Financial Mathematics
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Lecture-12
Equal Payment Series and Gradient Series Factors

Welcome to the lecture on equal payment series and gradient series factors, so yesterday I mean in the last lecture we had and our view of the compounding factors and in that we discussed about different type of factors like we have compound amount factor you add we discussed about the present worth factor. Then we also add the discussion about the equal payment series factors but in that we discussed about the equal payments is compound amount factor and sinking fund factor.

So, in the compound amount factor as we know that you have to multiply this factor with the equal payment annual payment and that will give you the future amount. And reverse of that factor is the equal payment series sinking fund factor where with factor will be multiplied with F, future you know amount and that will be giving you the equal annual amount. So, that is what 2 factors we discussed, today we are going to discuss further more about the equal payment series factors.

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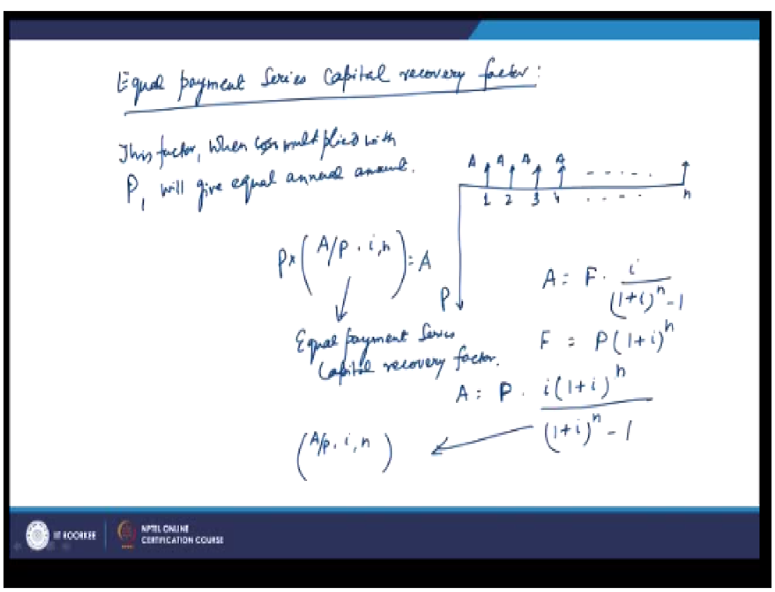

Among them there is a factor known as equal payment series, capital recovery factor many a times when we talk about the financial transactions we are concern to know that whatever we have a capital suppose somebody has a capital or the sum of money is there in the bank. Now from that amount since has kept in the bank if there is no interest earned then you know that it how long it will lost if you are taking annually certain amount.

Like if you have 1 lakh and every year you want withdraw you know 10000 rupees then you know that it is going to lost for 10 years and in 10 years it will be vanished provided there is no interest earned on the capital. However when we talk about the you know finance related issues or economics related matters, in those cases we know that whenever we have an investment we are going to earn on it.

Because that will earn certain interest, now this interest because once the time will proceed then there will be interest earned. So, we need to you know that what should be the equal annual amount which can be taken out. So, that as the time progresses in a certain amount of time the capital+the you know the interest earned on it all together you can use utilize it fully.

So, that is how this factor is defined that you are recovering the whole capital and since the capital is also earning interest. So, the you know the cash flow diagram will go like this you have you know 1 P at present and then you are going to have you know an equal amount every year. So, that it will after n years it will be you know completely over.

So, basically this is you know this is the factor which is multiplied, so this amount is A, so this factor when multiplied you know so multiplied with P will give equal annual amount. So, you know the capital amount that is P you know and this capital is to be recovered and that is how you need to know the value of this factor which will be multiplied with P and that will be giving you this you know the value of A.

So, how much you will be getting every time, so basically what you do is that here you have the factor that will be defined as you know A/P and i and n. So, this is the factor which is known as the equal payment series capital recovery factor and this factor when it is multiplied with P then it will be giving you the A. So, this factor is known as equal payment series capital recovery factor.

So, how to get the expression for this, so we know that earlier we discussed that A will be F*i upon 1+i raise to the power -1. So, we have discussed about the equal payment series compound amount factor n=payment series you know this present worth factor that was sinking fund factor was there. So, we are going to use that expression only, so this from this now we further know that F is equal to P
*1+i n.

So, this is equal payment, so this is basically the compound amount factor 1+i raise to the power n. So, if this F is replaced with this expression then it will be equal to A=F*i*1+i raise to the power n and divided by 1+i raise to the power n-1. So, basically what we see is that you know this is not F now because this will be P, so what we see that when P your P capital basically this is multiplied with this factor, this factor basically is that gives you A.

So, this factor is known as equal payment series capital recovery factor and that is why this factor will be equal payment series capital recovery factor that A/P i, n. So, this is something about you know you must know and there are many examples of such factors use like there are many cases when you know somebody will put some amount in the bank .

So, for the maintenance of suppose some garden or maybe certain colonies for a certain period. So, the amount suppose somebody has put I 5 crores of rupees for getting the monthly amount as well as you know so somebody wants to have annual amount. So, from 5 crore he will be getting the interest and what should be that amount every year for 20 years, so that you know society made it require it for many purposes like for giving the salaries for you know for making other infrastructures also.

So, that way in those cases you need to have the use of such factors like you have 5 crore here and then suppose rate of interest is defined, so it will be 0.1 10% interest is there. And then any will be 20, so you can use this formulas to get equal amount which you will be able to withdraw it every year and the at the end of 20 year this whole amount will be nil. So, that is how the use of capital recovery factor can be justified the next you know factor will be the equal payment series present worth factor.

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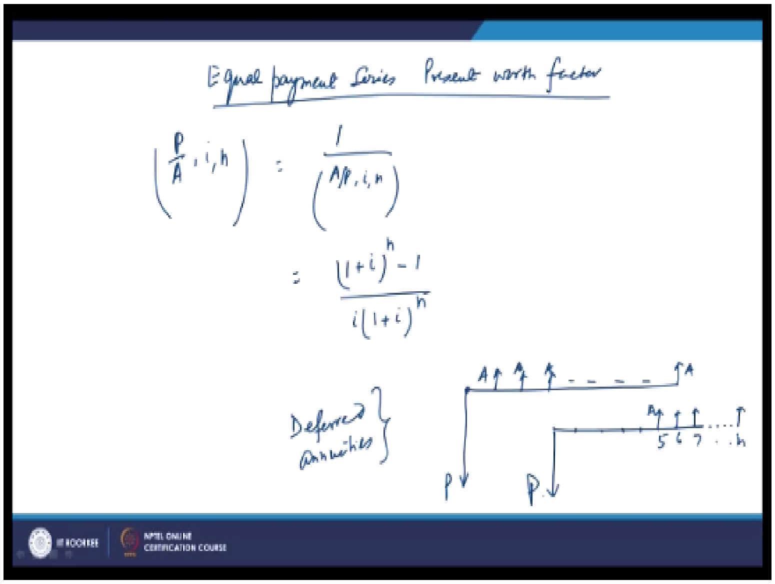

And for that again we will see that it will be the reciprocal of this factor, so it will be equal payment series present worth factor. So, in this case what we try to get is in this case you know the equal payment amount and you have to find the present worth. So, it will be basically P/A i, n, so this is the you know factor which is known as equal payment series present worth factor and this is reciprocal of A/P i, n.

So, A/P i, n we have seen that it was i*1+i raise to our n/1+i raise to the power n-1. So it will be reverse of that and it will be 1+i raise to the power n-1/i*1+i raise to the power n. So, this factor when multiplied with A it will be giving you the basically the present worth how much should be basically when you require to know that how much should be invested now, what should be the capital now.

So, that you are going to get certain amount of A=yearend payment for certain period. Suppose for the maintenance of the society and for the staff and all other things you require 10 lakh rupees every yearend. So, and that to for the 20 years, so for that if you how much you should you know have the capital now, so that from that interest will also come and because as the time progresses the amount will be withdrawn.

So, the interest earn will be slowly less but then the thing is that at the end of 20 years this whole amount are exhausted. So, in those cases when you have to find the present worth then you are going to use such factors. So, that will talk about you know finding the P when A is given and this series for the factor which is used for such calculations, so these factors are known as the equal payment series present worth factor.

Now as we discussed that you know these payments normally by convention thy start from the first year. So, you will be starting, so any amount suppose you have invested you will be getting return from the first year onwards. So, that is what the convention is and they are known as envities and that is what the amount is A, so you may have P and then it will go for you know n years.

Now the thing is that many a times what happens that your amount which is started the equal yearend amount which has to be started that is little bit delayed. So, this delay is basically known as the deferred you know time in those cases they are known as the deferred annuities. So, the thing is that the cash flow diagram will look like not here 1, 2, 3, 4 is nothing but it will start from the 5th one.

So, it will be again A and it will be you know 1, 2, 3, 4 is there so 5, 6, 7 and then which will be n. So, these are the typical examples are in the case of loans which the students take, so they tell that till they get the job this would be you know. They should not be told to pay, so maybe for 4 years they are paying and then start starting paying from the 5th year.

So, such are the cases of the deferred annuities and there are ways to find because now in this cases what you will do that for this you know series you may have either P here or F here equivalent value calculation. And then you can have you know the amount of P to be calculated, so such cases are the examples of deferred annuities and will have more such problems you know we will encounter when we talk about the economic equivalence how to calculate. In those cases the cash flow diagrams equivalent values how they are calculated that we will discuss.

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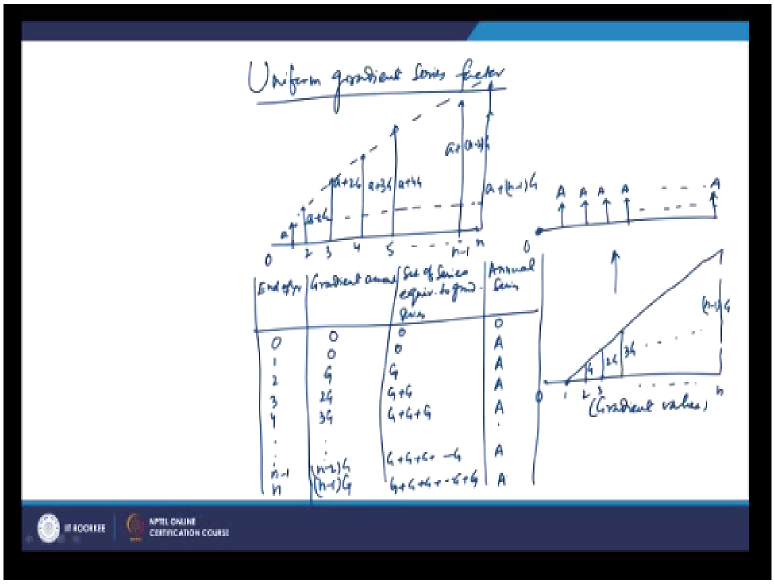

Now we will discuss about the uniform gradient series factor, so we are source that we are going to discuss about equal payment series as well as the gradient series factor. Now the gradient series factors as we discussed in our previous classes thereof 2 types 1 is uniform gradient series factor and another is the geometric gradient series factor. So, uniform gradient series factor will derive in this lecture.

Now what this uniform gradient series factor talks about is that when we are paying the amount you know by a gradient uniformly. So, suppose we are taking the example of a loan which is taken by the student to study. Now this person is going to pay the loan and in the first year he is going to pay maybe 1000 rupees or maybe 10000 rupees in a year. And from the second year onwards he wishes to every year a 1000 more.

Because he expecting that there will be salary hike, so on that basis he can expect that he will be able to pay more. So, in such cases how you should be able to find the factors how you should be able to find the basically what will be the present worth our future worth of such factors or what is basically the equivalent annual amount for any such type of series.

So, basically we are going to discuss about it, so in fact what we do is for such cases we find the equal annual equivalent amount for any such series. So, what we see is normally such series means you know in the so we will start from 0 time. In the first year it will be paid you will be paying A and in the second year you will be paying a+g. so, in the third year you are going to pay a+2g and in a fourth year you are going to pay a+ 3g.

And fifth year you have to pay a+ you know 4g like that it will be moving and in the n-first year you are going to pay a+n-2g and in the year you are going to pay a+n-1g. So, these are basically this will be further larger, so this is the example of so what you see is that there is a gradient this is maintained, this a straight line basically and every time as compare to the previous you know payment it is always you know larger than I mean by a fixed quantity.

So, it may be either larger or it may be smaller, so it maybe so that it is larger A, it maybe a-g, a-2g, a-3g like that. So, that also may happen so that will be decreasing series and this is an increasing series you will formally increasing series and that maybe uniformly decreasing series. Now the purpose is that how to represent such series and our job will be to find basically equivalent you know values.

So, what should be basically the equal annual amount for such series, what amount will be basically equal for such gradients. So, basically what we see is that you if you look at the this series you see that a is fixed here, a is always fixed but the gradient is changing. Now at this point this is 0, at this point basically there is you know the gradient amount is 0, so it is only a.

So, if you take a out of it, so it will be so the gradient value if you look at gradient value will be 0 here and then at 2 you know at this place it is so you are going to subtract a from every you know (()) (19:27) disbursements. And this point you will have g, so then at 3 you will have you know so that way your things will be moving, so this way your gradient will move.

So, you are going to have the come to this you know m and are so these are g 2g and then 3g and all it will be moving. And this will be n-1*g, so what we see is our job will be to have the you know cash flow diagram which will be talking about the equivalent value equal annual amount which should be same as the amount which is calculated from this place.

So, here it will be so basically what we do is we know that this is a, a is fixed but for this gradient amount which is being paid. Now onwards what should be the equal annual series for that gradient amount and that is why this is known as a uniform gradient series factor. And once we get that a then the capital A+small a basically will be talking about the whole you know equal annual series for the whole you know transactions.

So, that is what we are going to talk about and so what we can see by we can see by the in the table for if you talk about the end of year. And then if you go the gradient amount then if you take you know set of series equivalent to gradient series and then you have annual series. Now what we see is that your end of year is here we have defined, so it will be 0, 1, 2, 3, 4 and then it will come as n, so before that there may be n-1.

Now what we see is we know that anyway in the first year also gradient amount is 0, so anyway it is 0 here and a second year we start with gradient amount g. Then third year we get 2g then 4th year we get 3g, so it will be you know in the nth year it will be n-1*g. Now if you take of the set of series, so it will be 0 and then you have g here and you can see it will be written g+g, so it will be further written g+g+g.

So, if you take you know n-2g, so it will be g+g+g+g and in the last you will have all that extra g. So, this is these are basically the equivalent you know gradient series the g values and anyway we are going to have these particular a. So, we want the a to start maybe because in the first the 0 year we are not getting they are getting the first year end. So, we will have a, so what is there that we want to have this a, we want to find this a.

So, basically what we mean to say that we wish to have the this as equivalent to the capital you know A and that will be calculated by using the expression. So, if you look at this as you know you have the payment of a first year end to the nth year and if you take it is you know if you want to have the F. So, what is F, final future amount, so this a must be multiplied with the factor F/A i, n.

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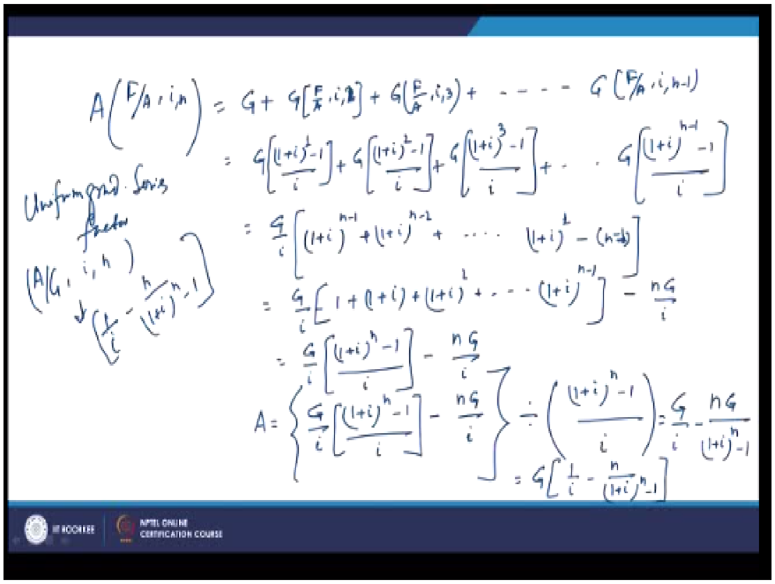

So, basically the a will be multiplied with F/A i, n this will give you the future amount that is the accumulated amount at the yearend. Now how much it should be equated how this should be equated what we see that. Now this is the value at this time, so at this time if you look at this g basically 1g is the this g has is not adding any value, so you have the simply add g. now if you come to the this 2gs, so here the amount will be g*F/A i, 2.

Because these are the 2 you know values and you are getting these 2 yearend values. So, it will be g/g*F/A i, 2, so this way it will go up to F/A i n-1, so you can have this as g it will be the last one. Then another g there are 2gs, so it will be g*F/A i, 2 similarly g will be F/A i, 3, so this will go up to g F/A i, n-1. Now we know that F/A you know this is i, 1, this is i, 2 ok.

Now so what we do is we will go further and what we see is that we can have g now this one we can write 1+i raise to the power 1-1/i, so it is g. Then as we know that for this factor is becomes 1+i 2-1/i+G 1+i 3-1/i, so like that it will go. And it will and the end it will go 1+i raise to the power n-1-1/i because we know that F/A i, n will be 1+i raise to the power n-1 i, so that is how it goes.

And this is what so this is similar as 1 because F/A i, 1 will be 1 itself, so we can further write we can take g/i. So, what we see is that it will be 1+i n-1+i n-2 and all that it will be moving. So, you are coming up to 1+i raise to the power 1 up to this we are coming and then ultimately we have this 1 term coming you know it is coming for n-1 times, so it will be n-1.

So, further we are getting g/i and then you have you will be coming to 1+ 1+i I think it will be there are this 1 is coming as n terms. So, I think this will be n, so you will have n here and then that is why your 1 will be coming like this 1+1+i and then so it will be similar there are n-1 terms basically. So, we have n-1 here and this 1 will be added, so 1+1+i+1+i you know 2, so it will go up to 1+i raise to the power n-1.

Then you will have –ngi so this n will be taking out and it will be ng upon i, so this is basically a geometric series. And we can write it as g/i and as we know that a is the first term and this is your common difference. So, it will be end if you look at you have n terms now, so 1+i raise to the power n-1 and divided by r-1, so it will be i. So, this is the term you are getting here –ng/i this is what you are getting.

Now if you look at this a, so this F/A i and we know that it is 1+i raise to the power n-1/i, so it will be divided. So, a will be equal to now we have this g/i and you have 1+i raise to the power n-1/i-ng/i and this all is to be divided with F/A i, n, so F/A i, n is 1+i raise to the power n-1/i. So, as you know that this factor will be when it is separately you know multiplied, so this will go and it will be g/i.

So, what you see is that you can write it will be basically g/i will be the first term and then you will have this one ng/i and into so this will be ng/i and that it will be i upon 1+i raise to the power n-1. So, if you look at it will be –ng upon i and this i will be vanishing. So, n*g/1+i raise to the power n-1, so what you see is you can have g as common and it will be 1/i-n upon 1+i raise to the power n-1.

So, basically this gives you that a and this a will be talking about the equivalent amount for the gradient amount. Suppose you have you are increasing you are depositing you are getting 700 in the first year. And from the second year onwards 100 each increasing, so this is for the 100 increase that value will be given the a and if you talk about the total you know equivalent annual amount it will be 700+that calculated amount for that extra you know 100 every year.

So, that way you can you know calculate these values using this factor and this factor basically is known as the uniform gradient series factor. So, you can have the name like uniform gradient series factor, so this will be basically multiplied with g that will be giving you a, so it will be a/g i, n. So, if you this factor is multiplied with you know g it will give you a, so this factor becomes 1/i- - n upon 1+i n-1.

So, this is the you know uniform gradient series factor, so that is how we calculate it and we can discuss some questions based on that when we discuss in the tutorial classes or are the assignment classes in the end, thank you very much.