Radiative Heat Transfer
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Lecture – 09
Radiative Heat Exchange between Black Surfaces

Hello friends, so far in this course, on radiative transfer, we have learned about the radiative characteristics of plane surfaces and we have also learned about the view factor now, in this lecture we will combine the knowledge of view factors and radiative characteristic of surface is to solve the radiative heat transfer problem between black surfaces.

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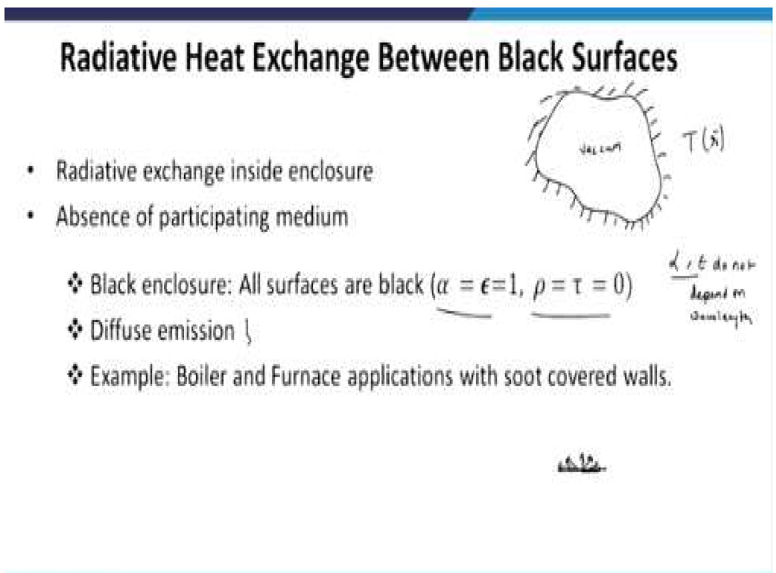

So, we will be studying the radiative heat transfer inside and enclosure, so there can be an arbitrary enclosure, okay and the surface is of this enclosure will be assumed to be black initially, we will assume that the temperature along the surface is not uniform that means, the temperature depends on the location *R* and we assume that the surfaces are black, so by assuming that the surfaces are black, we are basically removing the possibility of reflection from the surfaces.

So, the reflectivity or reflectance of the surface will be equal to 0 and these surfaces will be acting as perfect absorber and perfect emitter, so

$$\epsilon=1$$

we have also assume the surfaces to be black because we want to make sure that alpha and epsilon do not depend on wave length, although these relations are equally valid for gray surfaces also, we will see.

So, we assume that the surfaces are black, they are perfect emitter and absorbers, there is not participating medium, so medium inside this enclosure is vacuum, there is no participating media, we will extend this method to participating media where them enclosure is filled with some kind of gas, okay and the third condition is the surfaces are diffused that means, they emit radiation uniformly in all directions, so they are diffuse emitter, okay.

Such surfaces are although we may think that these surfaces are lot of restrictions is there on the black surfaces but still there are practical problems such as boilers and furnace where we have the walls covered with the small carbon particles called suit and in radiation characteristics of surfaces we have seen that the suit basically essentially acts like a black body, so this walls covered with suit of these furnaces can be approximated as black surfaces.

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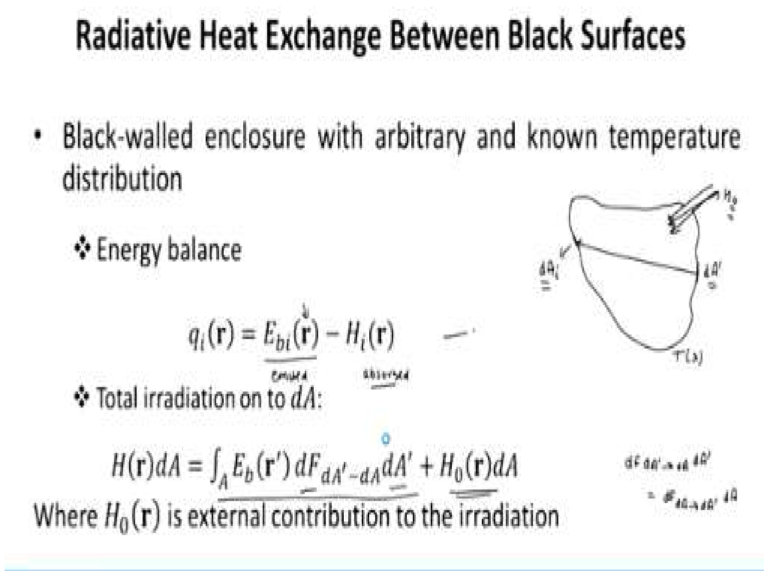

So, we basically are going to find out the relation between these surfaces, so here I am just going to take a special enclosure where we have small opening for through which some kind of radiation from outside can also come, okay, there is a small opening inside this enclosure and there is a small elemental area let us call this dA, okay, so we want to write energy balance for the surface dA, okay.

So, surface dA receives some radiation from rest of the enclosure as well as the outside radiation okay, H_0; H_0 is radiation coming from outside the enclosure, so the radiation received by surface dA will have 2 components, radiation received from the enclosure itself and radiation coming from outside that is H_0, okay in together with the radiation received by surface dA; surface dA will also emit some radiation.

So, this radiation emitted by the surface, let us call this; write it as E_{bi}, okay or write this small area as dA_i, okay, so the total amount of energy transfer or exchange by surface dA_i includes the energy emitted, so this is the energy emitted and this is energy absorbed, okay, there is no α because α = 1, so energy emitted which is same as the black body because you are considering black body in this case and it is the function of $\boldsymbol{r}$.

Because we have assumed that the temperature varies with surface okay, so the emissivity or emittance of the surface is also vary with surface location because the temperature vary with location, so the emittance or emitted energy is simply the black body emits a power as a function of R or as a function of temperature and then energy absorbed, so this energy absorbed is basically α times energy received, α = 1, so basically it is equal to energy received.

$$q_i(\mathbf{r}) = E_{bi}(\mathbf{r}) - H_i(\mathbf{r})$$

Now, energy received H_i is basically some of the energy emitted by entire enclosure, so we can write it in integral forms,

$$H(\mathbf{r})dA = \int_A E_b(\mathbf{r}')\, dF_{dA'-dA}dA' + H_0(\mathbf{r})dA$$

okay the energy received by the surface is equal to energy emitted by the enclosure at any location integrated over this area dA', so we have some area let us call this dA', okay, so energy emitted anywhere on this enclosure times the view factor $dF_{dA'-dA}dA'$.

So, we have to look at the view factor between this 2 areas, okay energy emitted by this surface times the view factor times the emissive power integrated over the entire area of the enclosure plus outside radiation H_0, falling on this area dA_i, so that is the total amount of energy received at surface dA, okay.

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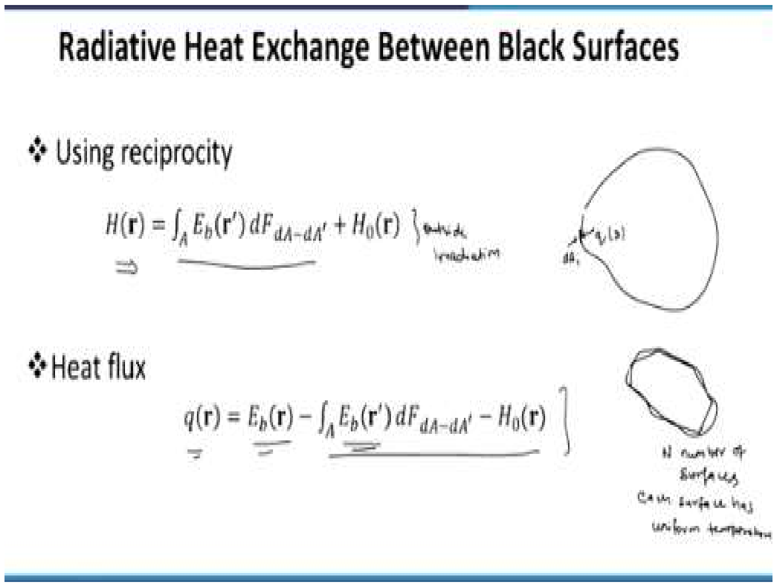

So, using the reciprocity theorem, so we; what we have is here the view

okay, so using the reciprocity theorem that we are substituted here, now cancelling out the dA term on the both hand side, what we get is the total amount of radiation on surface dA, okay.

$$dF_{dA'-dA}dA' = dF_{dA-d\ '}dA$$

So, we have just remove the dA from the both hand sides, so total energy radiative by on the surface dA or energy absorbed by the surface dA is simply

$$H(\mathbf{r}) = \int_A E_b(\mathbf{r}')\, dF_{dA-dA'} + H_0(\mathbf{r})$$

Where $E_b(\mathbf{r}')$ that is emissive power of the black body at any location times the view factor between the two surfaces, now substituting this value of $H(\mathbf{r})$ in this equation, we get radiative heat flux at any location.

So, we have this enclosure, this is surface dA okay, so the heat flux q = emissive power at this location – total irradiation from all the surfaces of the enclosure and outside irradiation, so this is basically the expression for heat flux on a black enclosure at any location ***r***, so heat flux at any location ***r*** depends on the amount of energy emitted from that location - total amount of energy received at that location from the entire enclosure and outside irradiation.

$$q(\mathbf{r}) = E_b(\mathbf{r}) - \int_A E_b(\mathbf{r}')\, dF_{dA-dA'} - H_0(\mathbf{r})$$

Now, what we will do is; now this type of problem cannot be solved okay, we have to do some kind of simplification to solve this integral, okay and what kind of simplification we can do is; first we will approximate these enclosure using some plane surfaces, so we will fit some plane surfaces inside this enclosure to make this problem solvable, okay, there can be *N* number of surfaces, okay.

So, we have to basically do some kind of approximation, we have to approximate that the enclosure contains some *N* number of surfaces and each surface has uniform temperature distribution, so there is uniform temperature on the surface, okay, there is no variation of temperature on that particular surface, from one surface to another, the temperature may change but or a particular surfaces, the temperature is constant or uniform okay.

So, by doing this E_b term can be taken out of the integration, okay, now the temperature is uniform on a surface, so it does not depend on the area of that particular surface, so it can the pulled out of the integral but it will be in the summation.

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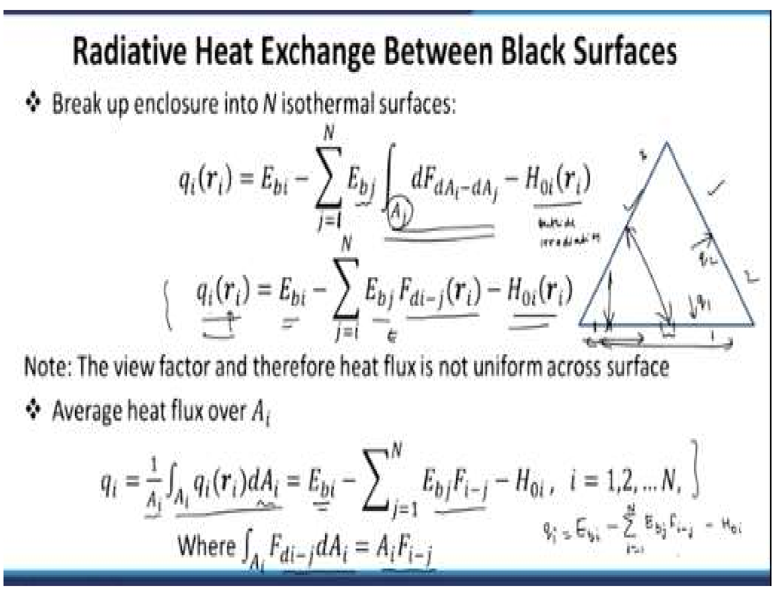

So, we have divide the enclosure into N number of surfaces, so summation 1 to j = 1 to N, E_{bj}, the emissive power of j^{th} black surfaces and then in the integration we are just left with this view factor - the outside irradiation, this is outside irradiation,

$$q_i(\boldsymbol{r}_i) = E_{bi} - \sum_{j=i}^{N} E_{bj} \int_{A_j} dF_{dA_i - dA_j} - H_{0i}(\boldsymbol{r}_i)$$

okay, now looking at the term in the integral, what we have is basically, dF the view factor between small element A_i and small element dA_j and we are basically, integrating over the entire enclosure, okay.

We have to integrate over the entire enclosure, so this can be represented as the view factor dA_j that means view factor of a small element dA_{i} or any location of the enclosure with the entire enclosure, so F_{di-j} represents the view factor of any location of the enclosure with the entire enclosure, okay and we have eliminated the integration from this equation, so all we are left with is heat flux at any surface q_i, okay.

Now, we are still using the dependence on $\boldsymbol{r}_i$, okay, because view factor does depend on location, so the view factor of this location on this particular surface is different from view factor of this location for the same two surfaces, okay, the view factor between these 2 and these 2 is different and that is why the heat flux will vary along the surface, so although the temperature is uniform, we have assumed that the temperature does not vary from one surface to another.

But the view factor does vary, the view factor is not uniform over the surface, so view factor is changing over the surface and that is why the heat flux is going to change that is why we have put that heat flux on surface i at a location $\boldsymbol{r}_i$, okay, it is going to be different at different locations = emissive power of the surface I because the temperature is uniform, it does not depend on $\boldsymbol{r}$ - summation over the all the surfaces, okay.

$$q_i(\boldsymbol{r}_i) = E_{bi} - \sum_{j=i}^{N} E_{bj}\, F_{di-j}(\boldsymbol{r}_i) - H_{0i}(\boldsymbol{r}_i)$$

E_{bj} is the summation over all the surfaces and the view factor from all the surfaces to that small element di, F_{di-j} is the view factor between small element dA_i and all the surfaces of the enclosure and this is going to change with $\boldsymbol{r}_i$ minus the irradiation from outside, okay. Now, so again, although this we have already simplified this problem by assuming that the enclosure can be represented by a finite number of surfaces.

But still there is a problem in finding the view factors F_{di-j}, so we have to do again some simplification, so what we will do is; we have heat flux varying along the surface that is q_i varying with the location on the particular surface I, we will try to find out average heat flux, average radiative heat flux, so what we do is; we integrate over the area dA_i,

$$q_i = \frac{1}{A_i}\int_{A_i} q_i(\boldsymbol{r}_i)dA_i$$

And then divided by A_i, so this is going to be now, average heat flux, so q_1 is average heat flux, q_2 is average heat flux, it is not a location wise heat flux, it is average heat flux over the entire surface of the surface i or j. So, now we just substitute the value of q_i, $\boldsymbol{r}_i$ in this expression, so E_{bi} does not depend on the surface area A_i,

$$q_i = E_{bi} - \sum_{j=1}^{N} E_{bj}F_{i-j} - H_{0i}\ , \quad i = 1,2, \ldots N,$$

So, this is the expression that we will use in solving our problems, okay so our final expression is q_i that it is heat flux on surface i = emissive power of surface i - summation j = 1 to N emissive power of surface j and the view factor between i and j, so this is the heat transfer between surfaces minus the outside irradiation a surface i, okay, so H_{0i} is the outside radiation, so this is the first form of the energy balance between black surfaces okay.

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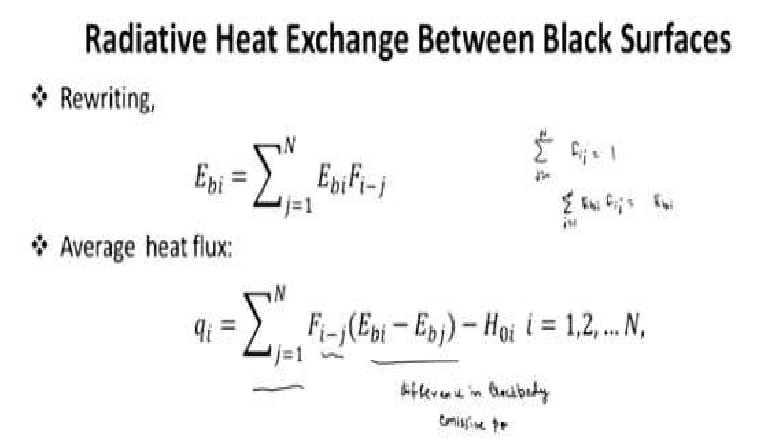

We have another form of this same equation, what we do is;

$$E_{bi} = \sum_{j=1}^{N} E_{bi} F_{i-j}$$

$$\text{Because } \sum_{j=1}^{N} F_{i-j} = 1,$$

Multiplying both sides by E_{bi}, we get

$$\sum_{j=1}^{N} E_{bi} F_{i-j} = E_{bi}$$

so if you write this in this form and substitute for Ebi in this equation here, we get

$$q_i = \sum_{j=1}^{N} F_{i-j}(E_{bi} - E_{bj}) - H_{0i} \;\; i = 1,2, \ldots N,$$

now we have the relation; the same relation we have written in another form in terms of difference in emittent; black body emissive power of the surfaces.

And then summing over, so we have this difference in emissive power; blackbody emissive power, okay, this is another form sometimes this form is preferred, sometimes the other form is preferred okay.

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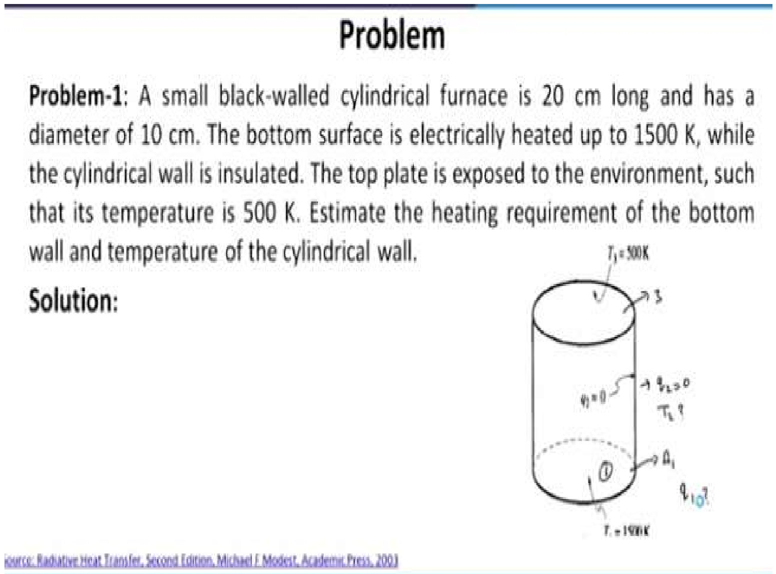

So, let us do one problem to understand this method now, in this method we have taken a small furnace, okay the furnace basically is cylindrical in structure, the bottom surface A_1 is electrically heated using some coils, okay and it is maintained at a temperature of 1500 kelvin, the cylindrical sides of the furnace are insulated that means the heat flux through these walls is zero, 0 let us called this surface 1.

The cylindrical surface is 2, it is insulated, so q_2 = 0 and the top surface is exposed to the environment in such a way that the temperature at the surface is uniformly maintained at 500 K so, we have to find out how much heating we have to provide to this electric heater at these surface in other words, we have to find out q_1 because the amount of energy in the electric heater is basically the heat flux.

And we have to find out the temperature T_2, so these are the 2 quantities that we have to find out so, we will use the theory of the blackbody enclosure that we have derived to solve this problem, so let us write down the solution of this problem, so we will use this expression.

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Solution

$$q_i = E_{bi} - \sum_{j=1}^{N} E_{bj} F_{ij} - \cancel{H_{0i}}^{\,0} \qquad i = 1, 2, 3$$

$$q_1 = E_{b1} - E_{b2} F_{12} - E_{b3} F_{1-3} \quad -(1)$$

$$q_2 = E_{b2} - E_{b1} F_{21} - E_{b2} F_{22} - E_{b3} F_{23} = 0 \quad (2)$$

From Eqn 2

$$E_{b2} = \frac{E_{b1} F_{21} + E_{b3} F_{23}}{1 - F_{22}}$$

$$F_{21} + F_{22} + F_{23} = 1$$

$$1 - F_{22} = F_{21} + F_{23} \qquad F_{21} = F_{23}$$

$$E_{b2} = \frac{1}{2} (E_{b1} + E_{b3}) \Rightarrow$$

$$\cancel{\sigma} T_2^4 = \frac{1}{2} (\cancel{\sigma} T_1^4 + \cancel{\sigma} T_3^4) \Rightarrow T_2 = 1265 K$$

Heating requirement

$$Q_1 = A_1 q_1 = \pi R^2 [E_{b1} - E_{b2} F_{12} - E_{b3} F_{13}]$$

$$= \pi R^2 \sigma [T_1^4 - T_2^4 F_{12} - T_3^4 F_{13}]$$

$$F_{13} = 0.0557$$

$$F_{12} = 0.9443$$

So, let first write down this expression, so we are going to use this expression

$$q_i = E_{bi} - \sum\nolimits_{j=1}^{N} E_{bj} F_{i-j} - H_{0i},$$

okay, there is no outside irradiation, so H_{0i} will be 0, okay, so we will have i = 1, 2 and 3, where i = 1 means the bottom surface, 2 is the cylindrical surface and 3 is the top surface, okay, so

$$q_1 = E_{b1} - E_{b2} F_{12} - E_{b3} F_{13}$$

let's call this a question one

Now

$$q_2 = E_{b2} - E_{b1} F_{21} - E_{b2} F_{22} - E_{b3} F_{23}$$

let us call this equation 2.

please note there is a term for F_{22} here because the cylindrical surface whatever it emits although some part of it will be received by bottom and top surface but some part of the emission from the cylindrical surface will fall on 2 itself, okay so there is a term for F_{22} here, there is no term for F_{11}, because they are flat surfaces, so no radiation emitted from surface 1 and 3 will actually fall on to themselves.

$$q_2 = E_{b2} - E_{b1} F_{21} - E_{b2} F_{22} - E_{b3} F_{23} = 0$$

So from, let us call this equation 2, so from equation 2, we can solve for E_{b2} because E_{b1}, E_{b3} are known, all the view factors we can look at these view factors from the book, these are standard configuration, we have already learned many methods to calculate the view factors, so E_{b1} can be written in terms of view factors and the emissive power as

$$E_{b1} = \frac{E_{b1}F_{21} + E_{b3}F_{23}}{1 - F_{22}}$$

Now, we can simplify this relation at little bit, from summation rule

$$F_{21} + F_{22} + F_{23} = 1$$

so

$$1 - F_{22} = F_{21} + F_{23}$$

Now what is F_{21} and F_{23}, F_{21} is the view factor between cylindrical surface and the bottom surface and F_{23} is the view factor between cylindrical surface and top surface which has equal, so

$$F_{21} = F_{23}$$

so with this, we get E_{b1} or E_{b2},

$$E_{b2} = ½(E_{b1} + E_{b3})$$

so, we have taken the this out.

so now E_{b1} and E_{b3} we already know, so we can solve for this okay, so

$$\sigma T_2^4 = \frac{1}{2}(\sigma T_1^4 + \sigma T_3^4)$$

σ will cancel out and from this we can get

$$T_2 = 1265 \text{ kelvin}$$

So the temperature of the surface 2 that is the cylindrical surface is 1265 kelvin, okay. Now, to find out the heating requirement that is amount of energy we have to supply to the electric heater.

So that the bottom surface can be maintained at a temperature of 1500 kelvin will be

the radius of the cylinder is already given, okay, the diameter is given as 10 centimetre, so R in this case is 5 cm or 0.05 meters, so R we know and then q1;

$$Q_1 = A_1 q_1 = \pi R^2 [E_{b1} - E_{b2}F_{12} - E_{b3}F_{13}]$$

so E_{b1} we know,

$$Q_1 = \pi R^2 \sigma [T_1^4 - T_2^4 F_{12} - T_3^4 F_{13}]$$

Now, all this view factors, we can calculate using the table in the book, I will just give you the value here, so F_{13} that is parallel disc

$$F_{13} = 0.0557$$

and F_{12} that is between disc and the cylindrical surface

$$F_{12} = 0.944.$$

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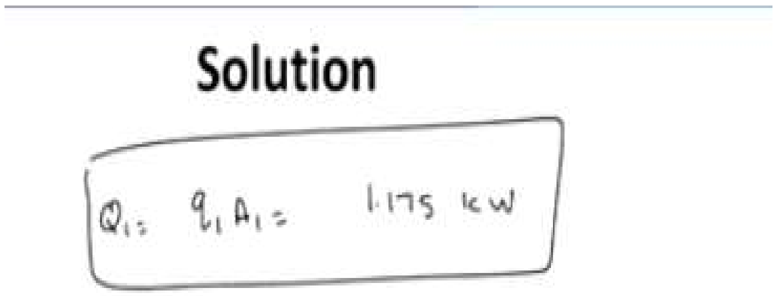

And with these values, we can substitute in the relation, we get

$$Q_1 = A_1 q_1 = 1.175 \text{ kilowatt}$$

So the total amount of energy that is required from the electric heater is 1.175 kilowatt, so this is a simple example on the radiative transfer between black surfaces, okay.

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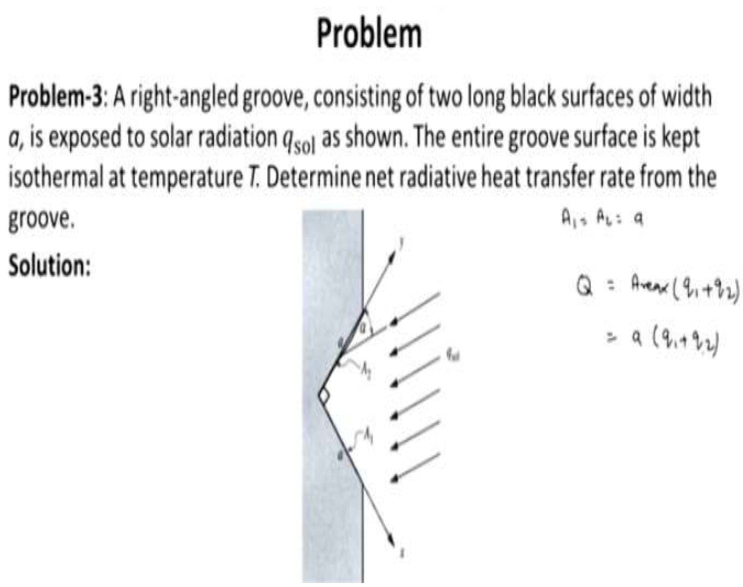

We will do one more problem, now in this case the problem also considers the radiation from outside that is we have radiation coming from outside the enclosure, so we have basically rectangular right angle grooves, there is a 90 degree angle, we have 2 surfaces, they have infinite width in the plane of the board, so the surfaces have a basically an area $A_1 = A_2 = A$, assuming unit depth in the plane of the board.

The angle between the two surfaces is 90 degree, okay, now radiation on this enclosure or groove is basically the solar radiation at an angle alpha as shown, the temperature of this enclosure is maintained, so it is an isothermal groove and the temperature is kept T, so we have to determine the net radiative heat transfer from the groove that means we have to find out total amount of heat transfer rate due to radiation.

That means

$$Q = Area(q_1 + q_2)$$

this is the quantity that we have to find out, so this will be simply, this quality we have to find out.

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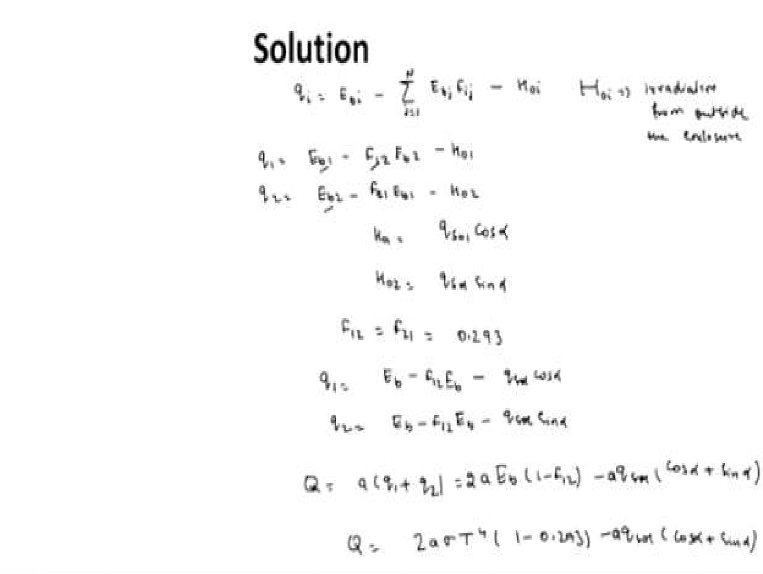

So, let us solve this problem, this problem is again very simple problem, so we will use this expression

$$q_i = \sum_{j=1}^{N} F_{i-j}\left(E_{bi} - E_{bj}\right) - H_{0i}$$

So this is H_{0i} is irradiation from outside the enclosure, so we write for surface 1;

$$q_1 = E_{b1} - E_{b2}F_{12} - H_{01}$$

Now there is no F_{11} and F_{22} because these are flat surfaces. And

$$q_2 = E_{b2} - E_{b1}F_{12} - H_{02}$$

Where H_{01} and H_{02} are basically the radiation falling onto this surface from outside, so

$$H_{01} = q_{sol}cos\alpha$$

$$H_{02} = q_{sol}sin\alpha$$

q_{sol} is the solar energy coming onto the surface at an angle α_1, so one plate makes an angle α and other surface will make an angle 90 – α, so what do we get; and the view factor

$$F_{12} = F_{21}$$

because of symmetry.

And we can find out again using tables, so there is the entry in the view factor table in the book for 90 degree grooves, okay and this fact

$$F_{12} = F_{21} = 0.293$$

if we calculate using the table. Now, all the quantities we know, E_{b1} and E_{b2} we know, they are same, σ T^4, the temperature of the 2 plates is same and F_{12} and F_{21} are also same.

So, what do we get;

$$q_1 = E_{b1} - E_{b2}F_{12} - q_{sol}cos\alpha$$

And

$$q_2 = E_{b2} - E_{b1}F_{12} - q_{sol}sin\alpha$$

Now, the total energy; total irradiation,

total radiation exchange from the groove

$$Q = 2a(q_1 + q_2) = 2aE_b(1 - F_{12}) - aq_{sol}(cos\alpha + sin\alpha)$$

So this becomes

$$Q = 2a\sigma T^4(1 - 0.293) - aq_{sol}(cos\alpha + sin\alpha)$$

T is the temperature of the groove.

So, this is how you can incorporate the irradiation from outside the enclosure into your problem okay, okay, so in this lecture, we develop relation for radiative heat transfer, radiative heat exchange between black surfaces in an enclosure together with irradiation from outside the enclosure, in the next lecture, we will consider radiation exchange between gray surfaces, thank you very much.