Financial Mathematics
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Lecture-01
Introduction to Financial Mathematics

Hello friends, welcome to the introductory lecture of the course financial mathematics. So this is the first lecture and this course as you know it is 30 hour course and in this course we are going to have the interaction we will have lot of problem solving assignments and today we are to start this course that is introduction to the financial mathematics, so that will be the title of today's lecture.

And the course is certainly that financial methods, so as you know that when we deal with you know finance or we deal with business many a times you need to know the basics and then further you also need to know the different concepts, like when you talk about business more importantly you deal with the transactions, you have either you take the you know you go for positive transaction, you go for negative transaction like you may have you may have the receipt, you may have the disbursement.

Then you have many things like interest rate interest which is often coming into picture you have the concept of you know percentages, most of the times you need to know you know that how these different percentages like rate of interest or depreciation**.** So how they will be you know affecting the transactions that is receipts and disbursements. Then what are the other you know terminologies like mortgage, debt.

Then you have risk also, so many aspects we will be covering in this course and today we are going to start on that you know in the direction and you will be dealing with the introduction to financial mathematics.

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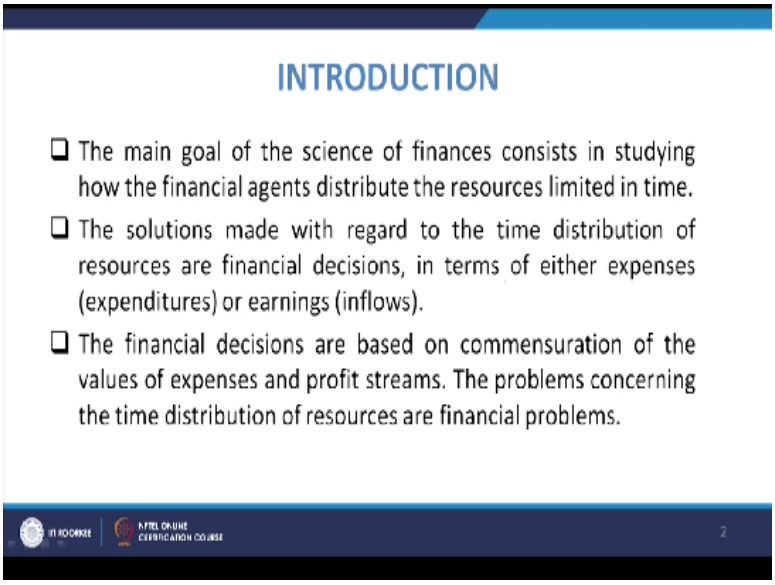

So the main goal of the science of finances consists in studying how the financial agents distribute the resources limited in time. Now the thing is that when we talk about the finance or science of finance so certainly as you know that we have limited resources. Resources are limited that is why this is lot of competitions. So you have the limited resources you have to utilize it.

You have to use a resources by you know in certainly the raw materials are there, they are converted into final product, then they have to be sold, in between you have transactions. So certainly all the fight is because of the limited resources and also time because time also place an important role when we talk about the business, when we talk about the finance, and time is equally important.

Because whatever as the time progresses the same thing has the different values, so you have time value of you know money or transactions or so. So in this you know courses what we intent to tell you that as it is written that the main goal of the science of finance which we are going to study it will be studying how these financial agents that is goal is involved the persons or the origins.

So how they are distributing that resources that is limited in time, then the solutions made with regard to the time distribution of resources are known as financial decisions, so in between whenever you have the certain problems coming or you have a stage where you have to take the decisions. So when you have to take those decisions or the solution which you

have come up with regarding the time distribution of the resources how you are distributing the resources with time .

So whatever decision you are taking there are the financial decisions and the decisions will be either in terms of expense or expenditures or earnings or inflows. So many a times we talk about the economical aspects or economical terminologies we talk about receipts or disbursements receipts means you are having inflow that is earning, similarly disbursement that is your expenditures or expenses.

So these are synonyms terms and these terms will be used as we go into the course. Financial decisions are based on commensuration of the values of expenses and profit streams. The problems concerning the time distribution of resources are the financial problems. So certainly as we have already discussed that you know you have the expenses and the profit streams that is to be involved into and with the problems which are concerning these terms distribution.

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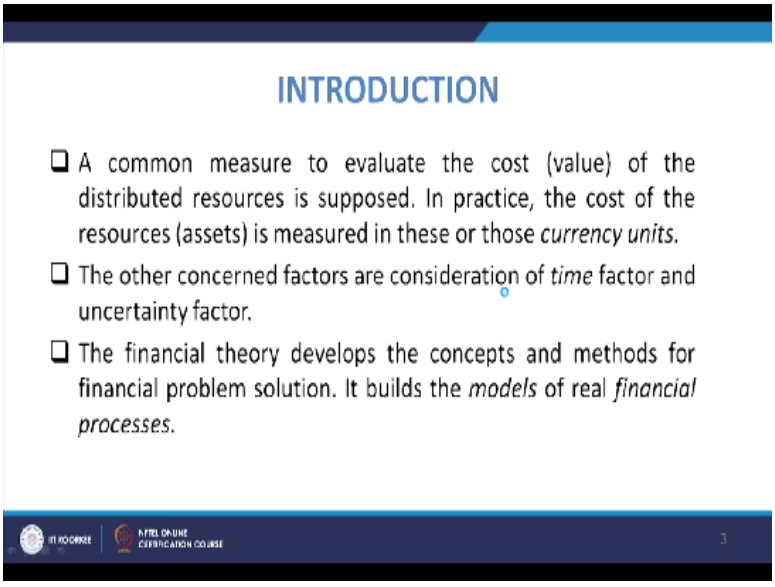

Now a common measure to evaluate the cost or value of the distributed resources is supposed and now in certainly you need a medium you need you know a way how to measure it and for that you have recurrence units. The recurrence units can be in terms of anything you may have the rupees. So as a you have rupees or dollars or pounds or so. So you are basically using these units currency units to measure to evaluate the cost of the distributed resources.

Because when at different stages you have these expenses or you have the incomes. So they are to be basically cotised that to be quantified and insert the units and that is normally the currency units certainly that is the standard way of during the transactions these days. The other concerned factors are consideration of time factor and uncertainly factor.

So as we discussed that when we talk about the transactions in a financial world then suddenly first comes the you know what is the amount of transaction, what is the amount of receipt or disbursement at any point of time. And certainly then you can have mapping at any point of time or if you neglect time at the present then you are thinking of the amount only that how much is the income.

And how is the expenditure like that, but equally important is intense of a financial transactions of financial world equally important is the time because at what time what kind of transaction is going on and it has certainly bearing on it. So it basically evaluated by discussing that we will discuss later and this tern value of money. So as we discuss that we are evaluating terms of currency units.

So suppose this money, then the factor is time factor, that is the time actually these transactions are taking place that time important, whatever you are doing the transactions today and if you are doing the transaction the same transactions after 1 year they cannot be you know taken at the same because there is a value of the you know time so you know the time is important.

But what time the transactions has been done because as the time is different its value would be different the 100 rupees which we have you know invested today it will not be same as 100 rupees after 1 year. So what it means to say that when we are trying to evaluate mostly when we are talking about evaluating alternatives or evaluating any alternative any you know offer.

So at that time the time is important at what time these transactions have taken place, so time is one important factor and that is why it will be seem that at what time suppose the interest rate is there. So, interest rate is what at what time and interest rate also may change, so basically because of these interest rates because interest is applied because you want to earn on certain of your asset.

So, if you have something which you are lending to someone so that he can satisfy his you know goals or he can get distraction from that. So, certainly you will not be seen that you someone to get the benefit at your own cost. So, I mean in financial terms, so you have the you will certainly try to earn and that is how the interest terms come into picture.

And since it is comes into picture, so unless it is 0 if the interest is 0 certainly the with different time you will have the sale value. But the interest is not only positive value and so you will have a different value at different time, so this time factor is important. Then another factor which cannot be load is the uncertainty factor because most of the time in case of financial decisions and we talked about the economy you know economical exchanges or economical you know aspects.

Then normally we look into the future we try to neglect what has happened in past, so certainly when you talk about the future you do not you cannot say anything with quite a good certainty. You know quite with 100% of certainty, so many a times you have the uncertainty involve and without taking uncertainty into the account you cannot say that you are going to have the decisions.

So, there will be uncertainty and these uncertainties are to be taken into account with that because and that basically gives you many definitions like or many areas like risk, you have risk in uncertainty. So, you have certainly you have risk in the financial area because you do not know what will happen what will be the rate of interest maybe after 1 year if it is variable then what will be have a happening maybe tomorrow or maybe after a week.

So, these are the you know the things on which you do not have any control they are the things of future and you do not have anything which you know with certainty. So, you have the uncertainty factor and that also is taken into account and many a times to you know as we do in the modelling practices. Many a times when you have very complex model with uncertainty you try to make it simple by making it deterministic, you try to you know **say** see that uncertainty should not be coming that way.

So, certainly when you have uncertainty there may be certain t also with decisions, the financial theory develops the concepts and methods for financial problem solution it builds

the models of real financial process. So, as we try to analyse these problems of finances, so you will have the concepts suppose you have to predict something the in that case you will have the methods of the solution, you will make certain you know prediction you have certain methods by which you will see that you are getting the solution of that financial problem.

And this way the real you know financial processes are being solved and for that you are making the models. Now most of the time these models they are you know using the mathematics, so that is why they are known as mathematical models, so that will come later.

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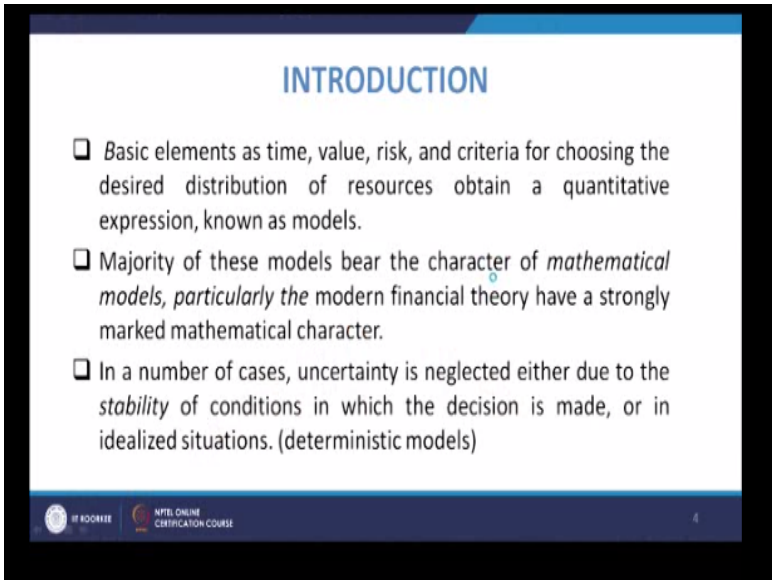

Now basic elements like time, value, risk and criteria for choosing that desired distribution of sources obtain a quantitative expression known as models. So, basically what are these models now you have the factors like time the values like what is are the amount of transactions how it is then what is the risk whether there is risk involve with that. so, you will have probabilistic component coming to it.

Then you will have what are the criterias what are the decision makers how you are going to decide. So, all these things for the distribution of resources they are giving you the you know a quantified expression in the expression and this expression which by which basically predict you **you** how to distribute the resources, how to utilize the resources to have maximum or optimum you know output, so that is basically a model.

Now as we discussed that most of these models when we incorporate or when we intend to have the you know effect these time or the value of the transaction or the risk or the criterias

based on certain conditions. All these normally have a mathematical character you have mathematical expressions most of the time and that is why they are known as mathematical models.

So, mathematical that is why mathematics you know is playing an imp role when we try to study that in concepts because they have certain type of correlations in between them where the mathematical you know terms are used, mathematical theories are used. And you get those you know output particularly in the modern financial theory, so they have a mathematical character you have lot of mathematical tools available.

You have lot of you know you have new ideas you have the you know think that there are many factors which are involved how to incorporate those factors. And every factor might have you know effect in it is own way, so they will they may lead to very complex kind of mathematical relations and which need to be solved. So, that you get the output, so you need to have the you know introduction to these mathematical tools you have to know the mathematical concepts .

Then how these you know with the time things change suppose interest rate will change or they will be other you know effects like there are many type of transactions where there are they combined effect of the parameters. So, all these can be dealt with the you know good mathematical tool which are available to us we have the equipment developed to us would computers available to us.

So, you can use them, you can predict and we can certainly do the justice by solving the problems in the way we desire. In a number of cases the uncertainty is neglected either due to stability of conditions in which the decision is made or in idealized situations. So, as we discussed that many a times you know is neglected uncertainty is neglected to due to the stability of because feel if you take the horizon if you see that there may be the stability of conditions in the future in the last in the larger time horizon.

So, in those cases this you know uncertainty is neglected and you can have the you know model as certain. So, you may have the deterministic model or you know what is the outcome coming to be. So, that way or in the idealized situations also, so in those cases in such cases

normally what we do is we take this so you will have at those places you will have the deterministic models.

So, what we saw that you will have lot of mathematical terminologies which will be coming up in this course you will be dealing with them. And you will learn how to use these techniques for the predictions of you know many financial decisions, economic decisions which will be helpful for you and the organization. So, when we talk about these you know mathematical finance then first of all we must be conversant with the basic mathematical you know concepts.

And for that we know that when we it was in this case mostly will be dealing with you know these you know computations you will have the use of numbers. So, if you talk about the you know mathematical you know concepts of the numbers. So, as we know that you have the all the numbers.

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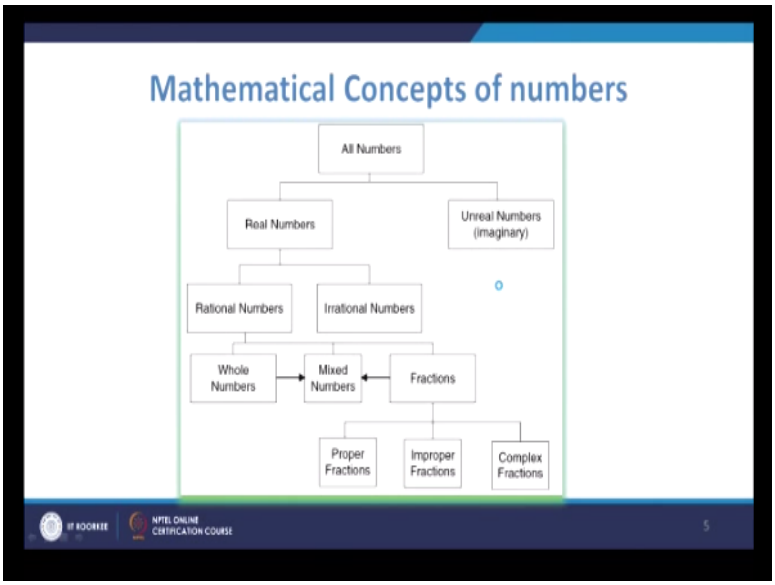

And this slide tells about you tells you about the numbers, so suppose you have all the numbers these numbers basically you have either real numbers or unreal numbers that is known as imaginary numbers. Then further you have real numbers or you know categorized as rational numbers and irrational numbers. Then you have the rational number further you have the whole numbers, mixed numbers and fractions.

And further you have fractions as proper fraction improper fractions and then you have complex fractions this is how the numbers are you know define. So, as we you can see that if

you talk about the, so you can start from the you know bottoms as we see that you know you have integers first then you have whole numbers that is -5 to -3, then you have 0, 1, 2 becomes a whole number. So, this is what is the elementary thing then most of the time you will be dealing with the fractions.

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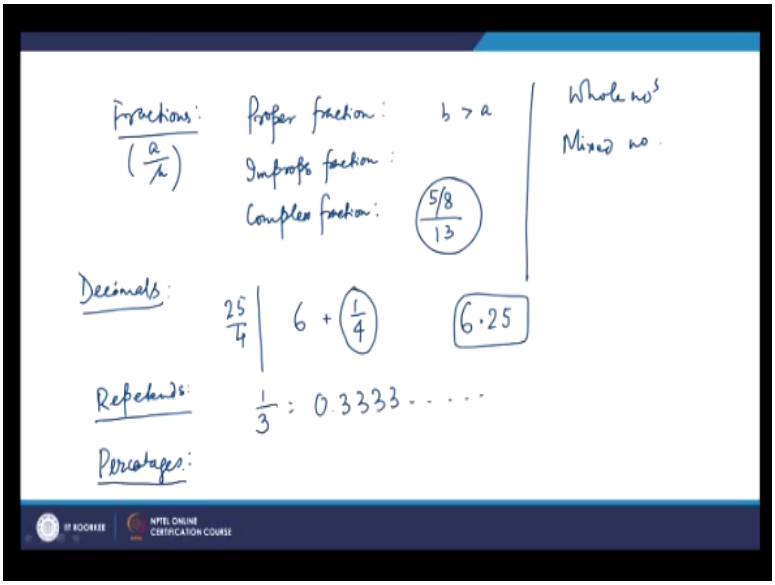

And if you talk about the fractions as we know that in the case of fraction you have terminologies like proper fraction. So, proper fraction as we know that, so in the fraction normally you denote these fractions as a turn a/b where b is not be equal to 0, so a/b is a fraction. And we know that a is known as numerator and b is denominator, so that is how there is a effects and it is defined.

Proper of fraction is the one where b will be more than a, so you will have that is known as the proper fraction. Similarly you will have improper fraction and improper fraction you will have b will be less than a, so that is improper fraction, so that is the one. Then apart from that you have the as we discussed that you have mixed fraction.

So, you have proper fraction, improper fraction and you have the, so then you have in those case of complex fraction what is there that you have in the numerator or denominator you have 1 as a fraction. So, suppose 5/8/13, so if it is one of this is there and this is this, so this is the example of complex fraction where you have 1 fraction is there in the case of numerator.

So, this is the example of the complex fraction when we talk about the numbers as you know you have the we discussed about the numbers as whole numbers then you have mixed

numbers. So, mixed numbers comes like you 13/force 1+3/force that is you have number whole number as well as you have the fraction. So, that becomes as a mixed number.

Then once you go to so that is what it defines most of the things like whole numbers or so. Now as you know that rational numbers or irrational numbers we know that rational number can be you know represented in terms of a/b where b is not equal to 0 fraction, certainly fraction is where you both have a and b that is defined and if it is you know if it has a value then it has a defined value.

Then it is you know a real number and if you I mean not rational number otherwise you have irrational number where you cannot uniquely define that is your irrational numbers. So, many a times when you take the example suppose you know like you have root 3. So, this comes as the irrational numbers, now we talk about the terminology I mean other term thing like you have many a times.

Now in this fractions why as they are important because this will be coming in your you know financial terms most often. So, what fraction of something is coming or through things are in this fraction or so or ratios. So, these things will be coming up in this, so that is why fraction needs to be understood, then comes the decimals, so as you know that when you are dividing the numerator by the denominator.

So, you know you get once you divide and if the quotient is not a you know whole number completely. So, in that case we will have reminder that reminder so suppose we divide 25/4. So, basically in this case you get 6 and then +1/4, so that is your 24/25/4, now this term is represented in terms of decimals and that is why you write it as 6.25.

So, that is how this you know when we divide so that is what we get that is so this point is, so that is why these or how we use these term as decimal. And you must have a good concept of the use of decimals because you know after decimal how many you know digits appear they are important certainly in the end whatever 0 you put in that does not matter but whatever you know digits are coming after decimal that you know they are known as the number of decimal places.

So, here in this case you have 2 number of decimal places, so that is how we define another terminology which will come most often is this repentance tens. Now repentance are there that when you many a times when you divide the numerator with denominator. Then in that case when you express it is in terms of decimal you will see that tem is repeated perpetually.

So, that is a case of repentance like if you take the example of 1/3, so this if you are trying to express this it will be 0.3333. So, basically this is the example of repentant, so in this case what we see is that the things are repeated continuously and that is why it is known as the repentant. Then comes something about you know percentages, so as you know that when you are expressing something.

And we are comparing it as compare to 100, so in the denominator you have 100, in those cases we try to tell that we try to you know say that this much of is the percentage. So, suppose something is 50 out of 100 then we call it as 50%, so when we basically you know try to denote a fraction which whose denominator is 100. In that case we talk about the percentage and it is very clear as you know that if it is if you are having the denominator as 60.

In that case the numerator has to be 30, so that same percentage is you know represented like then it is 50%. So, we discussed about the decimals, percentages, then there are certainly very elemental elementary you know tricks that you must know that how the decimal points are moved , what is how there will be effect when they are to be moved as a left or towards the right.

So, when you are basically multiplying with 100 then it has to move towards the decimal point has to move towards right and if you are dividing it then it has to move towards left. So, all these things are basically the you know elemental elementary information that you require to know. Then when we talk about the percentages.

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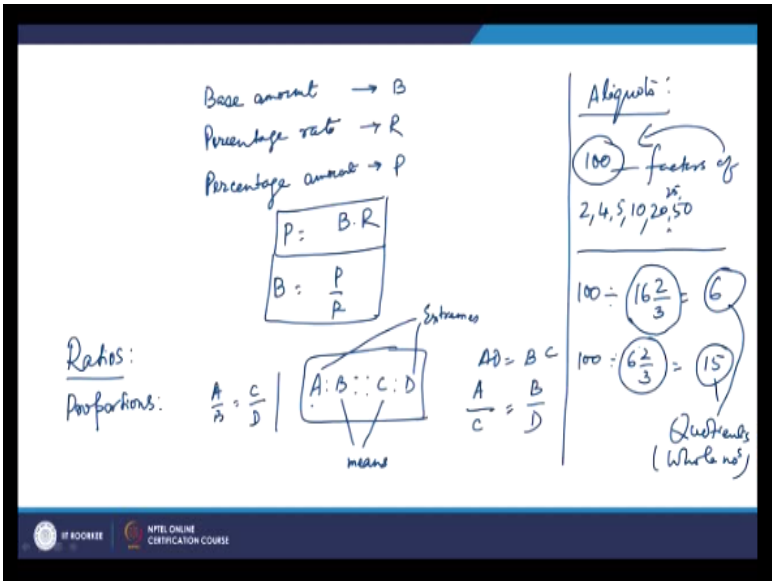

In that case you have few terms like you have base amount then you will have the percentage rate and then you have percentage amount. So, basically the if you try to see that the base amount is you define it as b and you percentage rate is your r and percentage amount is p. So, what is done is that the base amount will be multiplied with the percentage rate and that will be giving you the percentage amount.

So, the expression which will be holding for this it will be p=B*R, so that is how you calculate these percentage amount. Then if one of the things are known you can get the other like if you have to find the base amount and you know the percentage amount and also you know the percentage rate. In that case you can find B=P/R, so depending upon those you know whatever you have you know and whatever you have to know .

Based on these formulas you can find these you know values of you know a percentage rate, percentage amount or you know base amount. Similarly you have also you must know something about the ratios, so when we talk about the ratios, so what we see that we are basically comparing 2 values. So, if you have 2 values and you want to compare then we are basically presenting it terms of ratios like if you take this any room you have length and breath.

So, that can be you know compared in terms of ratios, so you have, so that way we try to have these you know ratios. Then many a times you have also like A:B:C also or you have also ratio of the 4 numbers like A:B:C:D. So, that way in that case A:B ratio is there then B:C

is there, there are C:B is there. So, those things are basically you know defined and that way you will have for the calculation of.

So, if you have 100, 200, 600 and 800, so you will have 1:2 then you have 100 to 200 standard is will be 3. So, like that it will go, so that way your ratio is are to be you know seen, so that we can see that how these are you know define. So, that ratio is also are used then along with that once we read ratio then you have you must have also studied about the terms known as the proportions and here in basically you are having the equality between 2 ratios.

So, suppose you have A=B=C whole D A/B=C/D in that case you write as A:B and that is proportional to C:D. So, that way this when this concept is used we talk in term of the proportions, so as you know that here you will have AD/=BC or what we can see from here also you also can get like A/C=B/D. So, when we talk about they this proportions then in that case these 2 are known as the means.

And then you these 2 are known as the extremes, so what we see is that, so you can have different type of you know relations like the ration of the first you know extreme to the. So, the end extreme of this is first extreme, so and this is the end mean, so like that, so that will be say you know first mean to the next extreme end extreme.

So, that way you can have the ratios defined and you can have the formulas these are known as the extremes and these are all known as the means. So, as you see you can have different types of you know formulas which are applicable in that. Another terminology which is most commonly will be used is you use of aliquots. Now what is there is that in this case you will have is an deviser.

So, by which when a dividend is applied there in that case you do not have any you know remainder. So, suppose when you know that when you talk about 100 suppose you practically say that it has a fractions if you talk about it is factors, factors of 100. So, why will to say that it will be 2 then you will have 4 then you will have 5, then you will have suppose 10.

Then you will have suppose so 20 and then 50 like that, so 250 then it also in between you will have 25, so like that. So, you will have these are the factors, now in this case factor means for that you divide and you get whole number as quotient and you do not have any

reminder. However if you try to see the you know other there are many other numbers by which if you divide they will not be reminder.

Like if you say 100 is divided by 16 you know 2/3, so if you look at this now this will be basically 6. So, here also you get the quotient as a whole number, however you are having this is a mix fraction mix number and you are dividing this and you are getting that. So, you will have this as the you know these are known as the elicits of 100.

So, you will have you know in that case similarly you will have many numbers suppose 6 2/3, so suppose 100/6 2/3. So, if you take that it will be (()) (34:20) say it will be 25, no not 25 it will be 100/20/3, so 100*3/25 will be 15. So, basically you are getting these quotients as whole number which normally these are the whole numbers and but what the deviser these are the devisers.

And these devisers are not basically the whole numbers however the quotient you are getting as whole number. So, they are the you know different type this they are the different elicots they are the elicots of 100. So, if you talk about the different you know elicots of 100 you will see that you will have like 33 1/3. So, if you divide it the with that so you will get quotient as 3.

So, 33, so if you talk about the elicot of 100 you will have elicots as you know you will have elicot as 33 1/3. Similarly you will be also get as 16 2/3 we have already seen then you have 14 2/7. So, that is also one another elicot by which you can have so if you divide it you will get quotient as a whole number. So, these are the different concepts which are used and they will be used for this.

So, this is what we need to know these are the elementary things we should know we should also discuss about certain other mathematical terms. So, functions or so in our next lecture, thank you very much.