Financial Mathematics
Prof. Pradeep K. Jha
Department of Mechanical & Industrial Engineering
Indian Institute of Technology-Roorkee

Lecture-14
Annuities Due and Deferred Annuities

Welcome to the lecture on annuities due and deferred annuities, so in the previous lecturers we discussed about the compounding factors and we also discussed about the factors related to F and A or A and F where A as nothing but annuity. So, annuity basically is the you know equal amount monthly or yearly which is paid you know in any type of cash flow diagram. Now there are cases, so **so** as we know that these annuities are normally assumed to be paid at the year end.

So, when we are discussing about you know cash flow transactions in the universe financial, you know sector then normally it is assumed that it is at the end of the time period. So the first payment will be at the end of you know the first year, so if we are starting from the from now there once 1 year is over then we are supposed to have first payment. So, that is the standard practice when we study these annuities.

Now many a times what we see that these payments are basically done, so basically at initial itself. So, that is what the annuities due that case belongs to, so annuities due means it is a just like ordinary annuity except that payments occur at the beginning of each term instead of at the end example insurance premiums, property rentals etc,.

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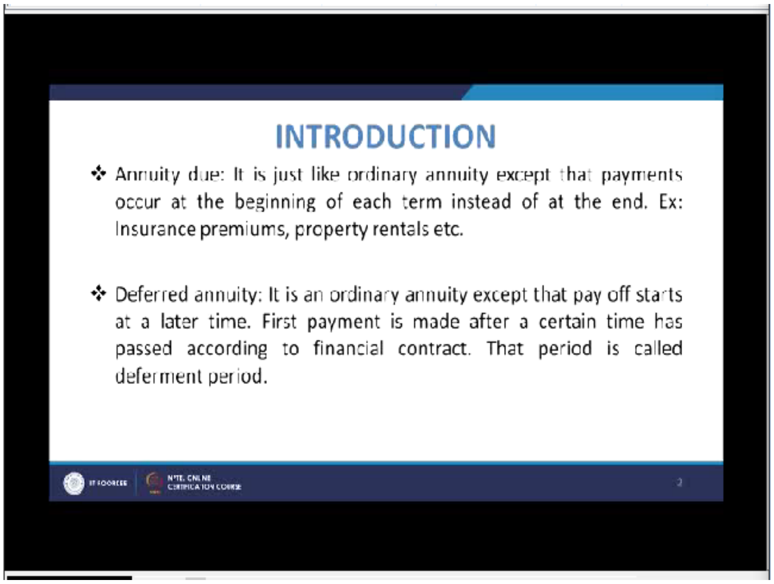

So, they are you have to have certain modifications. Now before that once we so as we know that you have to give, so just like insurance premiums you are giving the insurance you know amount at the beginning itself or rentals you know you are giving at the initial of the month itself. These are those examples, before that let us have a quick look out at the you know derivations which did for the annuity.

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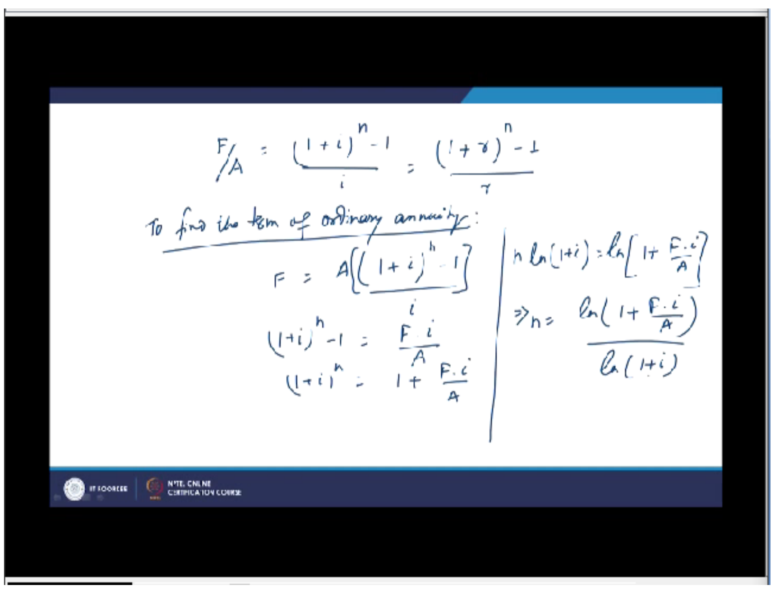

And we know that F/A factor was there and it was basically 1+i raise to the power n-1 upon i or many times you will see that it will be represented by term r also. So, 1+r raise to the power n -1/r, so that is what is the F/A and this is equal payment series compound amount factor and that is how we define it. Now if we have to find F then we know that F will be A times this factor similarly if we have to find A, A will be I upon 1+i raise to the power n-1.

So, that way you can have you know values of these any of the quantity but then we may required to find the term. So to find the term of ordinary annuity, so if we are talking about the annuities then to find the term means you have to find the value of n. So, as we know that it is F is basically A*1+i to the power n-1 upon i and you have to have the you know value of i. So, this A is basically whole divided by, so if you look at that what will be there 1+i raise to the power n-1 will be F*i upon A.

Then you will have 1+i raise to the power n will be 1+F*i upon A. So, you will be taking the log on both the sides and if you take the log, so n ln of 1+i will be ln of 1+F*i upon A. So, you can get n as ln of 1+F*i upon A/ln of 1+i. So, this way you can get the term of the ordinary annuities and there maybe questions based on some such kind of annuities and you can find the solution for such annuities like you may have you know for a business.

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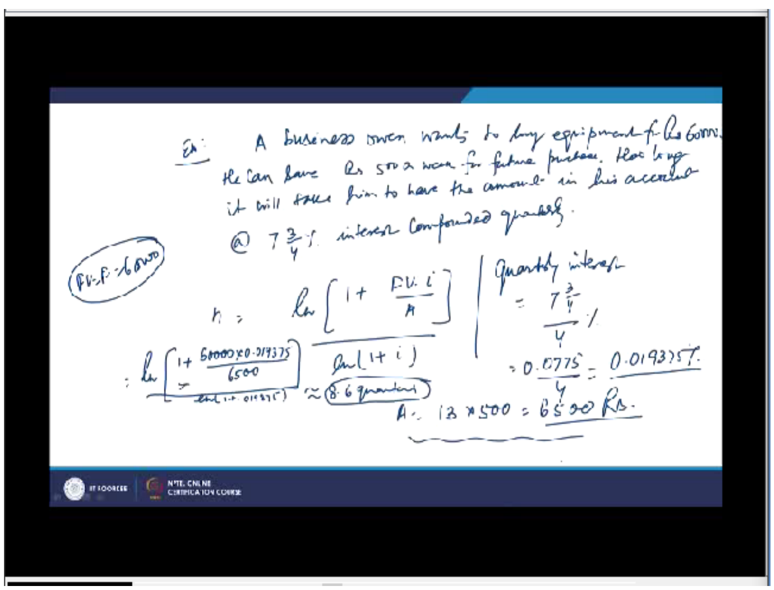

Business owner wants to buy equipment for Rs. 60000 and then you know he can save Rs. 500 a week for future purchase. So, how long you know it will take him to purchase to have the amount in his account, you know then it is given that at the rate of 7 3/4% interest compounded quarterly. Now these are there are certain some data which are you know where we need to have more of the its understanding maybe in our later lectures.

Because the interest rate is not compounded annually or not compounded weekly basically it is compounded quarterly. So, you will have to have these interest you know in the term of you know accordingly. Because you have given as it is 500 a week and then he has to have this you know amount for that is 60000 is to be deposited. So, accordingly we have to adjust the you know rate of interest. Now what we see that you have to have this 7 3/4% interest compounded quarterly.

So, for quarterly interest it will be 7 3/4 and divided by 4, so that will be basically the interest per 3 months, so that will be there. And then he has to deposit this amount 500 a week for the future purchase, so accordingly you have to have that now the thing is that again you have to have this n as we know that you must have the value of n to be fitting into ln of as we know that you will have to have n as ln of 1+FV*i upon A/ln of 1+i.

So, this I basically you have to have the effective interest and accordingly you have to put in. Now here you assumed that the his savings 500 rupees a week. So, basically his you know going for 1 quarter up to that his saving and then he is putting in the bank. So, that he can I get the interest, so based on that if you take quarterly interest. Quarterly interest will be basically 7 3/4/4%, so it will be point you know it is0 0775/4 further, so it will be 0.019375%.

So, this is for per quarter ok and then in a quarter we assume that his saving 13 times the amount, so the amount you know A, A will be 13 times 500. So it will be 6500, so 6500 rupees is saving for quarter enough, in every quarter he has, so that interest is basically this much and in that case he can you can put it here. So, it will be ln of you know 1+ you know this is F final value o F itself you can have FV is nothing but F that is your 6500 no not 6500 this is 60000.

So, it will be 1+60000 and then into I, so it will be 0.019375, then divided by A, so will be 6500 and then you will have this ln/ln of 1+0.019375. So, this way you can have these values you can calculate and what you if you calculate these values you will get something close to 8.6 quarters.

So, in 8.6 quarters basically that person will be able to get this much. So, if you look at you know in otherwise 6500 he is accumulating 1 quarter.

In normal case it will be something like 52000+3+4 something like 56000 and then suddenly because of the accumulated interest he will be getting 60000 so, this way you can have the value of the you know the number of terms you know for the ordinary annuities. Now we will talking about the value of the annuities due before that you can also have the finding of interest rates from the same you now formula. You can also have the equation for the interest rates also on the annuities, so that can be calculated from the normal formulas, now coming to the annuity due.

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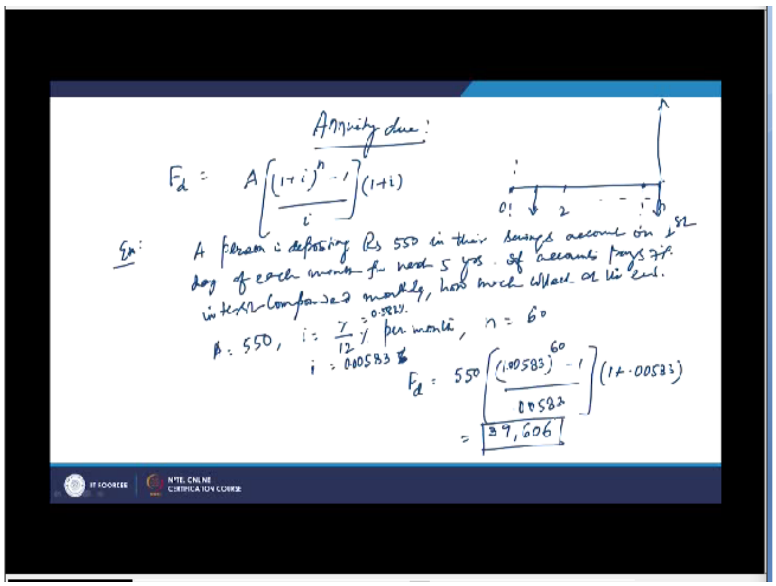

So, annuity due as we discussed that here now we assume that you are starting the value that the deposition or depositing the amount or transaction at the beginning of the time itself. So, in that case what will be the future value, so basically what is there that you have one more extra period and in that case the future value will be what you calculate using ordinary method. And if the r is the interest then you have to multiply it with 1+r.

So that value will be changing and this annuity due, in that case the future value Fdr or fvd that will be basically on normal case you have A*1+i raise to the power n-1 upon i, so this is in normal case. Now this you have n payment and you are starting from the beginning itself, so basically the you are getting in normal case what happens that you know you are not you are starting the payment from here and going till here and then getting final here itself.

In this case you are starting from here and going up to here, so that in the end you will be getting the same amount. So basically you will have the difference, basically whatever you have future value here being calculated it will be multiplied with, so it will have one more interest period to its account. So, you will have to multiply with F/p factor F/pin that is 1+r, so in is 1 itself, so in that case you will multiply with 1+i.

So, that becomes the case of annuity due and the we can see one example suppose that a person is depositing, so a person is depositing Rs. 550 in their savings account on first day of each month for next 5 years. So if account pays 7% interest compounded monthly how much they will collect at the end. Now this person as you know that he has started saving on the first day itself, so and he will be depositing that for 5 years.

And he will be in the end he will have some money which is deposited, so in this case what you see is that A is basically 550 and you know interest, so it is he is paying monthly. So, you will have to have the monthly interest now the interest is 7% compounded monthly, so 7/12% you know compounds per month. So in that case number of turns becomes it has 12 month. And then for 5 years, so 12*5 it will be 60, so now he will be getting that in the end, so you will be calculated and now this will be basically 0.0583%.

So, that will be the amount now if you calculate you put that you know in the equation, so this will be annuity due you know future value due in that due cases, so it will be you know 550, so it is A, so that is 550*1+i. so it will be 1.0583 raise to the power 60-1 upon 1, so that is i, i is 0.05, so it will be basically 7/5% per month is you know 7/5 is itself is a 0.583. So, it will be 0.0583 it is basically 0.583%, so it will be, so this is i is 0.00583, so it will be 0.00583.

So, this will be also 1.00583 raise to the power 60-1 by this and then again you will be 1+i, so 1.00583, so this amount being calculated that will give you the amount and this comes out to be 39606. So you can compute the value and you can get these values, so if normal circumstances if you look at 550*12, so it is 6 for 1600*5 it is 33000. So, but with interest you know earned he is able to accumulate 39606 rupees in this case.

So that is the you know basic you know difference when the cases is of annuity due and we calculate the values in such cases in this very manner. Now further when we have to calculate you know current value in the case of annuity due, so you know in that case if you have to find the current value.

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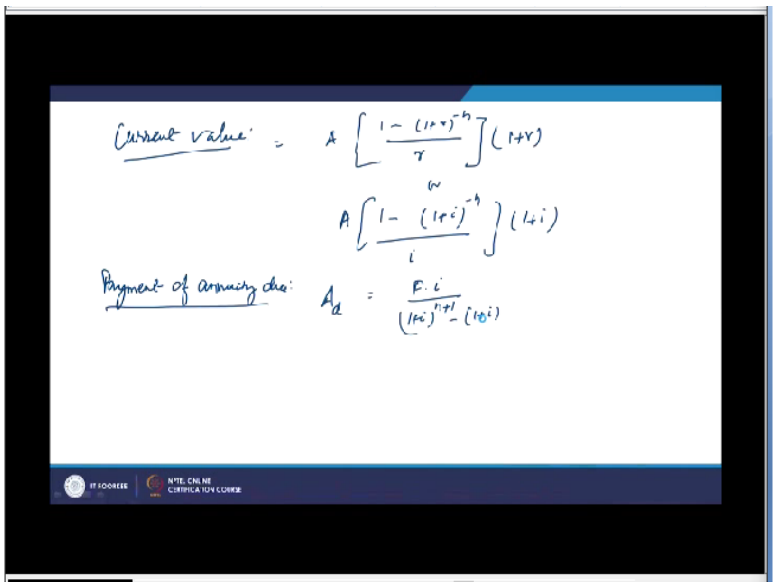

So, you know in the current value for these due cases will be you know A*1-1+r raise to the power -n/r*1+r, so basically it is nothing but you are you know dividing it by 1+r raise to the power n. And in that case you are getting that, so 1+i or 1+r whatever you can say or A*1-1+i raise to power – of 1/i*1+i, so what happens that here for getting the value you have to divided by 1+i raise to the power n.

So, in that case 1+i raise to the power n-1/i the into so and that too divided by 1+i raise to the power n, so it will be 1-this term and then this as same something other thing whole one. So, that way you can have the value of this current value in such cases being calculate. Now similar that we can also have the finding the you know payment of annuity due. So payment of annuity due and in that case you have to find the value of A.

So, as you know that if you know the F then you have to simply get the A from there and if you find A that is for due cases. So, we will have the subscript d for those cases and it will be F*i upon 1+i you know raise to the power n+1-1+i as you look at you know the earlier cases. So, now in this case as you see it will be Fd*i/1+i raise to the power n+1-1+i. So, that is what from here you can get these values.

Similarly you can have also the term of the annuity due cases also can be studied by finding the value of n in such cases. So, these are you know the only thing is that you have to have the concentration of 1+i or 1+r whatever you take that you know nominal interest term. So, that is to be taken into account and that is to be you know considered while finding other F or CV or ed, Fd or CVd or AD. Now we will talk about the case that is your deferred annuity case.

Now what is deferred annuity, now in the case of deferred annuity it is nothing but you know the same annuity ordinary annuity. But the pay of starts at a later time, so basically the payment which is starting its starting at a later time. So, you are deferring it by you know certain period, so first payment is made after a certain time has passed according to the financial contract and that period is called the deferment period.

So as you know that in many cases we talk about those periods where these you know we have to defer this time like we take the loan in many cases and we tried to pay the loan after few years. When we are thinking that we will getting job and then will be able to paid back, so that is the case of you know this deferred annuity. Now in these case what we have to do is that you know the simplest way will be that you find the you know current value or the principal amount or present value basically at the beginning time before taking deferred you know period into account.

And then also you take the you know annuity values for the deferment payment period and you can subtract you know the amount of P value which is resulting because of the deferment period payment. Because that has not actually taken place, so you can you know calculate these 2 values and then subtract one from the other. So that is your you know deferred annuity.

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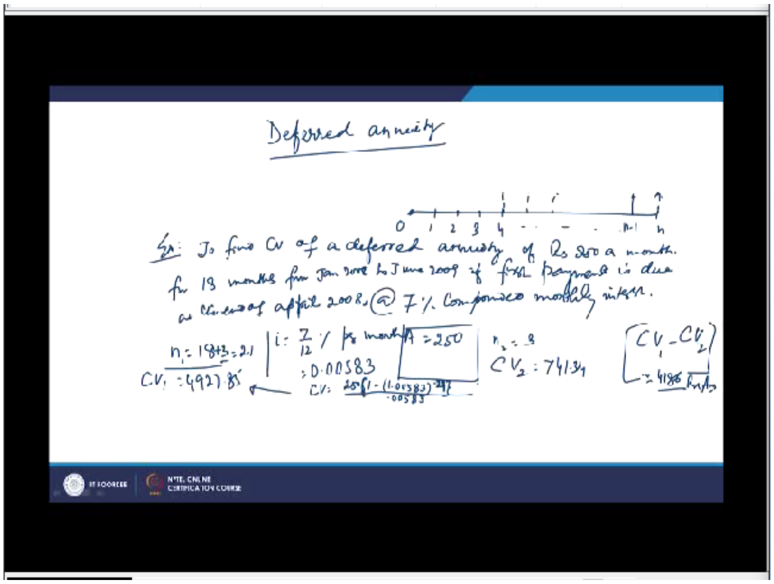

So what you have to do is that you know if the first payment is basically made at the 5th payment period then, so annuities set to be deferred by 4 payment periods. Because in the 5th payment you are doing, so basically you are deferring for 4 payment periods. So, you are suppose you have go to at ultimately total 20 periods, so will be finding the current value for 20 periods and then you will you finding the current value for 4 periods.

And the current value of 20 periods-current value of 4 periods will give you the current value for the whole deferred cases. Now this what I mean to say you know better that this is time 0 and this is 1,2, 3, 4 and it is going like n. Now in this case suppose it goes from here it you are starting paying like this, so basically are you know this deferred d by 3 periods, so you have to taken first of all you take for whole values.

Then you can have the for this value and then you subtract it, so that way you can get the present value, another way will be there are many ways and you can get the you know for such periods you can have the present value. So you will have be getting the present value here itself and then you can further you now map it to this place using the equivalence formula which you will we will we studying later.

But the whole concept is in that direction, so you can have one example and example is that you know to find current value of a deferred annuity of Rs. 250 a month for 18 months from January

2008 to June 2009. If first payment is due at the end of April 2008 and interest rate is at basically 7% compounded monthly interest, so what I mean to say here that basically this is the type of you know cash flow diagram which will be looking like and it has you know started from April.

So, it means it is going from the 4th period and n will be total 18 months, so basically n is 18 here you know n1 will be 18 and n2 will be maybe 3 something like that. And you have to first get the current value using this 18 value and then another value will be n2 will be 3, so we will be getting CV1 here and CV2 here and final value will be CV1-CV2. So, i if you look at i will be 7% compounded monthly, so 7/12% per month.

So, it will be 0.00583, so it will be i and you have to have this current value for the first case as well as the second case. And if you find the current values, so current value will be nothing but you know A*, so A is given as you know A is given as 550, A is 250. So, you can put in the formula A*you know 1-1+r raise to the power n/r, so that will give you the CV1 and CV1 will come out to be for approximately 4927.85.

And CV2 will come out to be approximately 741.34, so if you calculate that because you have to calculate it using you know CV will be A, so it will be 250*1-1+i that is 1.00583 raise to the power you know -18, so it will be this and that is divided by you know r. So, it will be 0.00583, so this way you will be getting this value 4927.85 and when this n becomes equal to 3 then in that case you will be having, so there is one thing which is to be you know seen in this case that the person is basically depositing for 18 you know periods.

So, this n will be basically 18+3, so that is 21 because he is giving for 18 periods and his due will be over in, so he will be getting at the end of you know September. So, it is ultimately it is 21, so then it is coming as 4927.85 and when you are putting 3 then it is coming 741 and so ultimately it will be, so as you know that since is deferring by 3 months. So, he will be going for a later 3 months ultimately he has to deposit for you know 18 you know periods.

And 3 periods is not depositing that is why it will be-21 here and so it will be if you subtract them it will be close to 4186 rupees. So, ultimately this is the answer, so that is how these problems related to deferred annuities are solved. There are many ways otherwise to solve in such cases because you can have the you know future values also got at certain time and then that can be you can have the equivalent amount at the present time or so. So, all these things can be done using these compound formulas, thank you very much.