Financial Mathematics
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Lecture-13
Geometric Gradient Series Factors

Welcome to the lecture on geometric gradient series factor. So, in the last lecture we discussed about the uniform gradient series factor and in that factor we were discussing about how the payments are made in uniformly increasing manner. So, the payments are increasing or decreasing by a fixed quantity and that is why it is known as uniform gradient series you know. And then the factor which is required to convert that gradient amount which is increasing every you know month.

So, gradient amount is not increasing basically that series is increasing, so for that you will have the equal annual series calculation. Now many a times what we see in our you know economic activities that the payment which you are making a yearend. They are increasing by a certain percentage, so that is known as so basically a geometric ratio is maintained. And that is why sub series are known as the geometric gradient series and the factors which are used for that cases are known as geometric gradient series factor. So, you know because in this case annual payments will be decreasing by a certain factor.

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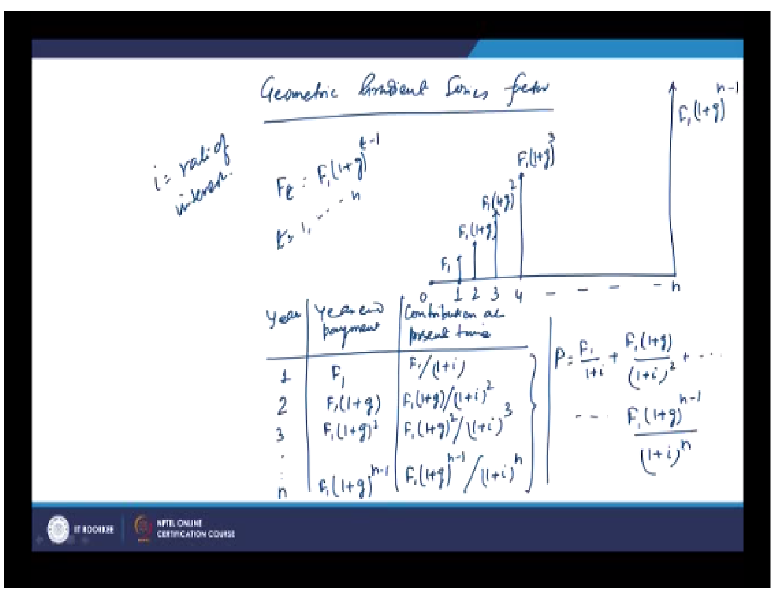

So, geometric gradient series factor, now if you look at the flow diagram of cash flow diagram of such you know cases what you see is that in the first year if your amount is what you deposit is F1. So, in the second year you will be you know going with F1*1 +g, so in the third year, so this will be 1st year 2nd year. In the third year it will be F1*1+g raise to the power 2 and fourth year it will be F1* 1+g raise to the power 3 like that.

So, in the nth year he will have F1*1 +g raise to the power n -1, so like that you will have percentage when there is percentage increase. Basically if support it is increasing by 10% it means if it is 100 rupees or 1000 rupees in the first year. In the second year it will be 1000*1+0.1 that is 1.1 x 1000 so 1100. So, that is how it is increasing, so that is why G is basically the percentage increase and that way this is increasing so you will have a basically not a linear increase.

But basically it will be increasing you know as is shown it is a nonlinear fashion. Now for that how to find the equivalent you know amount how to calculate and that is how you know how will do it that will understand you try to understand in this way. So, what we do it so what we see that in any year we get Ft at any T will be F1*1+G raise to the power T- 1.

This is what is your amount of you know financial transaction or money receipt or disbursement going on and T varies from you know 1 to n. So, that is how we go about it, now what we see is for such cases what we try to find, we try to find the equivalent you know P amount for such cases the present worth will be computed. Now how to do that, so you can do like this we have again we can find a table so it will be year.

And then you will have year and payment and then you will have contribution at present time. So, what we see is that in the first year yearend payment is you know we know that it is F1 and it is since we are talking about you know now here in this case i is the rate of interest. So, this yearend payment in the end of year 1 it is equivalent value at 0 time because this payment F1 is at the end of year 1, so it is value it is present value will be F1 upon 1+i.

So, that is what it is contribution is at present time, so suppose you take 2nd year 2nd year end payment will be F1*1+g. And if you look at this it is 2 you know there is a 2 years of time, so it is present value will be F1*1+g and divided by 1+i raise to the power 2. So, that is how it goes, so 3rd year it will be F1*1+g2 and it is again you will have 3 time periods which are there in between. So, it will be F1*1+g raise to the power 2/1+i raise to the power 3.

So, this way it will go and if you go to nth period then as you know that it is F1*1+g raise to the power n and -1 and this will be F1*1+g raise to the power n-1/1+i raise to the power n. So, this is how you are trying to get the present value of all these you know payments which are made at this discrete times and then ultimately were going to get the total you know it is contribution at the 0 time.

So, **so** this way we are going to have you know these values being equated and try to find it is value further. So, coming to the expression, so what we can see our expression will be so P will be basically as you see so P is here this amount at which it is component is taken. This will be the your present amount and that will be basically the summation of all these amounts.

So, it will be F1 upon 1+i+F1*1+g upon 1+i raise to the power 2, so like that it will go and it will go till F1*1+g raise to the power n-1/+i raise to the power n. So, you should you must be clear that what is g and what is i, the i is the rate of interest at which the capital or the investment has the rate of return. So, that is that rate of interest and g is the increase in the so, every time which is increased that is percentage increase.

So, g can be either less than i or it can be more than i or g can equal to i. So, the thing is that is the example is that there is 10% of increase in the every yearend payment and in the rate of interest may be 12% or even if maybe 8% or it maybe even equal to 10%. So, that is i and g is how it is increasing, so that is you know that is how it goes. Now in this case, so this is we are going to solve F1/1+i+1*1+g/1+i 2 and all that, now let us get it.

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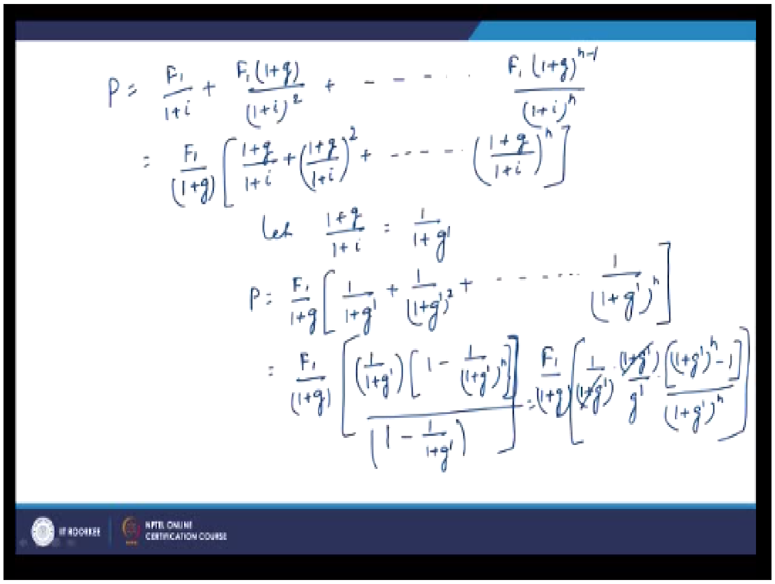

So, what we see is that will be further right P will be F1/1+i+F1*1+g/1+i 2, so it will go to F1*1+g raise to the power n-1/1+i raise to the power n. Now if we try to see that what we will do here we will take 1/1, 1/1+g common. So, that we get 1+g/1+i has uniform so in this place it will be 1+g upon 1+i+again you know 1+g upon 1+i raise to the power 2, so this way we will go up to 1+g upon 1+i raise to the power n.

So, we will have this series which needs to be solved, now what we do is in this case letter 1+2 upon 1+i we take it as 1/1+g prime. So, we are taking this is value as 1/1+g prime, so this you know P becomes equal to F1/1+g and then we get 1/1+g prime+1/1+g prime whole square+like that. So, it will be 1/1+g prime raise to the power n, so again it is a geometric series and it is first term is 1/1+g prime and it is common ratio is 1/1+g prime.

So, we have to add this you know series, so it will be F1/1+g anyway it is here. Now it will be a*so this will be certainly less than 1, so a*1-r1/1-r is a first term. So, first term is 1/1+g prime then 1-r n, so 1- 1/1+g prime raise to the power n so because common ratio is 1/1+g prime. Then so that will be it is sum then it will divided by 1- r, so 1/1+g prime. So, this is the you know expression is whole 1, so you have small 1 here.

So, further you will simplify it and what we get from here is F1/1+g and then if you see it will be 1/1+g prime. So, this 1+ g prime will be cut and hear it will be remaining is 1+g prime-1, so it will be g prime only. So, now what we see is 1/1+g prime*1+g prime/g prime and then we will have 1+g prime n-1/1+g prime n. So, this is what we are getting, now in this case so what we get is that this 1+ g prime and 1+g prime will go.

So, you will have 1+g prime you know n, so what we see is that F1/1+g 1+g prime n-1/g prime*1+g prime raise to the power n, so we can further write.

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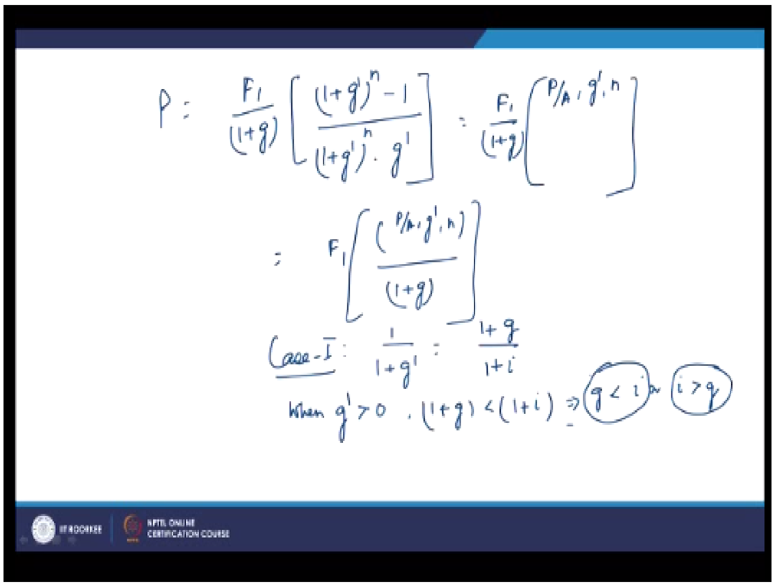

So, what we get is P=F1 upon 1+g and then it will be 1+g prime raise to the power n-1/1+g prime n*g prime. So, basically you must have this factor which is multiplied with the first year payment F1 and that will give you the present worth of all these you know values. Now if you look at this factor this is a very you know known factor which we have already derived and this is basically derivation of P/a i, n.

So, we can write this further as F1/1+g* and as you know that this is for the term g prime, so it will be P/A and this is basically interest rate will be g prime and this is n. So, we can further write as F1* you know factor P/A g prime n/1+g. So, basically in such cases of the geometric gradient series factors. We need to know the you know these g prime, so first of all what we do is we calculate g prime as we know that 1+g/1+i will be 1/1+ g prime.

So, we will calculate first g prime then we know n, so based on that we will have these factor P/A g prime n, so P/A g prime n will be this one. Now not necessarily that will be an integer g prime, so you will have to have these values from the table either you calculate or we will understand how we can refer to the tables for this. And once you get that then you multiply with the first year payment and divided by 1+g, in that case it will tell us and give us the value of the P.

And once you know P then you can have any value calculated, so this amount it will be multiplied with the factor A/P i, n it will give us the annual equivalent series or if you multiplied with F/P i, n it will give us the future amount. So, **so** that way we can calculate any value which is required further. So, that is how we calculate and we have to calculate first the g prime and then once we get the g prime then we will have to factor and that factor multiplied by F1 and divided by 1+g.

Now in such you know cases there may be cases when you have the you know g prime which is greater than 0 or less than 0 or equal to 0. So, suppose we are talking about the cases our now what we see is that in this cases we have 1/1+g prime is nothing but 1+g/1+i. Now when g prime is 0 I mean greater than 0, so g prime maybe greater than 0, so what does it means if it is greater than 0 it means 1+g will be less than equal to 1+i.

So, if the g prime will be you know more than 0, so this value will be less than 1, in that case this value has to be less than 1+i. So, this will lead to g is less than i or r i greater than g, so this will be the situation for the case and when g prime will be positive then you can directly calculate these values you may have the factor you know this factor P/g prime n. So, we know that this is 1+i raise to power n -1/i*1+i raise to the power n, so i in place of i it will be g prime, so you will be you know calculating that in the customary form.

And multiplying that factor with you know further with the first year payment and then divided by 1+g. Now the second case may be that sometimes you know g prime is 0.

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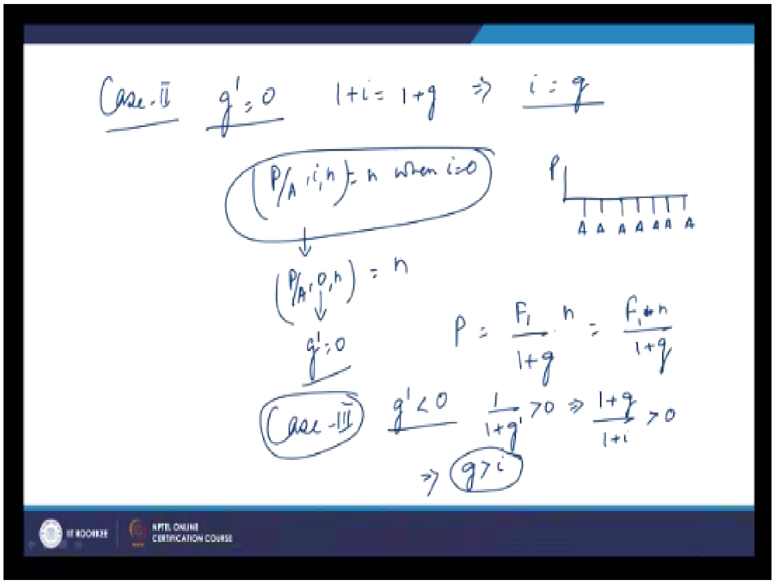

So, g prime 0 means that 1+i is by equal to 1+g, so basically your i is equal to g. Now in that case what will happen that we know that P/A g prime n we are talking about and P/A is so it will be P/A 0 n. So, you are giving you know so you are interest in fact rate of interest is 0, so you are basically if you look at this factor will be n itself. So, it is nothing but you see when you are giving A every year and rate of interest is 0.

So, basically this factor will be n because this P will be n, a if it is P then in that case it will be n, a. So, P/A you know p/A i, n will be basically n when i=0 because there is no interest rate earned. So, in these cases what we see is that P/A 0, n. so that is why we can write P/A 0, n that because we are calculating P/A g prime n, so, P/A g prime n this is nothing but g prime, g prime=0, so this will be basically n.

So, what we get is we are getting P equal to we have the expression P=F1 upon 1+g*P/A g prime n and that will be n. So, we are getting P/F1*n/1+g, so when you are talking about so this when g prime is 0 that will be the case. So, i and g both are equal rate of interest as well as the rate at which it grows they are equal. In those cases you can have such you know cases.

Now there maybe another case that will be the case 3 and in case 3 what we can have will be, so in 1 case g prime you have taken as positive in another case we have g prime as negative. Then another case we may have g prime as less than equal to 0 and g prime is less than 0 it means that you know 1/1+g prime will be more than 0. So, in fact it will be 1+g/1+i it will be more than 0 and in that case you will have you know g is more than i.

So, that way you will have those situations and we can discuss this situations we can solve the you know problems based on that and will be calculating these values and when we can find whatever we required to find out. Now in a nutshell we have you know we have discussed about the different type of factors which are used they are you know compounding amount factors, there we have used a the present you know single payment factors, we have used annual you know equal series equal amount series factors.

We have also used the gradient series factors among that uniform and you have the geometric gradient series factors. So, if you try to look at the you know these factors one by one what we see is that we will see that there will be some relationship between these factors.

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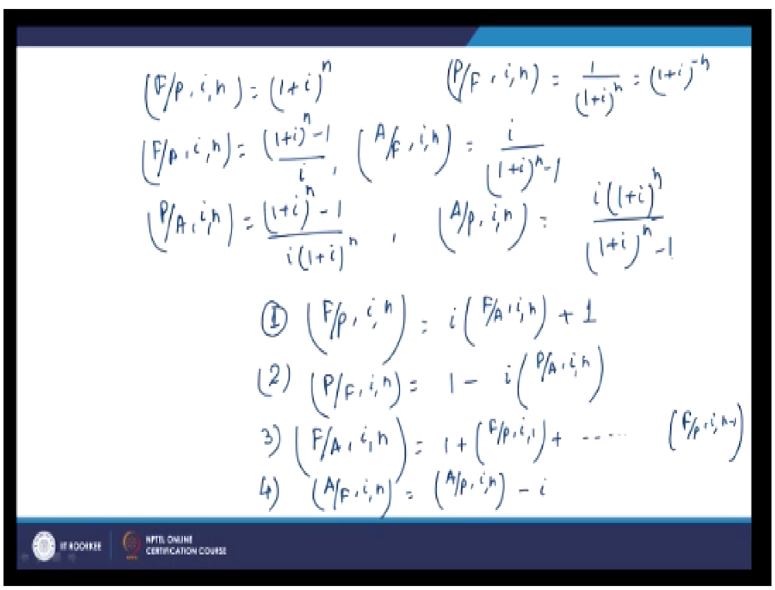

For example if you talk about, so what we have seen is that we have F/P i, n and this F/P i, n is as we know that it is 1+i raise to the power n. Similarly the P/F i, n is you know it is 1/1+i raise to the power n or you know 1+i raise to the power-n. Then you have F/A i, n and F/A i, n we know that it was 1+i n-1/i and A/F/i, n that was equal to a i upon 1+i and -1. So, similarly you have P/A i, n and that was 1+i n-1 upon i*1+i, n and similarly A/P i, n it was i*1+i, n/1+i, n-1.

Now if you look at these formulas what you see that there will be you can have certain you know relationship between these factors. For example if you look at these F/P i, n that is 1+i raise to the power i, n and then if you look at this F/A i, n this is 1+i raise to the power n -1/i. It means that if this factor is multiplied with i should it will be 1+i raise to power n-1 and then if you add it with the 1 that it will be same as this factor.

So, what we see is that to first you know expression is this factor multiplied by i, so we are equating F/P i, n this will be nothing but i times this factor and then so i times F/A i, n. And then we have to add 1, so that this 1 is cancelled, so this is how you can have 1 factor so if you know 1 factor and you have to know the second factor you can directly use, so if you know F/P you can directly find F/a i, n/i you know.

So certainly you know i and n, so by looking at this relationship you can find you know these factors. Similarly go to the next one you look at the factor P/F i, n, so 1+i raise to the power –n and you look at the factor P/A i, n that is 1+i raise to the power n-1/i*1+i, n. Now if you look at this expression like P/F i, n now this 1+i raise to the power n, now if this factor is multiplied with i.

So, it will be 1+i raise to the power n-1/1+i raise to the power n, now you subtract it from 1, so what will happen that 1+i, n-this one, so only one will be remaining, so it will be 1/1+i raise to the power n. So, that is how you get, so you are getting so what you do is you are multiplying this P/A i, n with i and you are multiplying with this. so, what you see is that this when it multiplied with I this i will vanish, so you will have this term and when it is subtracted from 1.

Then this 1+ i, n term go and you will have only one term remaining 1/1+i raise to the power n and 1/1+i raise to the power n is nothing but P/F i, n. So, you are getting this as the next relationships, so once you know 1 factor P/F i, n you can find P/A i, n. So, this expression is can be further used, similarly you have the expressions like F/A i, n, now this factor if you look at F/A i, n it is 1+ i, n-1/i.

Now this will be 1+F/P i, 1+ it will go and it will go up to F/P i, n-1, so if you see you know as you know that F/P is 1+i. So, it will be 1+1+i+ like that it will go and if you add it will be 1+ i, n, so it will be you know F/1 i, n-1 upon you know i. So, that you will be getting in such cases because it will be geometric series, so 1 you know a or n-1/r, so r will be basically so r-1, so it will be i remaining in the bottom.

So, this way you will have another so this way you if you try to find you will be seeing that A/F i, n, now this will be such A/P i, n you can see A/P i, n-i, A/P i, n if you look at this. And if you subtract i from this, so this term will be vanishing and it will be i upon 1+i. So, this will be A/F i, n, i upon 1+i raise to the power n -1, so we discussed, then next we can have further some relationships.

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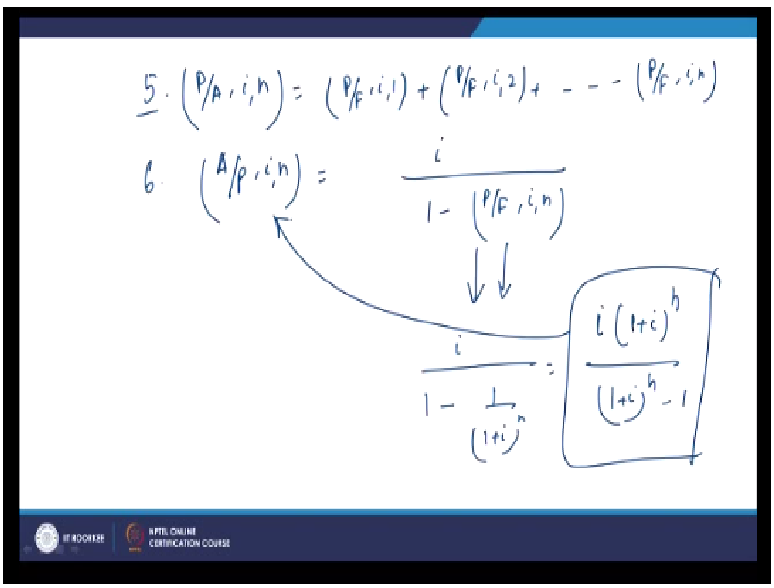

We may see P/A i, n now P/A i, n if you look at you know P/A i, n is this and if you try to see this in terms of P/F i, n, so you can write that it will be equal to P/F i, 1+P/F i, 2+ P/F i, n up to go it will go up to i, n. Now you can check from this that if your this series is increasing in that case 1+i will be you know going up like 1/1+i is will be going on.

So, you will have this as the first term and 1/1+i also the common ratio, so that will be further you know going into the bottom. And if you try to see by if you do the algebraic you know analysis, then you will find that will be coming as 1+i raise to the power n-1/i*1+i raise to the power n. So, this is another formula which will be utilized in the long run then you will have A/P i, n. Now this is equal to i upon 1-P/F i, n.

So, if you look at this u P/F i, n now this A/P i, n we know that this is you know i upon into 1+ i, n/1+ i, n-1 and this is P/F i, n, so P/F i, n we know that this is 1/1+ i, n. So, it will be multiplied 1+ i, n will go up and 1+ i, n- you know 1, so that way it will go and it will become A/P i, n. so, **so** as if you look at this one, so i upon 1-P/F is 1/1+ i, n, so it will be coming like i*1+ i, n/1+i n-1, so that is what it is coming as from here.

So, in fact you can do all these calculations and you can find these and these you know expressions will be useful for finding the factors. Although **n** you have to calculate this factors whenever required by using these formulas but as you grow as you go towards the chapters where you need the frequent values calculations and all that or you want to have many factors calculation. At that term you are going to refer to the table and that table is basically the interest table.

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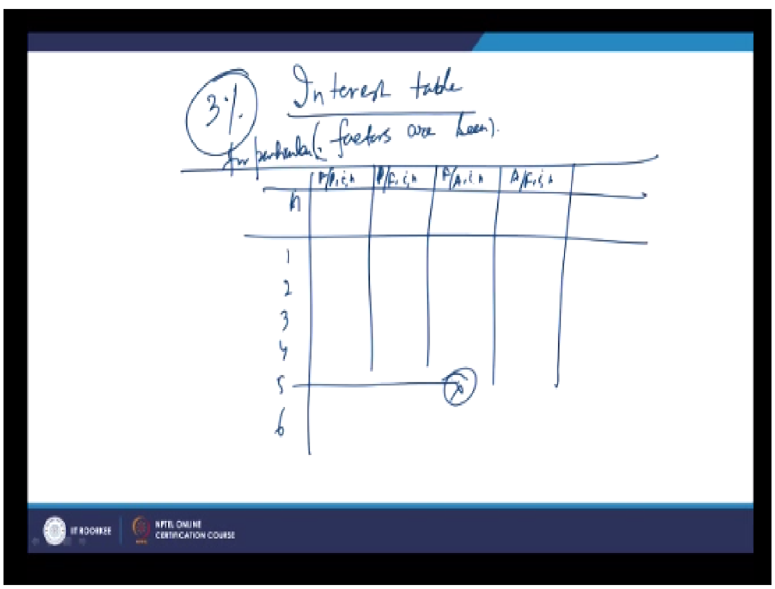

So, not interest table you can say (()) (31:42) factors table basically, so the factors you know are you know seen. So, what is there in that basically you will have a i you know n given like this and then you will have for particular i, i you will have a table. And all these factors F/P i, n and then P/F i, n then F/A i, n, A/F i, n all these values are basically given.

So, 1, 2, 3, 4, 5, 6 and you can go and for a particular interest suppose a 3% you can directly get the value from here. So, this value will be used from that interest table and we will see also that how to find directly these values from those tables. So, that way we will calculate will also talk about the different interests and that will be more clear when it when we solve the problems and use these tables for solving the problems, thank you very much.