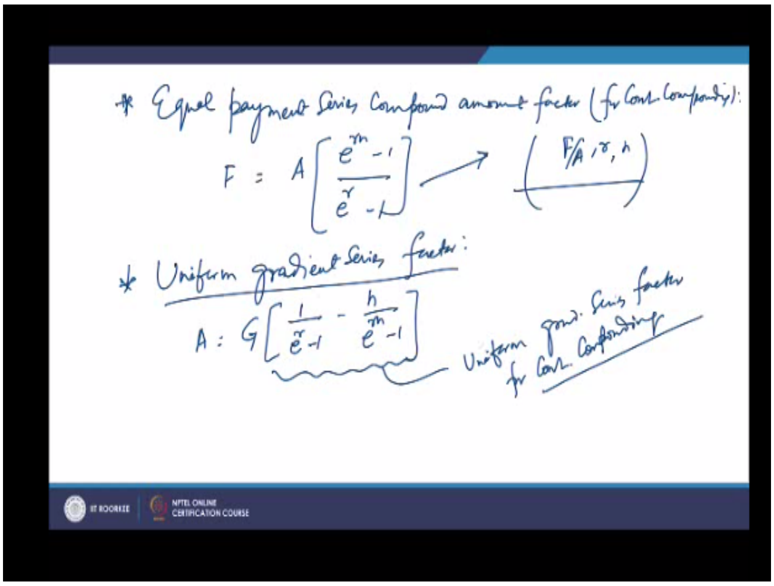

Then you as we know that equal payment series you know compound amount factor and that to for continuous compounding cases. Now for this as you know that it should be reciprocal of the equal payment series sinking fund factor and this factor will be multiplied with A to give you the present compound amount. So it will be reciprocal of the factor which has been found out and it will be e raise to the rn-1/A to the power r-1.

So this factor is basically denoted as F/Arn, so this the nominal interest rate r and n is you know the compound periods and that is your this factory is rn-1/er-1, so that gives you the value of you know the equal payment series compound amount factor for continuous compounding, we can have even for the you know gradient series factors and for example if you look for the uniform gradient series factor.

So uniform gradient series factor if you look at here you will get A between A and G so what we see that we can see have A=G*1/er-1-n upon e raise to the power n-1. So we can you know see the value of 1+I in those cases we compile to place it and what you see 1/I basically you what we see there 1/i-n/ you know 1+in-1. So that way it becomes 1/er-1, so that I that so basically this factor this factor basically will be the uniform gradient series factor for continuous compounding.

So you know in such cases what we see that you can have this you can also calculate this geometric gradient series factors also there also accordingly when you whatever expressions you have accordingly you can have the you know values being replaced 1+i omay be replaced with e raise to the power r. So you can have that expression for the uniform gradient series factors.

So all these you know factors will be used for calculating the value of the value of F or A or P or so and use of these formulas are basically you know suggested to be you know for the future you know applications, thank you very much.