Financial Mathematics
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Lecture-16
Compounding Frequency of Interest

Welcome to the lecture on compounding frequency of interest, so this is a very important you know aspect to be known to us that when we are dealing these financial mathematics and financial transactions. And we are involved we have to involve these financial institutions which are interest then you must know that how this interest is being charged.

So, when we study about the interest in normal cases what we talk is maybe 10% interest or 7% interest. And normally that is assumed that it is annually, so standard in a standard manner it is assumed that the interest will be always compounded once the year completes. So, after the year again the interest accumulated will be again a part of the principle amount for the interest calculation in the next year.

So, that is a normal you know notation or normal practice when we are dealing with the you know compound interest. But then in many cases the interest is basically applied many times is not that we are only doing in the years. So, **so** that **is** how the interest is you know applied many times and many a times that is why the interest is to be paid more frequently not necessarily after 1 year only means that you know after suppose 6 month the interest will be charged.

So, suppose there is you know amount of 100 and there is 8% of interest you now per annum compounded annually. So, that is in standard manner you know if you have rupees 100 and if the you know interest is to be calculated.

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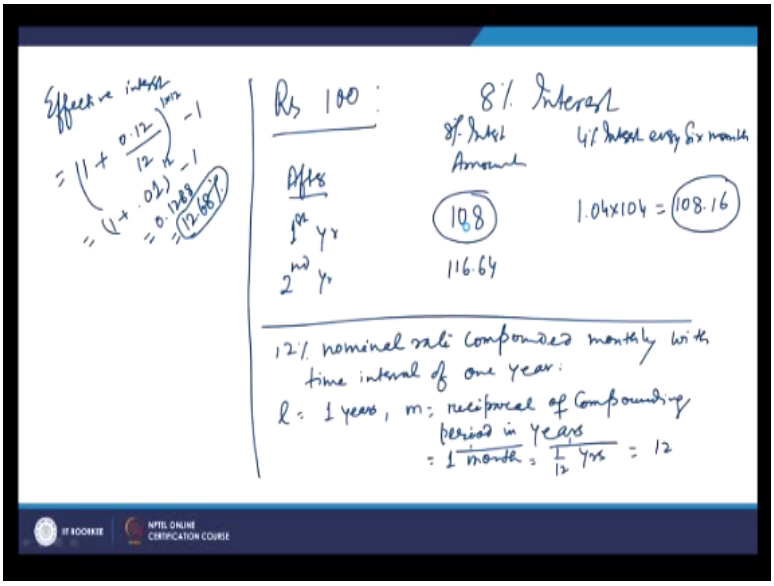

So, suppose 8% interest is there, so in a normal you know circumstance that means that after 1 year you know first year the amount you know becomes 108. Now so this is 8% interest you know per annum, so this is compounded annually in the second year again you know there will be 8% of 808 that will be interest. So, it will be you know 108*0.08, so it will be you know 8.64.

So, ultimately it will be 116.64 like that however if you just take the example that you are dividing this interest in 2 installments you are taking 8%in a year but then 4% in the first half and second 4% in the second half. So, that is basically the we are increasing the frequency of the compounding of interest that is what we mean in this case. So, we are talking about the compounding frequency of interest.

So, in that case if you are using 4% interest every 6 month, so what will happen that after you know after 6 month the 100 rupees will become 104 and after you know another 6 month. So, it will be again so, it will be 1.04*104, so basically it becomes 108.16, so that is basically this is 4% interest every 6 month and the terminology standard terminology what is used is that 8% interest compounded semi annually that is what we call it.

So, because we are basically compounding the interest in 6 month, so that is why we call it as compounding semi annually. And then you see that this becomes 108.16 instead of 108, so the interest amount which was earlier rupees 8 on rupees 100 that is the principle amount. So, that

became 108.16 when the interest was compounded semi annually or the frequency was 2 times in a year.

So, that is the basically difference between compounding frequency of you know compounding of the frequency of interest which is charged. So, if you go to further it will be further changed, so **so** what happens that the you know future amount will be changing. And many a times you know so when this is **d**one for suppose, so 2 times we can do it for 4 times, 8 times or 16 times or 52 times every week or we can do every day or even continuously that also we will discuss.

Now what is the effect of you know such changes of the interest which is done in the case of compounding you know frequency of interest. That is what we need to study in this you know chapter, now the thing is that so as we discussed that it will be varying from half of the year.

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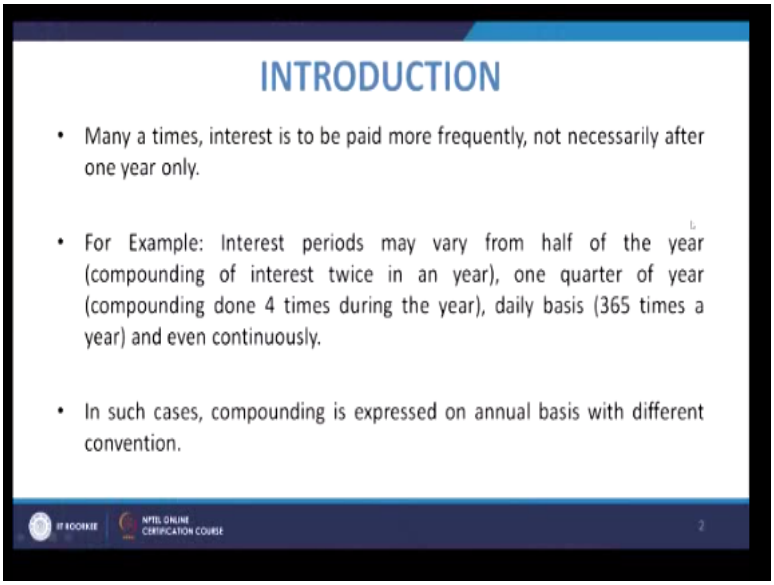

So, that is known as compounding of interest twice in the year then 1 quarter of year you may have the compounding. So, that becomes 4 times in a year then you may be doing the daily you may have 365 times in the year. So, that way you have the different you know notation of mentioning the names you know how the compounding is carried out. So, compounded semi annually or compounded quarterly, compounded weekly, compounded daily like that. You have different you know way of expressing these compounding frequency of interest.

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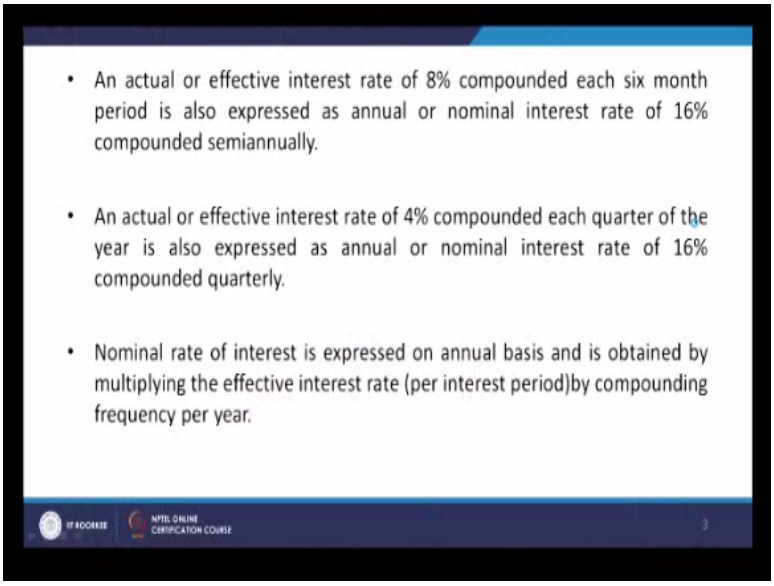

Now the actual or effective interest rate of 8% compounded each 6 month period is also expressed as annual or nominal interest rate of 16% compounded semi annually. So, what we see that there are 2 terms 1 is the nominal interest and another is the effective interest. Now when we talk about this term like you have 16% compounded semi annually, so that is what the nominal interest nominal term is and that is what we call it as the nominal interest rate.

So, in a year it will be 16% but then compounded semi annually, now if you look at you know the effective interest for the whole year if you look at in that case it will be different, it will be 16.64%, 85 every you know 6 month. So, in the 6 month 100 rupees will become 108 and then another 6 month it will become 116.64. So, basically when you call it as nominal interest rate of 16% compounded semi annually.

Then the effective interest rate for the year becomes different and that becomes basically you know 16.64%. So, basically you have 2 terminologies you know 2 terms of interest 1 is you know effective interest or actual interest rate and then you have the you know nominal interest rate what we I mean denote it as. Now nominal interest rate is expressed on annual basis and is obtained by multiplying the effective interest rate compounding you know by compounding frequency per year.

So, **so** that is how we basically express it on the annual basis, so that is your nominal rate of interest. So, basically compounded semi annually or whatever, so suppose the effective interest rate for 3 month is 3% it means nominal interest rate will be. So, it will be done 4 times, so that is what we are multiplying that, so it will be 3*4, so 12% and then compounded quarterly, that will be your nominal interest rate.

That is how we express the nominal interest rate, now how we are basically you know defining this nominal and effective interest rate.

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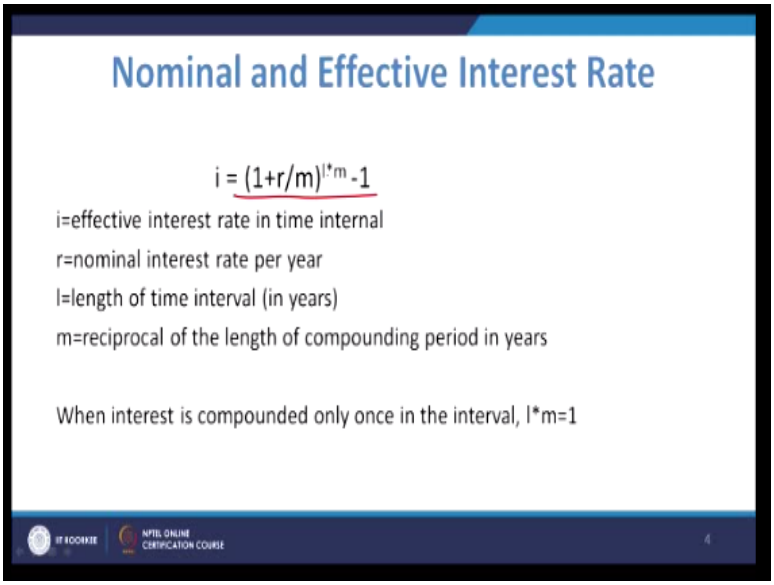

So, the effective interest rate it will be equal to 1+r/m and then raise to the power whole raise to the power l*m-1. So, that is what this is the you know formula for the effective interest rate, now what is what are the terms here. This r is basically the nominal interest rate/year, so that is what we discussed at 16% compounded semi annually. So, in the whole year you know whatever effective interest you have to find and you have to multiply with number of times.

That becomes your nominal interest rate value and compounding that many times that is how you use that term. So, that will be your nominal interest rate/year then your l is the length of time interval. So, in how much time you have to you know have this and then n will be the reciprocal of the length of compounding period in years. And when interest is compounded only once in the

interval, then l*m has be equal to 1. So, that is how you define this u nominal and effective interest rate s.

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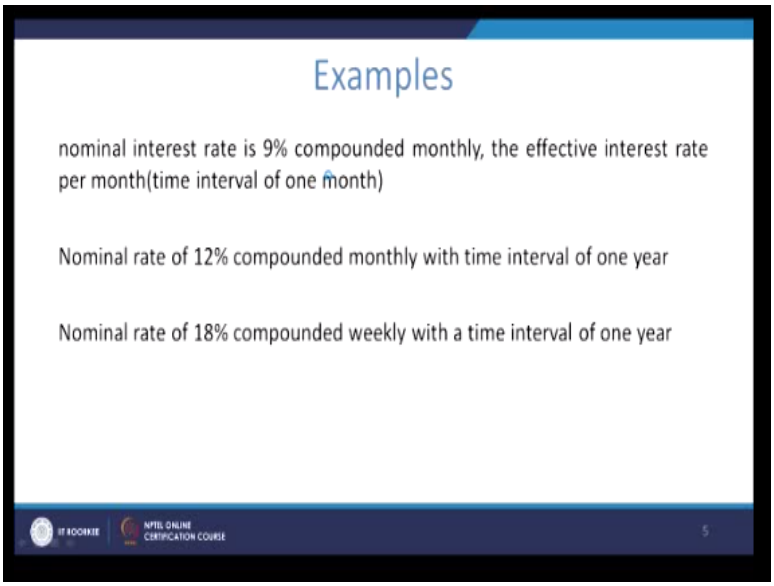

So, what we do is that we can see that how you know how we try to see that how they are changing. So, for example you can say that if suppose you have 12% of nominal rate and compounded monthly with time interval of 1 year. Now suppose you have to find basically the effective interest rate in a year and the nominal interest rate is 12% compounded monthly.

Now in this case if you look at this you know formula, so what we see is that your length is so if you see now in this case your l, l is your length of the time interval in years. So, from here you can directly write l will be 1 year because you have to find the effective interest rate in the year. Now m, m is reciprocal of the you know, so that is shown that m is basically the reciprocal of the length of compounding period.

Now compounding period is basically here compounding period is 1 month now n is reciprocal of compounding period in years. So, here in this case it is 1 month, so 1 month is nothing but 1/12 years, so it will be reciprocal of it. So, it will be you know 1/1 month basically 1/1/12 years, so it will be 12. So, your l becomes 1 and m becomes 12, now you have to fit in to the equation which is basically you know derived or which is given to you that the effective interest will be 1+r/m raise to the power lm and then -1.

So, if you take the effective interest, effective interest for this 12% interest compounded monthly and for the interval f 1 year it will be 1+your you know interest rate is nominal interest rate is 12%, so it will be 0.12. And m that is your reciprocal of the compounding period in years that becomes 12 and then it raise to the power l*m. So, we know that l is 1 and m is 12, so it will be 1*12 and then -1, so it will be basically 1+0.01 raise to the power 12-1.

And if you do that you will be getting 0.1268 that is 12.68%, so that is how you should not get confused with this way of presenting this interest that in this case if you see. Now this is your interest of 12% compounded monthly means every month end you will have the compounding, so basically 100 rupees if you have 1% will be compounding, so it will be 101 rupees after first month.

Then again 102.01 rupees in the second month like that it will go and if you calculate for the 12 month then in the end of 12 month it will be 112.68 rupees and that is why your effective interest for the period of you know 1 year it becomes 12.68%. So, that is how we basically calculate the effective interest for any you know period when your nominal interest rate is given to be calculated.

Now we are going to have the you know solution I mean examples, different examples of the you know different cases. So, if you go further and if you try to see then how it looks like so next you have nominal interest rate of 9% compounded monthly the effective interest rate/month. Now what will be the effective interest rate of 1 month, now if you try to see this question it is very clearly seen that the nominal interest rate is 9%.

And you have to find the effective interest rate in a month, so effectively you are so it is written that it will the interest will be you know 9/12% every month you have to use 9/12%. And since you have to apply only in a month, so is very simple that it will be 9/12% so it will be 0.12 % but how can we use the formula to get it.

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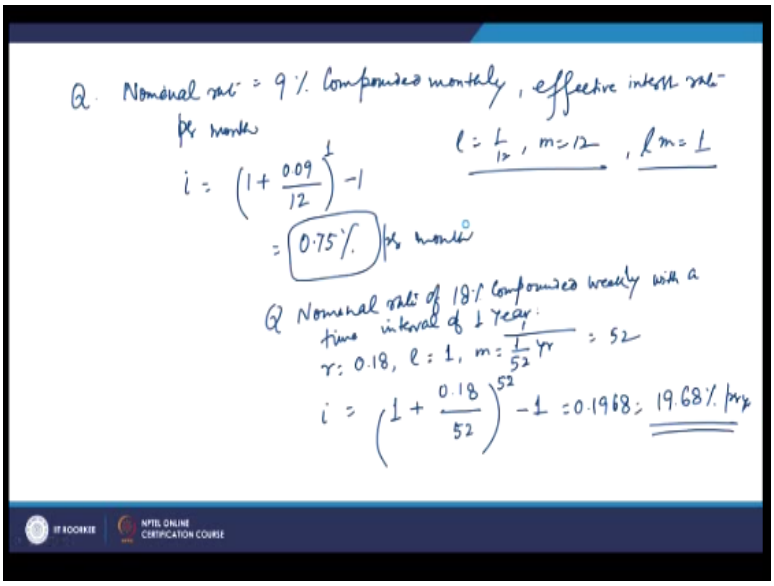

So, your you know question is that nominal rate is 9% compounded monthly and effective interest rate. So, you have to have effective interest rate/month, so in that case if you look at that, so effective interest rate will be you know1+. Now in this case your nominal interest rate is 9%, so it will be 0.09 then you have to have per month, so compounding period is you know 1 month.

And it is reciprocal will be basically you know, so it should be 1/12 years, so it will be 12. Then you know l*m, so l is your time that is your 1/12 because you have find effective interest rate/month. So, l is 1/12 and m is 12, so your l*m becomes in this case 1, so it will be 1 and -1, so if you calculate the value it will be 1+0.09/12-1, so it will be 0.9/12 that is you know 0.75%, so it will be 0.0075.

So, it becomes 0.75% as usual it is very simple that when you are defining itself that 9% interest rate and then compounded monthly. So, it is basically the effective interest rate for every month and only we are talking about every month. So, that is how we calculate it, next we can have the equation.

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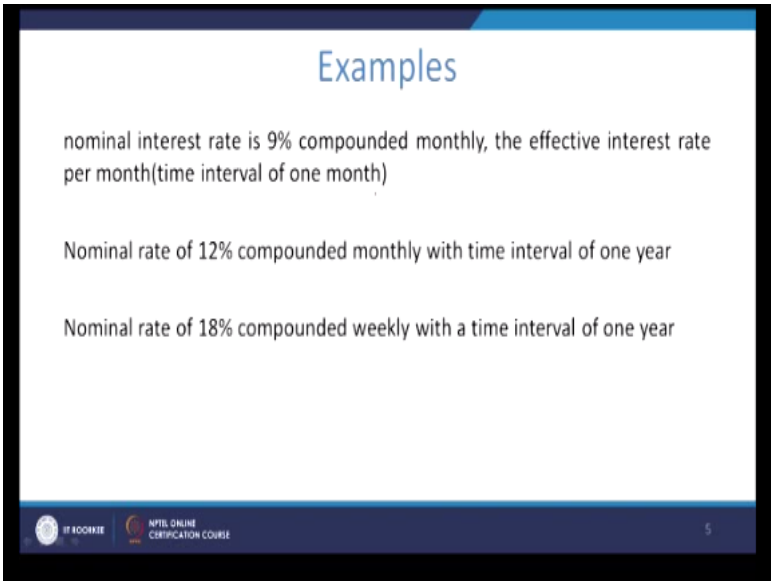

Next is that it is nominal rate of 12% compounded with time interval of 1 year we have already done it. We have see that 1+0.12/12 and then raise to the power 12-1, so it becomes 12.68%, next is that you have nominal rate of 18% compounded weekly with a time interval of 1 year. Now let us say in this case you have nominal rate of 18% then compounded weekly with a time interval of 1 year.

Now in this case as you see that you have r is 18%, r is 0.18 then you know l is given as you know 1 year, so it will be 1. Then m, now m is basically compounding period is weekly, so weekly means it is 1 week, so 1/52 you know year. Because if you assume that there are 52 weeks in that case you have m is 1/52, so this is reciprocal of this, so it will be you know 52. So, now you can see that your effective interest will be, now in this case 1+r, r is 0.18 then by m, m is your 52.

Now l is 1 and m is 52, so l*m is 52 and then -1, so this is how you are going to calculate the you know compounding you know compound interest for 1 year effective interest for the whole year. And that if you calculate it will be coming to 0.1968, so it will be 19.68%. So, if you see that in a normal case you are having 18% compounded you know 18% becomes if you compounded weekly. Then it is value becomes equal to 19.68%, so that is how we calculate these values, next we will further see the next question.

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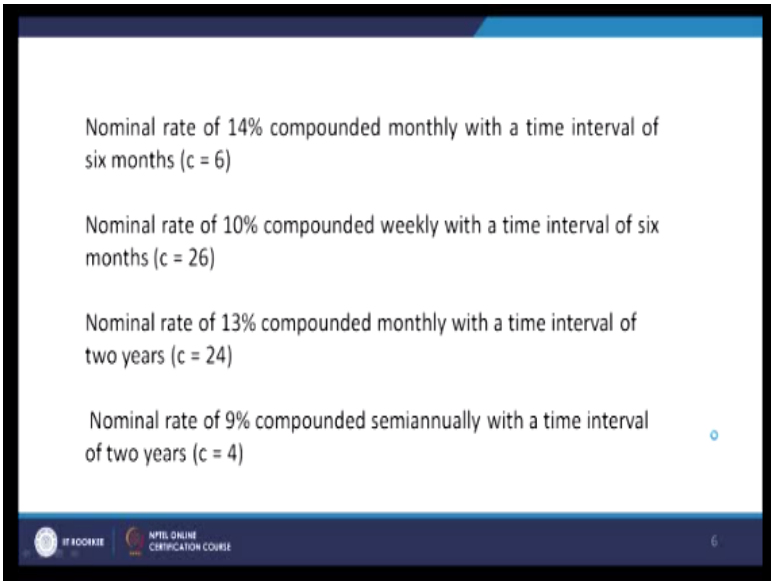

Then you have nominal rate of 14% compounded monthly with a time interval of 6 month. So, now we have to again see this different type of you know problem.

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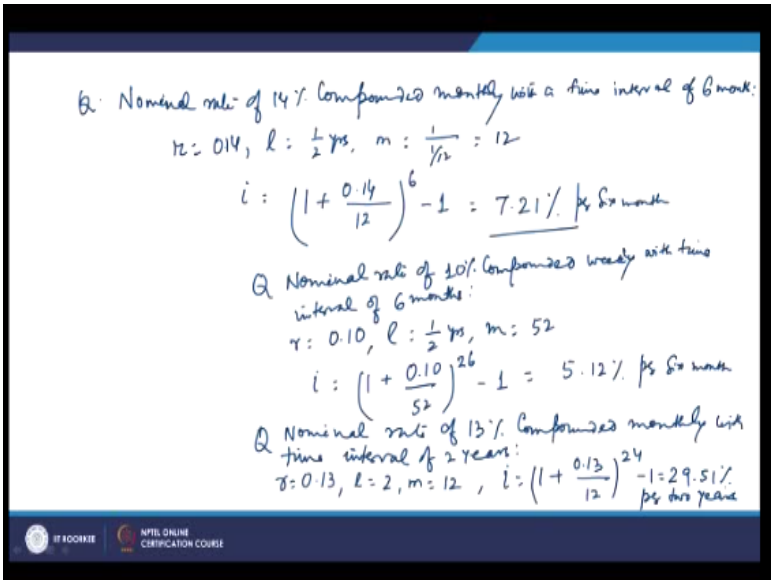

So, **so** this problem is that nominal rate of 14% compounded monthly with a time interval of 6 month. Now in this case as we know that your r is 0.14 and time interval we have taken as 6 month, so you have for this time interval you have to calculate. Now what happens that this is monthly know time interval is since 6 month, so it is half of the year. So, your r is becoming as 0.14 as usual then l you have to do for how much time you have to calculate, so that is 6 month, so it will be 1/2 years.

Now m, m is since it is compounding is done monthly, so it is 1/12 years and it is reciprocal of 1/12, so it becomes 12. Now based on that we can find the effective interest and it will be 1+you know 0.14/m, so it will be 12 raise to the power l*m, so it is half*12 that is becoming 6 and then -1. So, this becomes equal to you know 7.2%, so you know otherwise was 7% in the half year that becomes 7.21% when we talk about the you know compounding. And compounding is done you know monthly, so that becomes equal to 7.21%.

Then the next is nominal rate of you know 10% compounded weekly with a time interval of 6 months. So, this will have again in the different way we will solve. So, this is nominal rate of 10% and then this is compounded weekly with time interval of 6/months. Now as you see that in this case again you have r is 0.10, now is your time interval for which you have to calculate the effective interest is 6 month.

So, it will be 1/2 years 1/2 then m is it is compounded weekly, so reciprocal of that in years, so weekly means 1/52 years, so m will be 52. So, you can have the effective interest rate as 1+r/m, m is 52 raise to the power l*m, so l is 1/2 and m is 52, so 1/2*52 is 26-1 and this becomes equal to 5.12%. And this is per month we will have to write this effective interest, so here it is again per 6 month.

So, this is the effective interest we must write down because here also we are this is per year. Now similarly we are this is per month, so this will be per month like that we are calculating every time. So, you have to basically mention what for what you know time you are calculating the effective interest. Now next we may have the you know further you there is a problem of nominal rate of 13% you know compounded monthly with a time interval of 2 years.

Now again we can see another type of you know example where it is rate is 13% compounded monthly and you have the time interval of 2 years. So, you know nominal rate of 13% compounded monthly with time interval of 2 years. So, in this case what we see is that your r becomes certainly 0.13 and compounding is done monthly and your time interval is 2 years, so, l becomes 2 years.

Then m it is done monthly compounding, so m will certainly be you know 12 because it is 1 month, so compounding is done every 1 month. So, 1/12 of the year and that becomes you know 1/1/12, so it will be 12, so if you find the effective interest you know it will be becoming 1+0.13/r/m, m is 12 raise to the power l*m, l is 2 and m is 12, so it will be 24 and -1, so this will come out to be 29.51%/2 year.

So, basically we are interested to know the value of the you know this effective interest rate in the span of 2 years and for this span of 2 years your effective interest rate becomes you know 29.51%. In normal case you assume that 13%, so 2 years 26% per since compounding is done monthly so it is increasing that much full and it becomes 29.51%/2 years.

So, when your you know that period for which the effective interest rate is to be computing it becomes more and more effectively it becomes the computed value becomes more and more, that is how you calculate those. Then you have another example of you know nominal rate of 9% compounded semi annually with a time interval of 2 years. So, you can again see that if you have a problem of you know.

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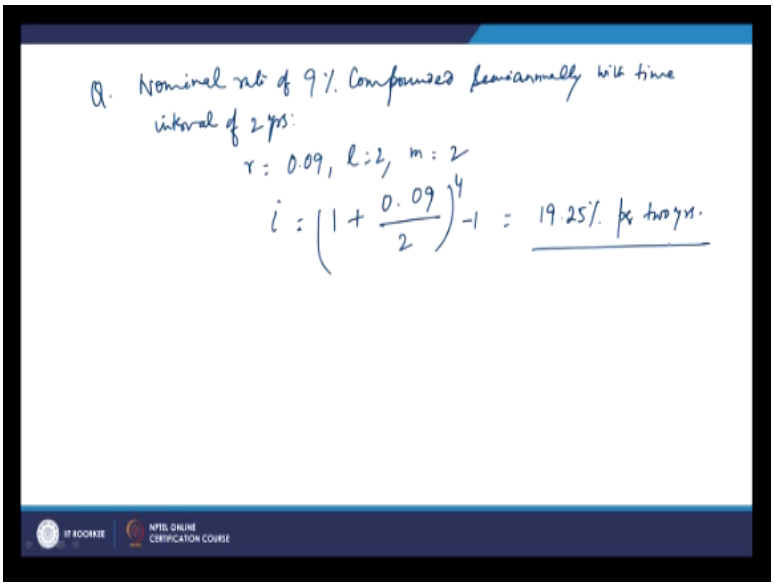

Nominal rate of 9% compounded semi annually and then you have with time interval of 2 years, so now in this case again one of things are I think hopefully you have got the confidence to talk about it. So, your r becomes 0.09 and you have again l as 2 and since it is done semi annually, so

m will be reciprocal of ½ years, so it will be 2. So, effective interest rate will be 1+0.09/m so 2 and then l*m will be 4 and -1, so it will be 19.25%.

So, basically for the 2 years it will be 19.25% that is how you are getting this is for 2 years. So, this is the you know different type of examples we have seen how to compute these effective interest values. Then you know what we see that in such cases we can quite you know clearly visualize that how you know this effective interest rates are computed and based on that. So, basically we can also do for daily basis.

So in that case you have to divide it by 365 and then you have to have the appropriate you know exponent is to be taken based on the different conditions and then you have to compute. Now in the next lecture also we will discuss about the case of continuous compounding where basically when there is infinite number of times your compounding takes place then it is a case of continuous compounding. So, that we will discuss and how the interest rate varies, so that we will see in that lecture thank you very much.