Radiative Heat Transfer
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Lecture - 06
View Factor

Hello friends. In the previous lectures, we discussed about radiative characteristic of plane surfaces and their properties and their evaluation from basic fundamental laws. Now in this lecture, we will discuss about radiative transfer between two surfaces. The first topic on this series is based on view factor.

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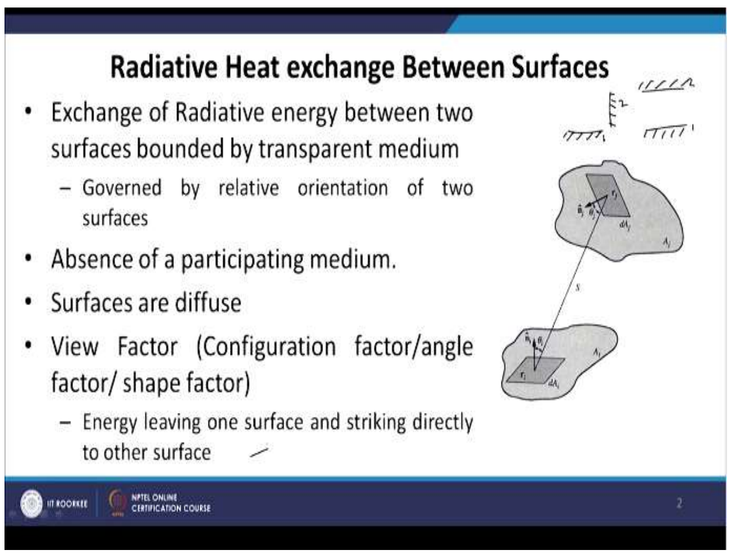

So it is pretty much very much clear that amount of energy exchanged from one surface to another will depend on orientation of the two surfaces. So for example, we have two surfaces having the same area and similar temperatures. If their orientation is not same, the amount of energy transferred from one surface to the other will not be same.

For example, if we have surface 1 and surface 2 in this configuration having the same temperature and area and we have this configuration, the two plates have the same area, similar temperatures but the amount of energy transferred between these two surfaces will not be same and the reason is that the orientation of one surface with respect to other is different. Now to account for this effect, the angular dependence or the orientation, we talk about view factors.

Now on view factors, we talked about exchange of radiative energy between two surfaces bounded by transparent medium. So the condition for calculating the view factor is that the medium surrounding these two surfaces should be vacuumed. We can also calculate similar factors when the medium is not surrounded by vacuum or it is surrounded by some kind of gases that absorb and emit radiation but that part we will reserve for later lectures.

In this part, we will assume that the surfaces are surrounded by vacuum and we also assume any absence of participating medium. The second assumption in this evaluation of view factor is that the surfaces are diffuse. That means they emit radiation equally in all the directions. They also reflect radiation, they may or may not reflect radiation but if they do they reflected in diffuse manner.

So view factor is basically the orientation between the two surfaces, sometimes it is called configuration factor, angle factor or shape factor. So we define this view factor as energy leaving one surface and striking directly to other surface. So what we mean by directly striking the other surface means that reflected radiation from some other intermediate surfaces is not accounted.

So view factor means that radiation is emitted by one surface, it is diffused and it is absorbed by the other surface directly and there is no intermediate reflection that is taken into account in the evaluation of view factors.

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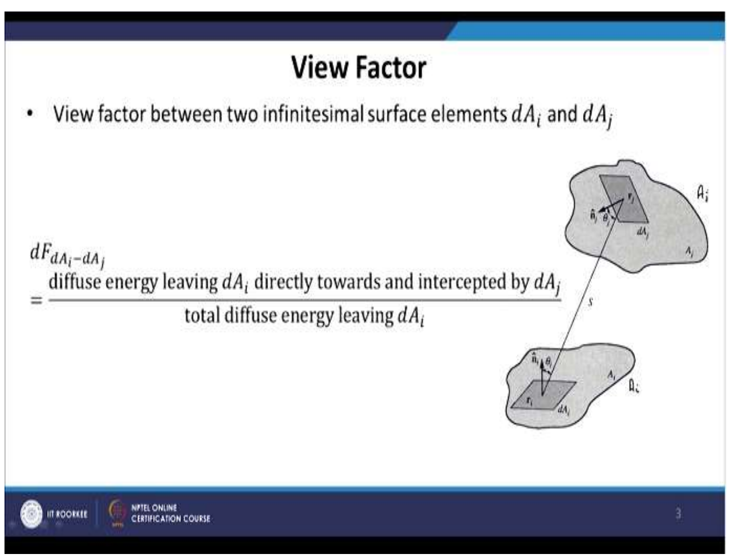

So the definition of view factors goes like this. For two surfaces, A_i and A_j let us consider some infinitesimal areas dA_i on surface A_i and dA_j on surface A_j. Thus, view factor between these two infinitesimal surfaces is defined as
View factor = diffuse energy leaving surface dA_i directly towards and intercepted by surface dA_j / total diffuse energy leaving surface dAi. So total energy will be leaving surface dAi in all 2π solid angle.

But only a fraction of this energy will actually reach surface dA_j and this is governed by the orientation, the solid angles and relative areas of the two surfaces.

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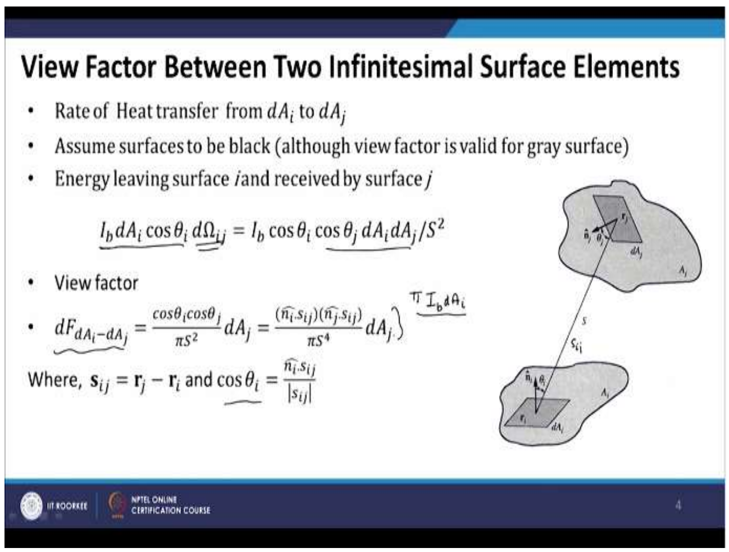

So how do we evaluate this view factor mathematically? So view factors basically gives you an idea how much rate of heat transfer activity place between surface A_i or first surface and surface A_j the second surface. So although the definition of view factor is equally applicable when the surfaces are gray. That means their emittance and absorptance does not depend on wavelength.

But in this particular derivation, we will assume that the surfaces are black. That means they emit radiation equal to the blackbody radiation and they absorb all the radiation falling onto them. So the view factor definition that we will derive will be based on the gray blackbody radiation; however, the view factors are applicable to gray surfaces as well. So energy leaving surface A_i and travelling to surface A_j is calculated based on the intensity of radiation of blackbody Ib.

So the amount of energy emitted, total amount of energy emitted by surface A_i depends on the blackbody intensity I_b which is we know is energy transmitted or emitted per unit solid angle per unit area normal to the rays. So we have to multiply by the projected area dAi cos theta i to calculate the total energy and multiply by solid angle between surfaces i and j that is d omega ij.

We can write the value of solid angle as $I_b \cos\theta_i \cos\theta_j \, dA_i dA_j/S^2$. So what we have done is we have written the solid angle in terms of angle θ_j. So $d\Omega_{ij}$is basically given by $\cos\theta_j \, dA_j/S^2$. That is the projected area along the direction s_{ij}. s_{ij} is the direction joining surface A_i and surface A_i. So along this direction, the projected area of dA_j is $\cos\theta_j \, dA_j$.

And then divided by the distance between the two surfaces S^2 gives you the amount of energy leaving surface dA_i and reaching surface *j*. So the view factor is basically defined as amount of energy that is emitted and received by surface j/total amount of energy emitted by surface A_i. So total amount of energy emitted = $\pi I_b dA_i$. This is the total amount of energy that is emitted by surface A_i is dA_i.

And when we divide the amount of energy received by surface A_j dA_j by this total amount of energy emitted, we get the view factor between these two infinitesimal surfaces that is *dF* the small view factor between surface dA_i and dA_j is equal to $\frac{cos\theta_i cos\theta_j}{\pi S^2} dA_j$. We can also write this view factor in terms of vector notation where we have the angle cos theta *i* written in terms of the normal vector n_i,

$$cos\,\theta_i = \frac{\widehat{n_i}.s_{ij}}{|s_{ij}|}.$$

This is the angle $cos\,\theta_i$, so we replace the $cos\,\theta_i$and $cos\,\theta_j$ with this vector quantities and we get the value of view factor

$$\text{view factor} = \frac{(\widehat{n_i}.s_{ij})(\widehat{n_j}.s_{ij})}{\pi S^4} dA_j.$$

So we can sometimes use vector notation to evaluate the view factors or sometimes we can use the trigonometric angles to calculate the view factors.

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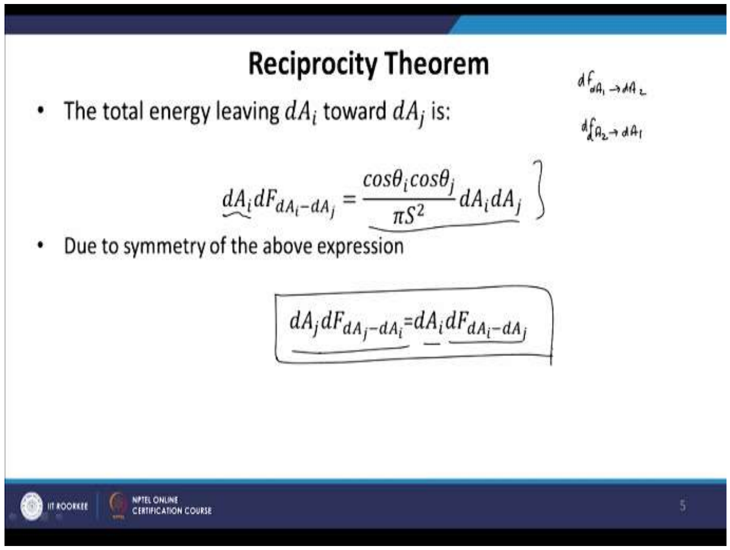

Now the view factors evaluated from one surface to another that is surface 1 to 2 $dF_{dA_1-dA_2}$ can be used to evaluate the view factor from surface A_2 to surface A_1 that is $dF_{dA_1-dA_2}$. So we have to basically apply what we call reciprocity theorem. In reciprocity theorem, the view factors from surface 1 to 2 is related to surface 2 to 1 using their areas. So what we do is in the previous definition of view factor $dF_{dA_i-dA_j}$, we multiplied by the area of the surface dAi that is we multiplied by this area.

And what we get is expression of view factor multiplied by the small element area dA_i and this expression is symmetric i and j. So if you replace i with j and j with i, this expression will not change. So what we conclude from this expression is that dA_j $dF_{dA_i-dA_j}$, that is view factor multiplied by area of surface j is equal to area of surface A_i small element area dA_i times small view factor from surface A_i to surface j that is the reciprocity theorem okay.

So if you know view factor from one surface to the other, you can calculate easily the view factor from the other surface to the first surface using this reciprocity theorem. So this exercise was done to calculate view factor between two infinitesimal surfaces dA_i and dA_j. We can extend the same logic to find out view factors between a small infinitesimal surface and a finite area surface okay.

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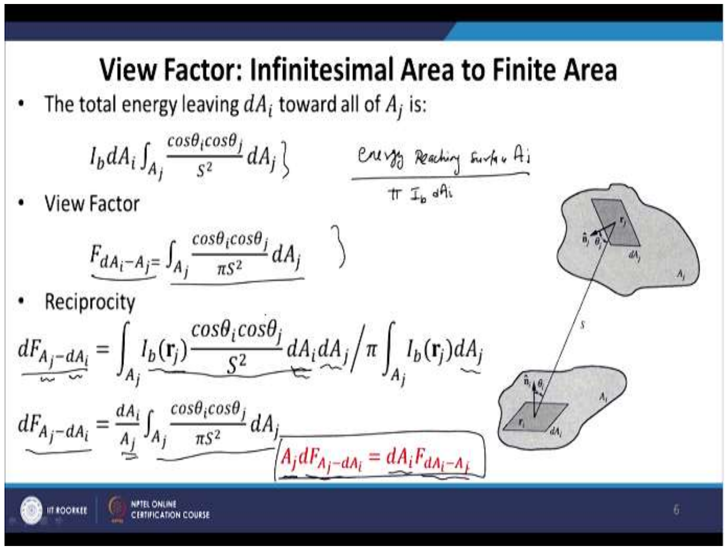

So if we want to find out the surface area or this view factor that is view factor from a small infinitesimal surface dA_i with whole surface A_j we have to basically extend the same logic, the total energy leaving surface dA_i towards the entire area A_j. So all we have to do is use this previous expression that we basically used. We used this expression $I_b dA_i \cos\theta_i \, d\Omega_{ij}$ and just multiply or integrate over the entire area dA_j.

So this basically gives you the total energy leaving dA_i towards all of A_j. So all we are doing is just integrating over the area A_j and we define view factor again by dividing this energy with total energy. So this is energy reaching surface the entire surface A_j/total energy emitted that is ($\pi I_b dA_i$) okay. So we just divide by the total amount of energy emitted by surface A_i and we define this as a view factor $F_{dA_i - A_j}$ and this expression of this view factor is very similar to the previous one.

Just we have an integration here over the entire surface A_j. So the expression is integration

$$\int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi S^2} dA_j$$

Now again this will satisfy the reciprocity theorem that we discussed. So dF we write dF from the entire area A_j to small area dA_i is equal to the amount of energy emitted from the surface okay and integrate it over the entire surface A_j.

So the first quantity is basically the amount of energy emitted from surface j total surface j reaching surface element small surface element A_i and then we just integrate over the entire surface okay and divided by the total energy $\pi I_b dA_i$ and then the view factor $dF_{A_j-dA_i}$, this A_i is constant over A_j. The dA_i is not changing, we can take it out okay and this quantity becomes πI_b, the A_j becomes in the denominator.

$$dF_{A_j-dA_i} = \int_{A_j} I_b(\mathbf{r}_j) \frac{cos\theta_i cos\theta_j}{S^2} dA_i dA_j \Big/ \pi \int_{A_j} I_b(\mathbf{r}_j) dA_j$$

So we get $dF_{A_j-dA_i}$ that is the view factor from the entire surface A_j to small element dA_i is equal to the area of small element dA_i which we have taken out from this integral divided by A_j

So what we get is this reciprocity theorem, the total area of surface j, A_j multiplied by the view factor from the entire surface j to small element dA_i is equal to the small area of surface A_i and view factor from small element A_i to the entire surface A_j. So this satisfies the reciprocity theorem. Now just like the view factors between an infinitesimal area and a finite area, we can calculate view factor between two areas of finite magnitude.

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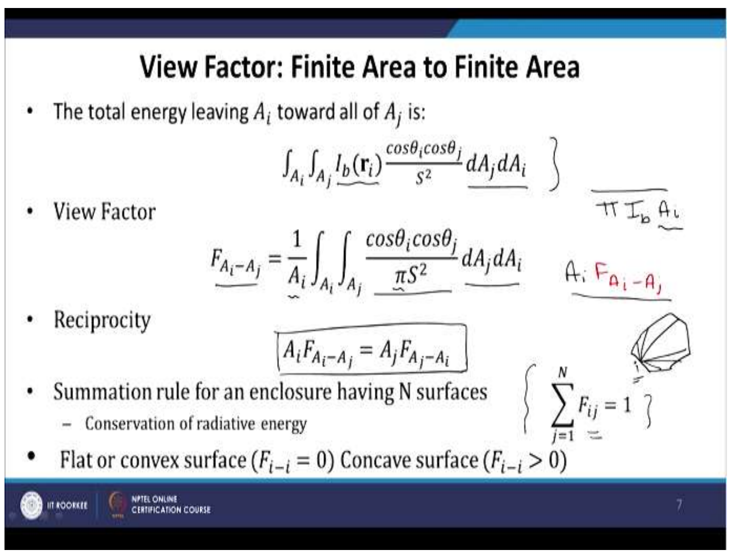

Now in the case where we calculated view factor between infinitesimal areas, we just integrated over area dA_j. Now when we are trying to find out view factors between two finite areas, we have to do double integration and this double integration has to be taken place over the entire surface A_i and surface A_j. So the total energy emitted from surface A_i is $I_b cos\theta_i dA_i$ and this surface energy basically reaches the surface A_j okay.

So this is the total amount of energies between surface A_i and surface A_j. Now we should keep in mind that the temperature of this surface should be uniform okay. If the temperature of the surface is not uniform, the intensity I_b will vary over the surface and then the definition of view factor will change. For this purpose, we will assume that I_b is independent of the surface okay that is it is uniform over the surface.

So this total amount of energy when we divide by total energy emitted by surface A_i in all the solid angles that is $\pi I_b A_i$. So this is the total amount of energy emitted by surface A_i over all the solid angles. We divide this total energy reaching surface A_j by this quantity, we get the definition of the view factor $F_{A_i-A_j}$ which is equal to now this A_i will basically come in the denominator and the π will basically come in the denominator of the expression of the energy reaching A_j.

So the view factor is basically

$$F_{A_i-A_j} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{cos\theta_i cos\theta_j}{\boldsymbol{\pi} S^2} dA_j dA_i$$

So this is the view factor between two finite surfaces. Now this expression the $A_i F_{A_i-d\ \ j}$ is symmetric okay, this expression is symmetric that is when we change i and j indices, the expression does not change. So from this we can conclude that

$$A_i F_{A_i-dA_j} = A_j F_{A_i-dA_j}$$

That is the reciprocity theorem is valid for view factors on finite areas as well. So other than these reciprocity theorem, we also have summation rule. So if we take an enclosure of any number of surfaces okay, the total amount of energy leaving any surface i including itself okay, so the total amount of energy leaving surface i and going to all the surfaces including itself should satisfy the conservation of radiative energy okay.

And we can basically based on this argument conclude that the summation of view factors over all the surfaces j is equal to 1 to N will be equal to 1. That means total amount of radiative energy is conserved okay. So all the energy leaving surface i should reach one of the surfaces from 1 to N and the summation over the view factor should be equal to 1. So what is basically view factor?

View factor is nothing but a fraction of how much energy leaving surface 1 is reaching to other surfaces. So this basically gives you an idea of that the fraction should add up to 1. Based on the same thing, same argument, the flat or convex surface will have a view factor of 0 to itself. That means if you have a flat surface, the energy will be emitted in all direction and this energy will not strike itself.

The surface will not receive any radiation that is emitted by itself okay because it is flat and similar argument the convex surface also will not receive any radiation falling onto itself okay. So for these two surfaces, the view factor F_{ii} that means the view factor to itself will be equal to zero. On the other hand, for concave surfaces, the view factor will not be equal to 0; it will have some finite value.

Because some amount of radiation leaving this surface will actually fall upon this surface itself okay. So

$$F_{ii} \text{ for concave surfaces will be } >0$$

So these are the definition of view factors between infinitesimal areas, infinitesimal area and a finite area and between two finite areas. Now how do we evaluate these view factors? So there are number of ways by which these view factors can be evaluated.

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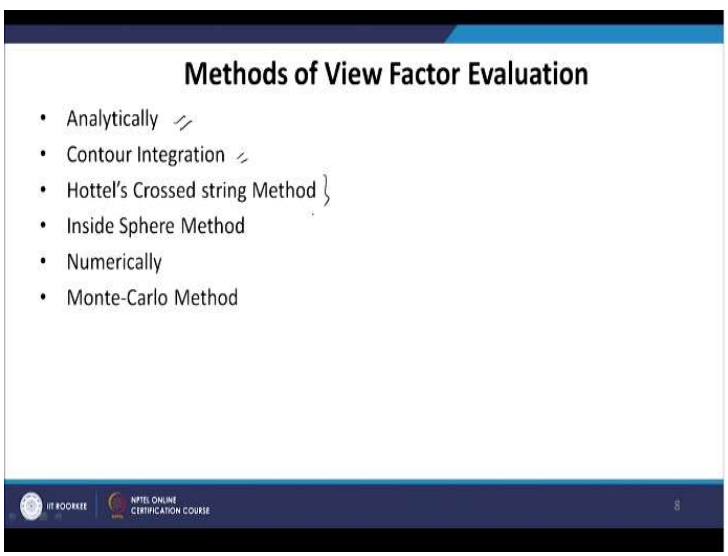

We can do double integration that is called analytical method. There are simplified methods like contour integration which basically simplifies the double integration, double surface integral into line integral, the path integral. Then, we have Hottel cross string method, inside sphere method. We can use numerical methods where we divide the surface into a small number of infinitesimal surfaces.

And we can do calculation based on some numerical scheme and then Monte Carlo method can also be used. Monte Carlo method based on the ray-tracing scheme can also be used to evaluate the view factor. So in this lecture, I will focus on the analytical methods.

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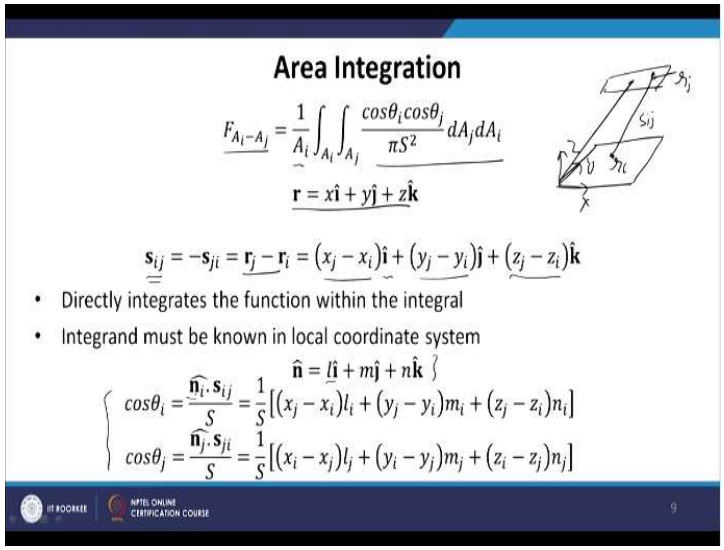

So we have the first method, the first analytical method is the area integration where we have to basically integrate over the surfaces to evaluate the view factor.

$$F_{A_i-A_j} = \frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{cos\theta_i cos\theta_j}{\pi S^2}dA_j dA_i$$

So this was the expression we already developed for the view factor F_iA_j between two finite surfaces. Now to evaluate this integral, we use Cartesian notation.

For different problems, we can use different notations but just to give an idea in the Cartesian notation, we can represent any point on the surface, first we have to find out the origin, we can put an origin on the surface okay and then we can represent any vector r either on this surface or on the other surface from this origin will depend on the x coordinate, y coordinate and z coordinate of the surface okay.

And we represent the vector sij as the vector joining two points or two infinitesimal areas between two surfaces. So this is s_{ij} and s_{ij} can be represented in terms of the coordinates of this point which is r_i and coordinate of this point which is r_j. So $\mathbf{s}_{ij} = -\mathbf{s}_{ji} = \mathbf{r}_j - \mathbf{r}_i =$ and the direction $\mathbf{s}_{ij}$ is just with the minus sign okay. So ultimately what we are doing is taking the vector difference between the two coordinates.

$$\mathbf{s}_{ij} = -\mathbf{s}_{ji} = \mathbf{r}_j - \mathbf{r}_i = (x_j - x_i)\hat{\mathbf{i}} + (y_j - y_i)\hat{\mathbf{j}} + (z_j - z_i)\hat{\mathbf{k}}$$

So other than this, we also need the information for angles so let us say we have been given the normal vector to the two surfaces represented by

$$\hat{\mathbf{n}} = l\hat{\mathbf{i}} + m\hat{\mathbf{j}} + n\hat{\mathbf{k}}$$

the direction cosines l, m and n. So l is the direction cosine in x direction, m is direction cosine in y direction and n is direction cosine in z direction. So $\hat{\mathbf{n}}$ is the normal vector in the sense $\hat{\mathbf{n}}_i$ will be normal vector to surface i and n_j will be normal vector, unit normal vector to surface j okay.

And we can find out $cos\theta_i$and $cos\theta_j$ using this expression okay. So this is the information that we need to find out the view factor using this analytical method. So of course, this method is going to be little involved as we will see with the help of an example. There is going to be lot of mathematics, we have to take care of lot of vector quantities to simplify this method, although this is not always the simplification.

For some geometries, you will find that contour integration is little easy to do while for some quantities for some configuration actually the contour integral may be more complicated than the area integration but for some applications this will be easy. So contour integration is based on Stokes theorem that basically converts a surface integral into a line integral.

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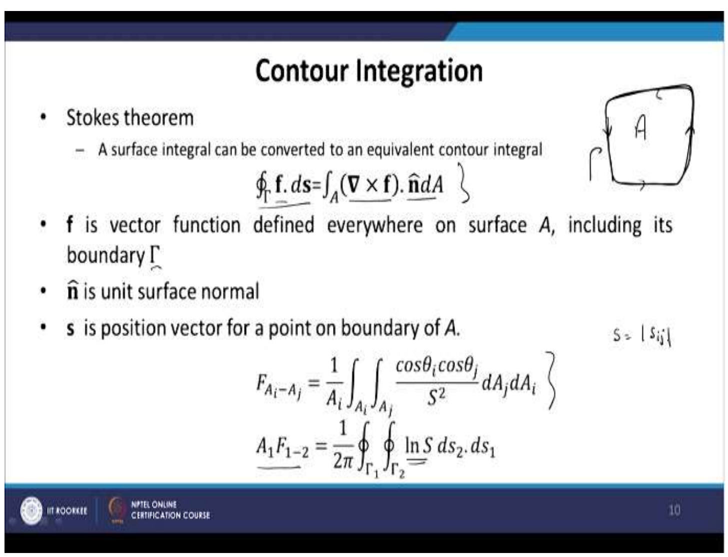

- So the Stokes theorem is given by this, so let us say we have f a function, a vector function which is defined on surface A including the boundary Γ okay. So there is a function f. The Stokes theorem says that the surface integral of the curl of f and dotted with the area vector and integrated over the surface is equal to the function f evaluated using the line integral over the path around the surface.

So if you have the surface A, this is the path that we have to follow okay. So this is basically the boundary of the surface. So we have surface A and we have this path of this around this surface that we have to evaluate okay and n is in the unit normal vector to the surface okay. So with some mathematical manipulation, the surface integral, the double surface integral in the finite area view factor can be converted into double line integral by applying the Stokes theorem twice.

And what we get is A_1F_{1-2} that is basically the view factor between surface 1 and 2 or i and j is equal

$$A_1F_{1-2} = \frac{1}{2\pi}\oint_{\Gamma_1}\oint_{\Gamma_2}\ln S\, ds_2.\, ds_1$$

S is the same s_{ij}, the magnitude of s_{ij} is basically the S okay. So this is how we will evaluate.

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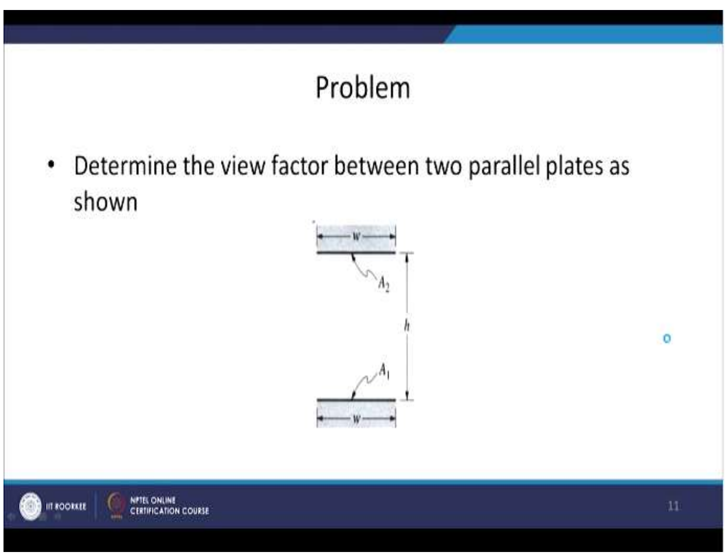

So let us conclude this chapter with one example. So we have two parallel plates, the width of each plate is w and the distance between the vertical distance or the separation between the two plates is h and the plates extend to infinity that means the thickness of these plates in the plane of the board or paper is infinity. So we have to evaluate the view factor for this configuration.

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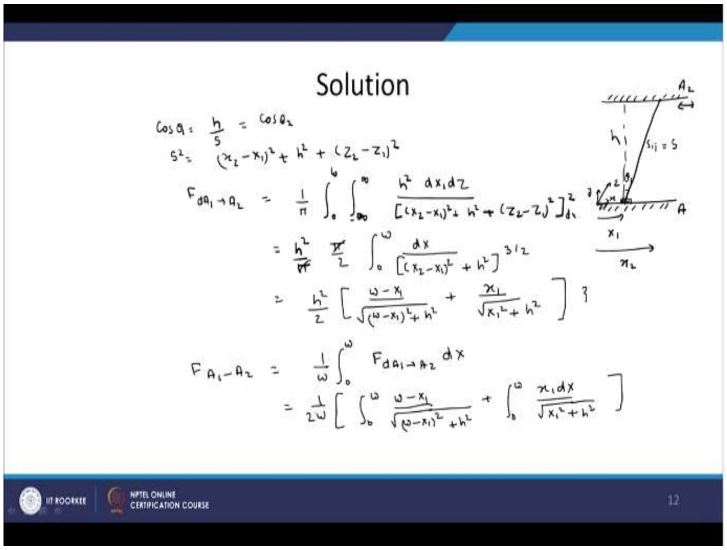

So let us apply the area integration method for this okay. So we have this is surface A_2 and this is surface A_1 and any point we can just put our coordinate system here, this will be y direction and z direction will basically be in the plane of the paper. So we will start with the z and then we can put z tending to infinity for the thickness of the plates reaching infinity in the z direction okay.

So this will be s_{ij} okay, so this will be x_1 and the coordinate of this point will be x_2 and similarly y_1 and y_2. So what we write is the first thing we will write is the angles, cos theta 1 okay, this is the angle theta 1 so$cos\theta_1$ is simply equal to and this will be h/s okay. So

$$cos\theta_1=h/s$$

okay where h is basically this distance. So from this triangle s_{ij} is equal to S,

$$h/s=cos\theta_1$$

$$cos\theta_1=h/s=cos\theta_2$$

are equal in this case in this particular example. So we write

$$S^2=(x_2-x_1)^2+h^2+(z_2-z_1)^2$$

Now we are assuming that the plates are of finite depth but we will eventually put z tending to infinity. So we define first a view factor a small element dA_1 to entire surface A_2 okay.

$$F_{dA_1-A_2}=\frac{1}{\boldsymbol{\pi}}\int_0^W\int_\infty^\infty\frac{h^2dx_1dx_2}{[(x_2-x_1)^2+h^2+(z_2-z_1)^2]^2}$$

So this expression we are going to use, so this can be defined as where z limits are from infinity to infinity we assume that the z direction is infinite. We have to integrate over the two areas and then in the denominator we have, so remember we have S^4 in this because the S square is already there in the expression but we have S in cos theta also. So in the denominator we end up with S^4 okay.

$$F_{dA_1-A_2} = \frac{h^2}{2}\int_0^w \frac{dx}{[(x_2 - x_1)^2 + h^2]^{3/2}}$$

So this is the view factor between small area dA on surface 1 and entire surface A_2. Now to calculate the view factor from entire surface A_1 to A_2, we just have to integrate over the entire surface A_1.

$$F_{dA_1-A_2} = \frac{h^2}{2}\left[\frac{w-x_1}{\sqrt{(w-x_1)^2+h^2}} + \frac{x_1}{\sqrt{(x_1)^2+h^2}}\right]$$

So these two integrals again can be solved easily, also it is worth observing here that the two integrals are pretty much the same okay.

$$F_{A_1-A_2} = \frac{1}{w}\int_0^w F_{dA_1-A_2}\, dx$$

We can make change of variables; they will be same okay.

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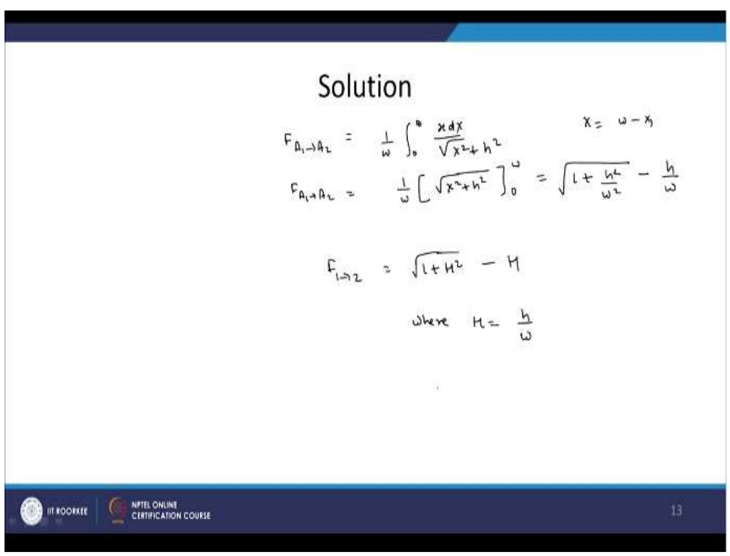

$$F_{A_1-A_2} = \frac{1}{2w}\left[\int_0^w \frac{w - x_1}{\sqrt{(w - x_1)^2 + h^2}} + \int_0^w \frac{x_1 dx}{\sqrt{(x_1)^2 + h^2}}\right]$$

$$F_{A_1-A_2} = \frac{1}{w}\left[\int_0^{\infty} \frac{xdx}{\sqrt{(x)^2 + h^2}}\right]$$

Here

$$x = w - x_1$$

$$F_{A_1-A_2} = \frac{1}{w}\left[\int_0^{\infty} \sqrt{(x)^2 + h^2}\right]_0^w = \sqrt{1 + \frac{h^2}{w^2}} - \frac{h}{w}$$

$$F_{1\to 2} = \sqrt{1 + H^2} - H$$

Where $H=h/w$

So this is the view factor basically okay. So F_{1-2} for this configuration using the area integration technique is simply 1+h square-h where h is equal to h/w okay. So this is how you can apply the area integration technique and find out the view factors. As you have already observed the mathematics involved in this calculation is pretty involved and many people, many researchers have already found the view factors for standard configuration.

And they are available at the back of the textbook okay, so please try to solve a couple of configuration yourselves using this method. Thank you for your time. In the next class, we will use another method to find out the view factor.