Financial Mathematics
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Lecture-15
Problems Solving on Compounding Factors

Welcome to the lecture on problem solving on compounding factors, so in this lecture we are going to discuss certain problems which you have you know to solve and it is related to the topic which we have discussed this week. And it is related to compounding factors, so we discussed about many types of discrete compounding, discrete payments factors and among them you had initially the single payment factors.

So, you had the compound amount factor single payment compound amount factor single payment present worth factor. Then we discussed about the equal payment series factors in that we had the you know equal payment series compound amount factor, equal payment series sinking fund factor. Then you had the present worth factor all that factors there also we discussed about the different types of you know gradient series like uniform gradient series and the geometric gradient series.

And how to see this you know type of problems how they are solved and how even to refer to the tables to get these factors values from the tables. So, let us start with the problem on you know 1 question.

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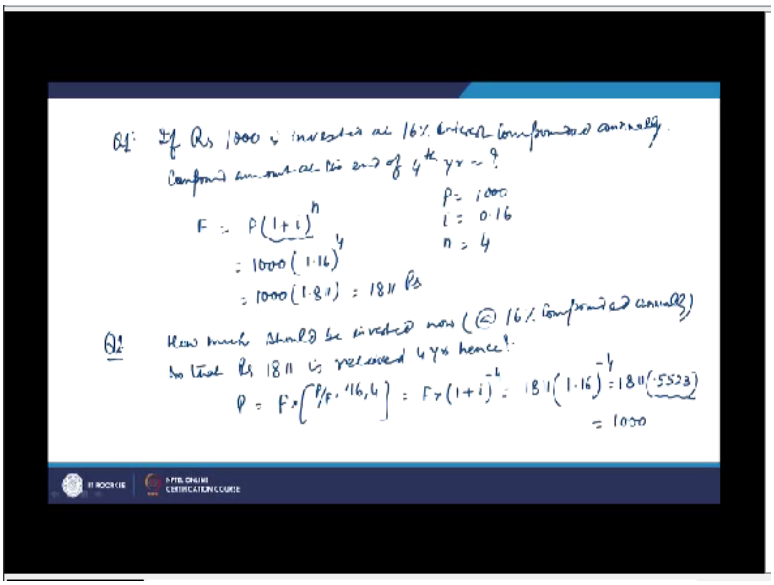

And question is that Rs. 1000 is invested at 16% you know interest compounded annually and in that case you need to know, so you know at the beginning of your one it is done. So, your compound amount, so compound amount at the end of 4th year will be what, so suppose a man has now invested Rs. 1000. And he wants to know that what will be the compound amount at the end of 4th year.

So, in that case what you do is normally this is the case where you have to use the formula F=P*1+i raised to the power n. So, basically you will have a P as 1000 and i is 16% interested compounded annually, so it is per annum interest, so it is 0.16 and n is 4. So, you can have 100, 1000 and multiplied by 1.16 raise to the power 4. So, that is how you will have the value of you know 1000*1.811 and that is 1811 rupees.

Now this is how the you know problems related to this of the compound, so this is basically the factor here 1+i raise to the power n is the single payment compound amount factor that is 1+i raise to the power n and that is multiplied with you know 1000 you get 1811 rupees. Now if you try to see you know for this now as we discussed that it is a factor which is single payment compound amount factor F/Pin and if you look at you know if you these are the tables which you can refer. So, this is basically a table which is showing the 16% you know interest value is there.

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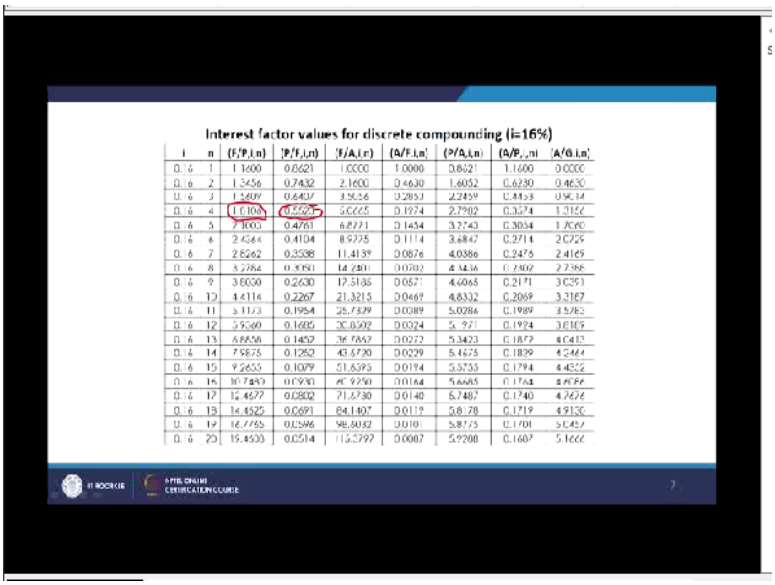

And in that this is the F/Pin factor is there and your n is 4,so for 4 basically you get the value 1.8106 that is 1.811 you can take. So, simply the P is to be multiplied with this factor and that value will give you the you know value of the compound amount that is F. So, that is how you solve such problems contrary it way you know contrary to, so this is question 1 and question 2 will be just opposite to question 1 you can have.

And it may be that how much the should be invested, now at 16% compounded annually interest rate, so that Rs. 1811 is received 4 years hence, now suppose such you know question comes in those case this is just you know the factor which is being used will be just opposite to it. And here you will have the factor use factor which is that will be used will be single payment present worth factor and that is 1+i raise to the power –n.

So, you have to get P and for that you will be multiplying F with P/Fin, so i is you know 16%, so 0.16 and you know, so you can have also value as 16 also we write in so, it will be 16 only and n is basically 4, so it will be F* you know 1+i raise to the power -4, so F is given as 1811. And this will be 1.16 raise to the power -4. Now this 1.16 raise to the power -4 comes out to be 0.5523. Now this 0.5523 this will be coming as 1000 that is what you get.

Now this factor is single payment present worth factor and this you can again refer it from here this is what you see is 0.5523. So, it is P/Fin, so that is single payment present worth factor and

that factor is used to give you the present value when you are given some future amount rate of interest and also the time. So that way you can calculate those values, next we will have a question number 3.

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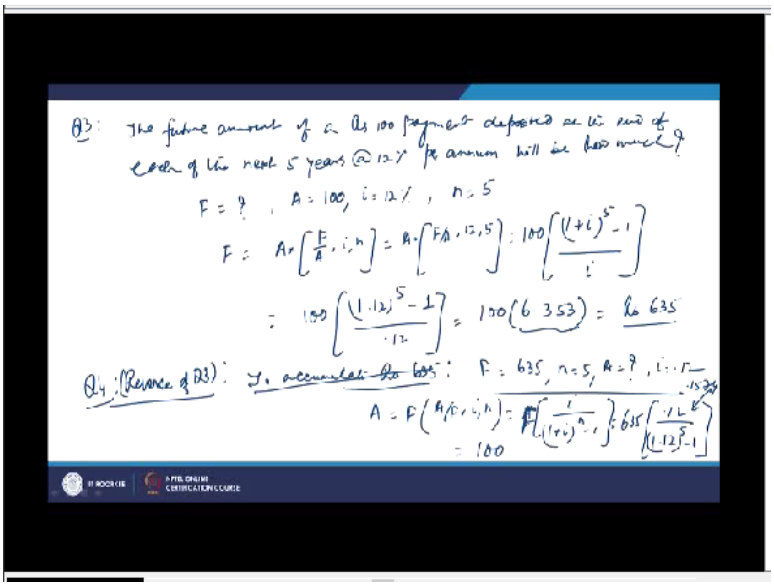

Now in this question will use the different you know factor and in this basically what we see that you know the future amount, the future amount of A Rs. 100 payment deposited at the end of each of the next 5 years, 5 years at the rate of 12% per annum will be how much?. Now in this case you need to have the future amount you are depositing rupees 100 every year end and that too for the next 5 years.

So, this is the example of the annuity basically equal payment series factor and in this case you are going to use the equal payment series compound amount factor. And we know that for that the equation has such form, so you need F and you have A as 100 i is 12% and is 5. So, in that case F will be A*F/Ain and this is basically you know equal payment series compound amount factor, so it will be F/Ai you know that it is 12 and it is 5.

So what you will do is 100 and this factor is basically 1+i raise to the power 5-1 upon i that is 100*1.1 to raise to the power 5-1/0.12, this is the basically factor which is coming out to be you know by taking this you know formula. And that comes out to be you know 6.353, so all together it is coming as Rs. 635. So, what we see that when you have to calculate how much you will you

know your final amount will be once you are depositing some amount every you know year end for certain time period.

Then in that case you are going to use this equal payment series compound amount factor and that factor value is coming as 6.353. You can further verify it when you are going to solve the equations later on you can verify this value from the you know table of the interest and that you can refer for it is for 12%.

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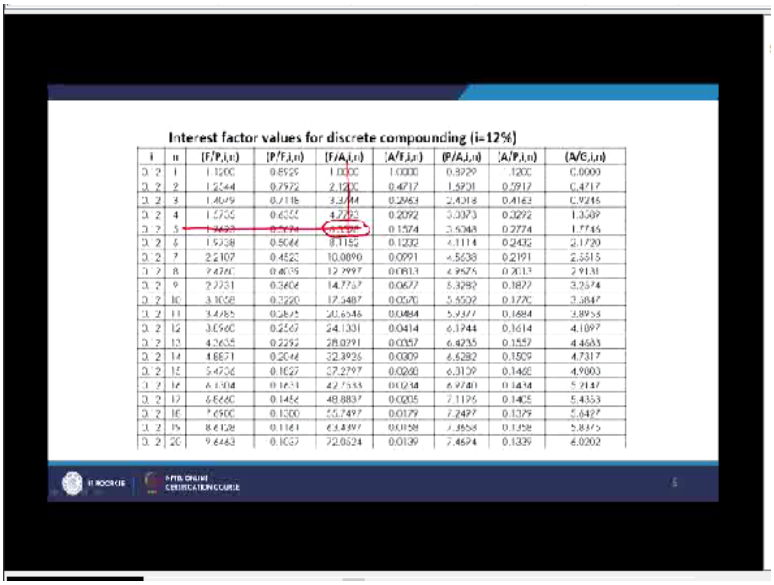

So, you go for the 12% table and for 12% table and is 5 and the factor is F/Ain, so this is your F/Ain and you will go to 5 this is your 5 and this is your F/A factor. So, this is you see this is 6.3528, so 6.353 is taken and that is how you get this 6.353, so directly you can have the value of 635. So that is how you get the future value calculation, then you will get you know you can have any again the reverse of this you know problem.

So it will be reverse of question 3 and in this case the question maybe that you know that you have to accumulate suppose to accumulate you know Rs. 635. So, basically how much you are going to deposit you know every year end for 5 years when the interest will be 12%. So, in that case you know, so in that case F is known as 635 and n is you know 5 and A is to be found out and i is 0.12.

So, in that case you are going to use A as F*A/Fin, so it will be A*i upon 1+i raise to the power n-1. So, you will be having you know this will be F basically this will be F and so you will have 635* then you will be using that 0.12 of 1. 1.12 raise to the power 5-1. So, basically if you do that you will be getting 100. Now in this case the factor which you will be getting will be 0.1574, so this factor comes out to be 0.1574 and that you can see by looking at this number 0.1574.

So, **so** this way you can use these tables also to find these factors but primarily you must know that how this value comes. So, these value are coming because of certain formula that formula you know we have studied and based on that formula you can have the value of these factors being calculated. Now next will be solving a question, now this question is talking about you know finding the value of you know A when the piece given like Rs. 1000 is invested at 15% interest compounded annually.

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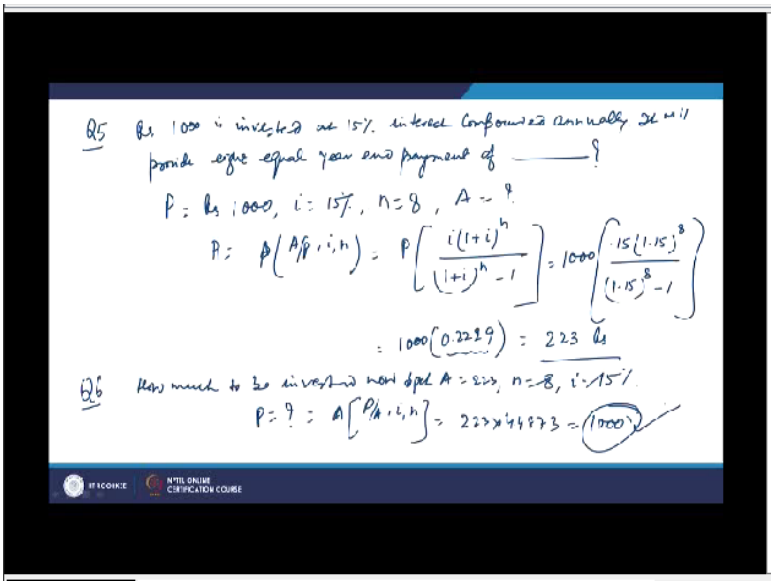

So it will provide 8 equal year end payment of how much, so you know you required to see that you have invested 1000 rupees and you want to have the equal year end payment. And the interest which you are getting is 15%, in that case how much you know year end payment will you be getting. So for that to you know you have to find, so what is given is your P value is given as Rs. 1000 you know i is given as 15% and 8 equal year end payments, so it how much will be there.

So, you have to have A as what so we know that P will be A* so A will be basically P*/A/Pin, now as you know that A/pin will be this factor is basically given as i*1+1 raise to the power n/1+i raise to the power n-1. Now this factor if you try to find, so it will be 1000* i is 0.15 1+i is 1.15 raise to the power 8 and then divided by 1.15 raise to the power 8-1. So, this is what the equation will all factor will look like.

And this factor if you calculate it will be 0.2229, so it will be something like 223 rupees, so what is series that you are going to get 223 rupees every year. And if you are investing rupees 1000 now, now this factor as you know this is A/Pin and i is 15%, so further you can refer to how to see that.

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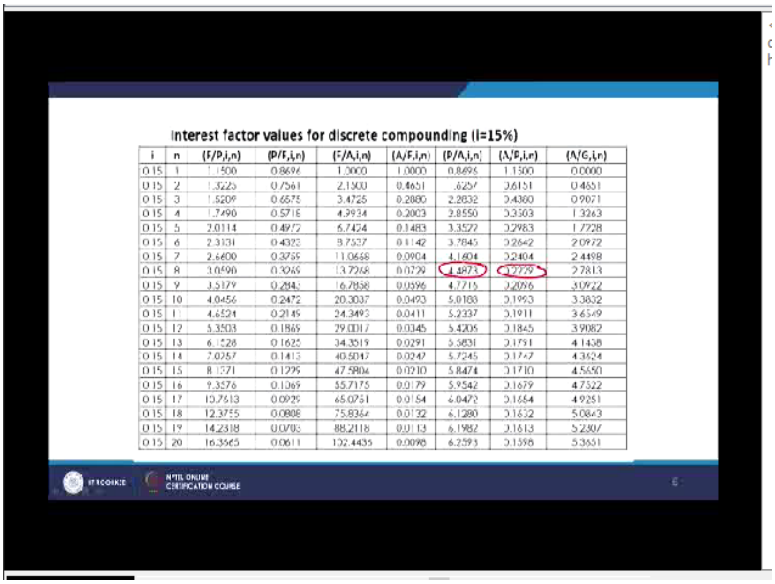

So, you go to 15% interest payable AP/in and n is basically 8, so this is 8, 8 will come here and this you are getting. So, this way you can get the factor and you can use it for calculating, so you can have you know just reverse of this question also, you can use you know for finding that if you are you know you have you are giving this 8 series of 223 rupees payment how much should be you know invested now.

So how much should be invested now for that your P required, so how much to reinvested now, so that you get A as to get A as 223 for 8 payments, or 8 you know year end payments and i is 15%. So, in that case what you do is you have to get P and P will be A*P/Ain, so in that case this

will be should be reverse of that and that will be 0.4873 I mean 4.4873 and so once you multiply this number you know 223 with 4.4873 it will give you 1000 rupees.

So this is reverse of you know reciprocal of this, so it will be giving you 1000 rupees, so and that also you can see if you look at the P/Ain. So, P/Ain will be 4.4873, so basically this is how you make use of such factors and try to get you know the values calculated. Then now we will have the use of the gradient problems, now let us say that you know a person is planning to save Rs. 1000.

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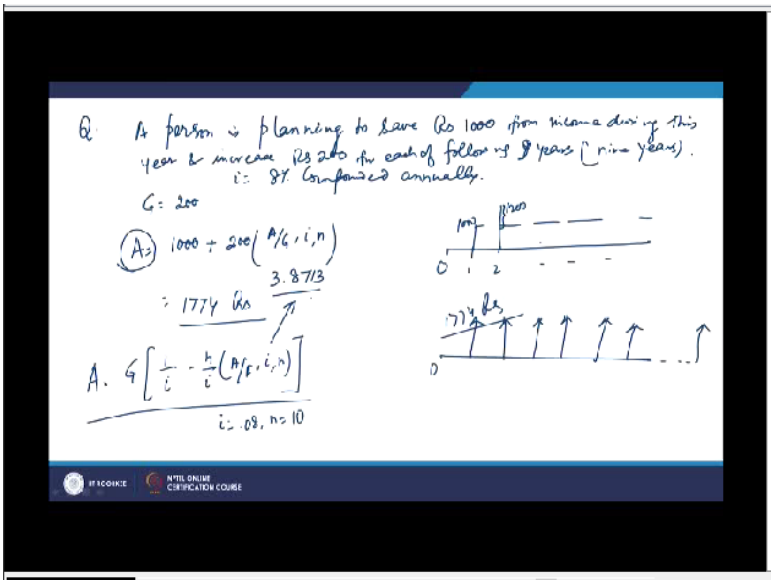

So, a person is planning to save Rs. 1000 income during this year and increase rupees 200 for each of following year following 9 years. You know as we know that this person for this year and he is going to save 1000 rupees and then next year he will be 1200, next 100 1000, next year it will be 1400 like that. So, he is going to save 200 rupees more every you know year. Now for you know 9 proper years for 9 following years in each of the following 9 years he is going to save.

So, all together he is going to save for 10 years and how much he will be you know saving in the end and basically we like to know that you know what that should be equal to you know his saving you know Rs 1000 every 1000 and then 1200 and then you know 1400 and so. Now this should be basically equated to how much every month of the every year of the savings.

So, you have to get basically the equivalent you know annual series annual amount for particular series. Now in that case what you do is that, so as you know that you will have this series, so you will have 1 and 2 and that, so in that first one is saving something like 1000 and then you will have 1200 also. So, like that it may go savings, so it may go like bottom most also but anyway we can think that ultimately how much it will be accumulate in the end that is what we have to calculate.

Now for this as you know that in this case you know that your G value is you know 200, so your increase of amount is basically 200. Now the thing is that the equivalent annual amount will be 1000 is the base, so he is going from there and then this G is basically 200, now what he had to do is that this G value is to be multiplied with A/Gin. So, you have to multiply this 200 with A/Gin, i is you know 15 i is given as 8% compounded annually.

So rate of interest is 8% compounded annually, so in this case what you have to do is you have to find this A/Gin that also already we have seen and if you look at this A/Gin.

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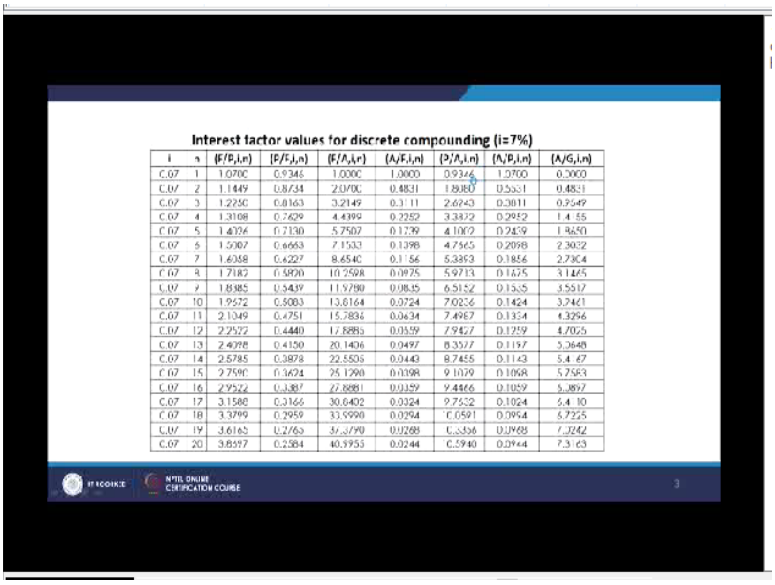

Then you can go to this 8% interest table and this 8% interest table is anyway not given, so but you can calculate it and this interest is 7% is given.

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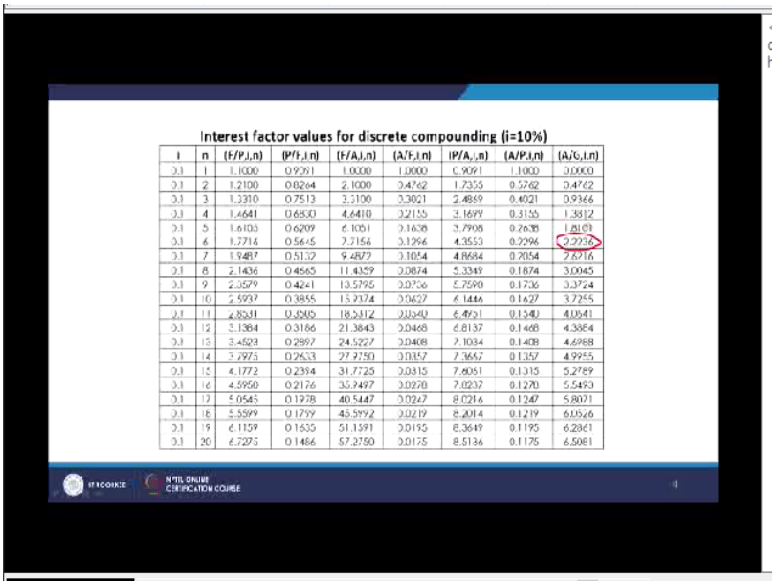

And so you can have that table of 8% and 8% table basically is coming out to be 3.8713, so ultimately it will be coming as 1774 rupees. So, basically this 1774 rupees for the 10 years will be same as 1200, 1400 up to go up to 10 years. So, it will be same as that and that is how you can have this A, then you can find its you know present value or its future value that can be calculated. So, this is how you calculate for these arithmetic gradient series method.

And that way this is the equivalent annual series, so it will be same as you know it will be same as of 1774 rupees. So, it will be going up to n=10, so it will be same as this 70 1774 Rs. 1, so basically this way you can equate this value. So, even this as you know that this A/G factor basically you know that A=G*1/i-n/i*A/Fin. So, that from here you get this A/Gin, so that G is multiplied with this and this value will come out to be 3.873 from here for you know.

So, A/F you know that A/F will be a upon 1+in-1, so that way n/1+i n -1, so if you put that i as you know 0.8 and so it will 0.08 and n will be you now as used to me that this is 10. So, you will get this factor and that can be calculated and if you see this a A/Gin that is seen from this table. They are from you will get this 3.87713, so that way you can have the calculating these values, then you can have similar problem for the geometrically decreasing series.

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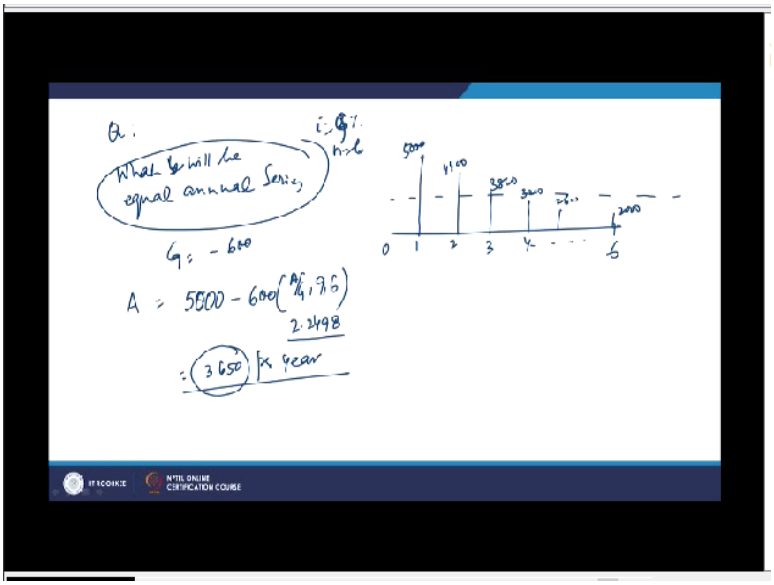

And you may have a question like you have to have the equivalent annual series and for that you know cash flow is shown like this that in the first year your payment is 5000 and second year it is 4000 you know 400 and third year it is, so 6600 it is less 3800 something like that. So, it is going up 6 years, so in the 6th year certainly it will be decreasing 3200 then you will have you know again 2600 and then 2000.

So, it will be 4th is 3200 then you have 2600 and then you have 2000, now for this what will be the you know equivalent annual series, what will be equal annual series. So, in this series as you know that the amount is not constant it is decreasing, so what will be that series which will be giving the same equivalent value. But it will have the equal payment it will be something like here but you need to know by looking at the factor.

So, again for that you know in this case you are going to have the value and here i is given as 6% compounded annually and as we know that n is 6. And your G value is basically – you know 600, so what you say is in these cases what we get is that you will have you know equivalent annual amount this will be, so in this case your base amount is 5000 basically. And in that it will be decreased 600, so it will be again A/Gin. So, it will be a A/G 9 6, so for 6% whether we have the interest you know rate yes it is given.

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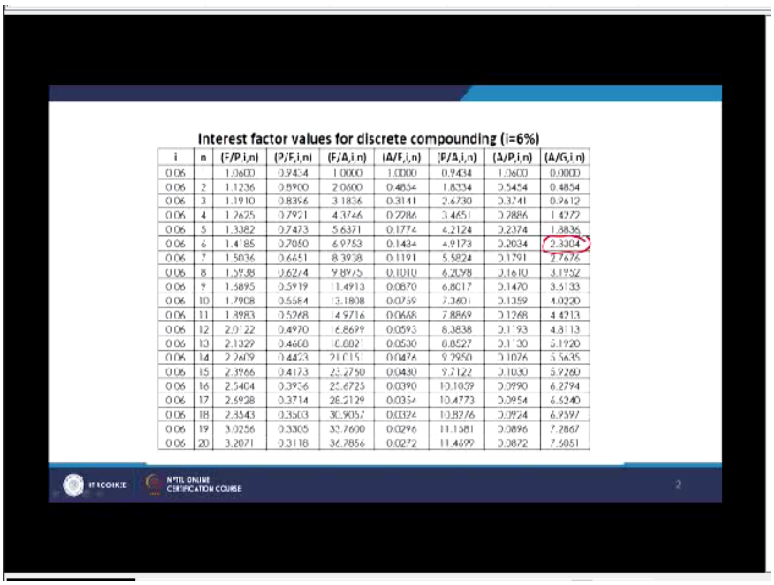

So, if you look at the A/Gin for 6% it is here 2.3304 A/Gin now if you look at this it will be so A/G this is for 9 you know i is 9% not that it will be i is 9% and n is 6 i is 9% is given. So, for 9% we have to see, so will go to 7 and 10 any way we have still not 9 % and now10% if you look at 10% for 6 it is given 2.2236. And for 8% for you know 10% it is 2.2236 but if you calculate for 9% it will be 2.2498.

So, it is this value which we are getting and if you calculate that it will be 3650 per year, so basically what we see is that I mean again you can calculate it from that formula 1/you know we have seen that 1/i-n/I A/Fin. So, if you calculate this for that 9% value then you will get this factor value as 2.2498 and that value basically come out to be 36500. So, from here you can get that this feeling that for if you use deposit this 3650 every year.

And it will be same as the same series 5000 and decreasing series of 600 rupees every month and so that way you can solve these types of problems and get yourself you know more and more acquainted with you no such kind of problems and more and more confidence building will be occurring in due course of time, thank you very much.