Financial Mathematics
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Lecture-11
Introduction to Discrete Compounding and Discrete Payments

Welcome to the lecture on Introduction to Discrete Compounding and Discrete Payments, so we discussed about the different types of interest typically simple interest and compound interest. And based on that you have other terminologies like discount rate or so, now when you have the dealing with the investment agencies or the banks or so. So, we normally come across the loan terms or you know when we take something from any bank.

So, there you have to deal with these compounding you know at different time you are going to pay in a different way. So, you have certain factors they are known as compounding factors and this factors are basically used to get the future value or current value or even we will talk about the terms like annuities. So, these needs to be you know understood to you and for that we will discuss about you know the discrete compounding and discrete payments in this lecture.

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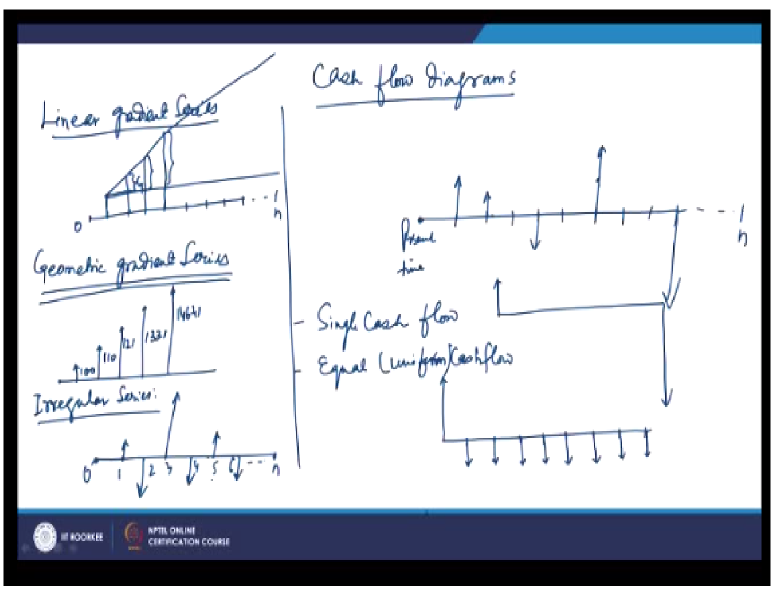

So, first of all we discussed about the cash flow diagrams that is quite clear to us that in the cash flow diagrams this is a diagrammatic representation of the cash transactions or the transaction in the terms of a receipts and reimbursements at different time step you know discrete time values.

So, that is how the cash flow diagram is represented and you know you have , so this is the your present time and then you time you know these compounding periods are you know defined.

So, 1, 2, 3, 4, 5, 6 like that, so it will go to n and so that is your so where sometimes you may have some flow like this. So, what this kind of you know transactions will be talking about the cash flow diagram. So, this diagrammatic representation is known as cash flow diagram, now the cash flow when we talk about there are different types of cash flows and that different types of cash flows are like single cash flow.

So, single cash flow means you know you are going to invest something now and you are you know you have received something today as a loan and you have to pay at the end something. So, this is how you know this is a case of single cash flow because ultimately you have only one transaction how much you have to pay maybe after n you know some years. So, when compounding is going on that is a kind of single cash flow then there are also cases of equal or uniform cash flow.

Now this in this cash flow what happens that many a times what we do is we receive well known from the bank or the financial institution. And then we are trying to paid in the equal you know series manner even on monthly basis or yearly basis, so such are the examples of equal uniform flows you have the transactions during the span of the time. In a single amount that is you know same amount which is going on.

So suppose you have taken from the bank certain amount today and then every year end or every end of this period compounding period you are going to pay some amount to the bank, So, ultimately this amount which you are paying every year , so basically that is known as enmity also. So, this is equal or uniform cash flow because here you know it will be something like going on every you know year end or so.

Then there maybe you know cases like linear gradient series, so the cases maybe linear gradient series. Now what it mean linear gradient series means you know you have started you know you started from 0, 1, 2, 3, 4, 5, 6, 7 or 8 or like that you are going. Now you started paying some amount you know you are receiving or paying at this and then every year you will be having some increase an increase will be by a fixed you know quantity.

So, basically this is the increase and this increase is you know fixed to every year you will have an increase in the amount you are paying. So, this gradient is known as the g, so basically it is a linear gradient because it is a linear profile. So, we talk about the linear gradient series suppose you are going to deposit 1000 rupees in the year end then the next year end 1500 after that 2000 or 2000 then 2500 in the next year or low.

So, in that case is a case of linear gradient series and you know this may happen because someone wants to save his money every year and since every year he is expected to get some increment. So, maybe the increment from the increment you also he will be putting towards saving, so suppose he is getting increment of 1000 rupees. So, he is increasing his saving every year by 1000 rupees, so something like that will be a base amount and then this will be the increment every year.

So, that from the base in the first year it will be second year will be g increased then is the 2g increased, 3g increased are like that. So, this is a type of a linear gradient series and we may deal with such kind of series where we have to find ultimate the ultimate m will be to calculate the you know the total equivalent value it may be equivalent to certain uniform flow series. So, this gradient series must have been must be represented by 1 equivalent uniform flow series or it may have an equivalent future amount or equivalent current amount or so, this also we will discuss.

Then you have geometric gradient series now what happens in this geometric gradient series that here also you are increasing or decreasing. Now in the case of linear gradient series not necessarily always we are increasing, we can also decrease. So, suppose we are giving the 10000 rupees in the first year and the second year we are giving 9000, third year we are giving 8000. So, increasing 1000 rupees every year, so in that case g becomes negative as –of 1000, so that way if there will be increasing or decreasing series.

Now in this case in the so we are seeing that we are increasing that amount by a certain you know fixed quantity. Now thus the same thing maybe in terms of increasing or decreasing order but by certain percentage. So, suppose you are giving you know you are you know in the first year and you are giving you are getting something like 100, second year and you will be getting 10% increase, so it will be 110.

Then you will have again 10% increase from their 121 further increase, so it will be 133.1 like that so further increase 146.41 like that. So, it will be 160 points you know something like so it will go on, so such series they are not basically the linear one basically they are having some you know parabolic type of profile at different profile not done linear. So, this is known as the geometric series, gradient series and in that basically also again the purpose will be to represent these or to know this.

So, that you have to ultimately find equivalent uniform cash flow series or they future value or the current value of that cash flow. So, that will be your geometric gradient series then most commonly you will also get irregular series. Now irregular series means among the timeline you do not know when will be what, so there maybe some in here, some increase then further it may go then come down then it may be here.

So, such series are basically you know they are they known as the irregular series. Now in such series you have to if there is a pattern by looking at this you can use those formulas you know respective formulas to find you know the equivalent amount. But otherwise if there is completely there is no pattern in those cases you have to find the equivalent amounts at a particular time. And then ultimately when all the values I mean equivalent of all these transactions are calculated at a particular time.

Then you can basically come to any other you know timeline or so, so that way you have the different you know types of such type of the you know cash flow series which is likely to you know be encountered in our you know future studies. So, we will be talking about the different types of so basically what happens that we normally deal with the terms known as the factors.

So, they are the so normally what we do is we normally are interested to find some factor we should be multiplied with either the principle value or the future value.

And then in that case we tend intend to get you know other terms like maybe future value from present value or maybe annuity value from the you know present value. So, all these things are many times required to be calculated and that is what we will try to see further that what are those how these factors are being you know named. So, what are the name of these different factors that we will see, so among them we will discuss starting discussing about the you know single cash flow series.

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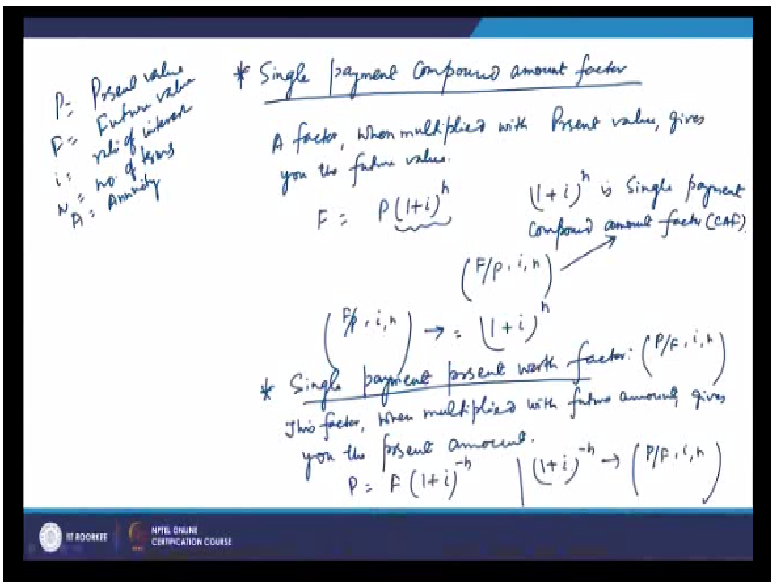

So, in that we have the single payment compound amount factor, now this single payment compound amount factor this is a factor when you will multiply this with the present value or the current value in other terms or present value will now talk about some terminologies. So, you will have p as present value then we will have f as future value then you will have i as rate of interest n is number of terms or years, how many times so what is a.

So, these are the normally terms which will be required at this stage we will also come to know about A, A is annuity value or equal uniform you know cash amount, so annuity. So, this single payment compound factor this is a factor when multiplied with present value gives you the future value or the compound amount basically future value is nothing but the compound amount because there is compounding going on in during that period.

So, we know that when we invest any amount p now it will be p*1+i at after 1 year p*1+i raise to the power 2 after 2 years, p*1+i raise to the power 3 after 3 years. So, basically your f becomes you know p*1+i raise to the power n for the rate of interest i and n you know be the number of years. So, basically this 1+i n raise to the power n, so this 1+i raise to the power n is single payment compound amount factor, that is also known as CAF.

So, that is how this factor, so this factor you can calculate and when you will study you know in our later lectures. So, when large amount of these values are required to be known you can even refer a table, that table talks about these values. So, now this factors since it is being multiplied with p and this gives you f, so basically they are known as you know F/P i and n. So, this is these single payment you know compound amount factor, so multiplied by p it will give you the f.

So, F/P i, n we can write this basically is the notation and this is nothing but 1+i raise to the power n. so, suppose you are required to be to tell about the you know value which will be accumulated after suppose 10 years. So, in and you are investing now 15 lakh rupees, so 15 lakh is to be multiplied with the compound amount factor that is single payment you know compound amount factor.

And that will be 1+i and rate of interest is suppose 10%, so 1+1.1 raise to the power you know so suppose number of years and that will be the factor multiplied with p simply give you the amount of F. So, this is the single payment compound amount factor , now similar to that you have, so opposite to it or reciprocal of it will be single payment present worth factor. So, these factor is basically represented.

So you have to find p and given F, so this factor is represented by this symbol P/F i, n means this factor when multiplied with future amount gives you the present amount or principle sum. It means the factor is to be calculated and this when multiplied with F will give you P, so we know that P will be f*1+i raise to the power –n or F/1+i raise to the power n. So, basically this factor 1+i raise to the power –m.

This becomes single payment present worth factor and we also call it as we calculate we find you know P given F, so P/F i, n. So, it was F/P i n and this is P/F i n and this is 1+i raise to the power –n, so that is how you calculate these you know single payment compound of amount factor or you single payment present worth factor, so and you calculate all these values.

Now we will further discuss about the other series which is the uniform cash flow series and when we talk about the uniform cash flow series we talk about the payment which is made you know every year end and that amount is constant. So, how to find such cases, so basically these amount which is made at the yearend or whatever you receive at the yearend they are known as annuities.

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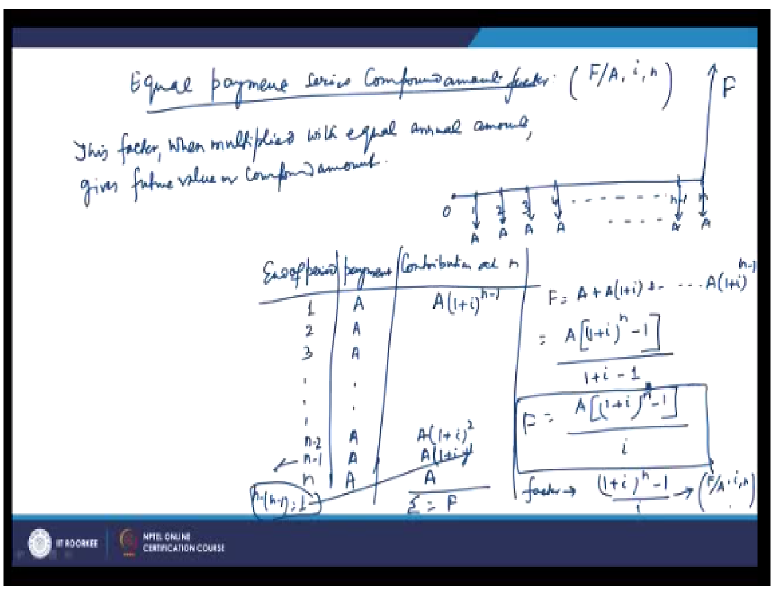

And based on that you have a series known as equal payment series compound amount factor. Now this factor basically is in this case you find F this future amount compound amount, compounded amount provided A is given every yearend what you are getting that is given and based on the rate f interest and the number of times they compounding as taking place or years.

So, you can see by looking at the cash flow diagram basically it is nothing but in every yearend you are you know giving and it will be n-1 and this is n. So, every yearend you are depositing certain amount that is basically fixed amount. So, it is all is A and ultimately this year also you are giving A and here also you are giving A and ultimately you will get 1 amount that is your future amount that is F.

So, that is what is the example of you know the see equal payment series compound amount factors. So, this factor when multiplied with equal annual amount gives future value or compound amount you know so certainly the rate of interest and this is known, so if you look at the series what we see that we have A going on here then A is going you know every year.

Now what you see is that if you try to see it is value this A which is given at the end of n years this A is as usual it is do not getting any interest. So, if you see end of period payment and it is contribution at n, so end of period payment you know 1, 2, 3 to n basically we are you know paying. Now you know end of period payment, so end of period and this is payment, so end of payment is anyway we are paying only A.

Now it is contribution at n is how much, so this A which is being paid here it is contribution at this point is only A. Because it is not getting any time to you know compound, so it is value will be only A, now this point at n-1 it is getting you know 1 compounding period. So, it will be getting, so if it is n-1 it will be getting a*1+i, so you know this way it is going on and ultimately what we see at the first time and this is your you know nth time.

So, this way you will have for n-1 it is 1+i only so for the first one it will be coming as 1+i raise to the power n-1. So, if you look at this and this sum so if you sum them so that way your it is value will be you know calculated for you know every you know value. So, what we see is for the first case, first case is nothing but n-1, so that is why it will be you know A*1+i raise to the power n-1.

So, this is nothing but n-n-1, so it is power 1, so n- so that way it will have you know 1+i raise to the power -1, they here this is coming power 1. So, this is nothing but this is corresponding to n-n-1 1. So, this 1 is going here, so this way for n-2 it will be again A and it will be A*1+i raise to the power 2, this 2 comes from n-n-2. So, this 1 it will be n-n-1 is 1, so it will be at 1+i raise to the power n-1.

So, all together all it is sum, this summation should be equal to F, if summation must be the same F, so what we see is F will be A+A*1+i+going on up to A*1+i raise to the power n-1. So, all together there are n terms and that is why it will be it is a basically series of the geometric series where the compressed term is A and common ratio is 1+i a* and term is total term is m, so 1+i raise to the power n-1 and divided by r-1 that is 1+i-1.

So, it will be A*1+i raise to the power n-1/i this is the you know expression which basically equates the F value you know in terms of A i and n and this factor. So, what we see is this factor, so factor is basically 1+i raise to the power m-1/i now this factor when multiplied with A this will give you the F. So, that is why this factor is basically known as the equal payment series compound amount factor.

And that is why we call it as since we are calculating F based on i for a particular i and n. So, this is represented by the factor F/A i, n. So, you will have a table and whenever you have you know certain A then for some periods it is being been accumulated. So, that n if you know at rate of interest you know and according to that as per the table you can just calculate this values and calculate ultimately the F value. So, that way you know we find and we give the name to these factors, now on the contrary you may be required you know to find the A when F is given.

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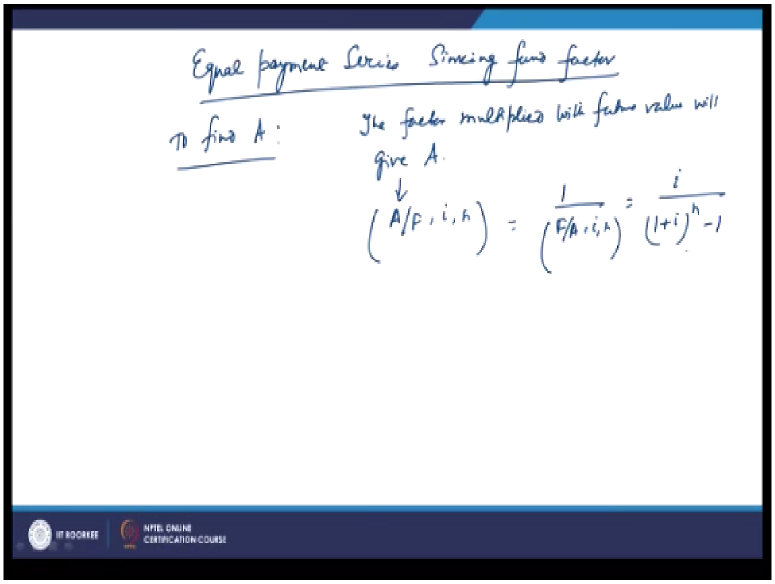

So, that factor is known as basically equal payment series sinking fund factor, now what we see here is that in this case this factor you have a fund which is getting sunk. And basically you want to fin, so you want to find A, so the factor multiplied with future value will give A. So, basically A is like you know you have a future in future you have certain which is there suppose to be deposited and how much you will be drawing every year equally.

So, that the equivalent amount ultimately it is replenishes, so it is gone, so for that you know this factor is, so that factor is basically you know you have to find A provided F for i and n. And it is basically the reciprocal of the F/A i, n, so this factor will be 1/you know F/A i, n that is equal payment series compound amount factor and in that case we have calculated.

So, it will be i upon 1+i raise to the power n-1, so you know like you may be told that this amount will be there deposited at the end of so. And what should be basically the every year amount which can be taken out, so that in the end it becomes you know 0, so that way you have equal payment series sinking fund factor which is defined and that is your i upon 1+i raise to the power n.

So, you can have different type of questions will have it when we deal with the question solving session we will see that how these factors are utilized to calculate the value of the you know F or

A or P or F or so. So, we will have other more factors in our coming lectures, thank you very much.