

**Radiative Heat Transfer**  
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**Module - 8**  
**Lecture - 36**  
**Radiative Properties of Particulate Media**

Hello friends, I welcome you all to this course on radiative heat transfer. In the last few lectures, we discussed radiative properties of gases. We found that the radiative properties of gases vary very erratically with the wavelength. And we represented this erratic behavior using approximate models, especially we discussed narrow band and wide band models. We discussed k-distribution models. And we found that some of these models are very accurate, while other models, they are based on simplified assumptions and cannot be directly applied to non-homogeneous path.

Now, today we will discuss radiative properties of particles. There are large number of applications where radiation is basically governed by the presence of particles. And in fact, in many applications, particles are the major species radiating and absorbing the gases, absorbing the radiation from the walls. For example, in furnaces, radiation is mainly governed by soot and ash particles.

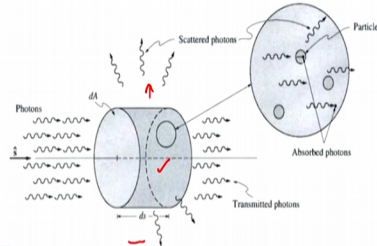
In atmospheric radiation, we have number of particles ranging from small molecules to aerosols. Compared to gases which have spectral behavior organized into lines and bands, the spectral behavior of particles is rather smooth. Although it is smooth, but the variation of this spectral behavior is very difficult to calculate. So, even though the spectrum is relatively smooth, finding the properties of particles is not straight forward.

And the reason is, there is no guarantee to understand the size, shape and distribution of particles. So, if you know the size and shape of the particles, the calculation may be relatively easy. But, because in a particular application, we may not know the size and shape of the particles, the calculations may turn out to be more difficult than gases.

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## Radiative Properties of Particulate Media

- ❖ Gas molecules, ash, char, soot, aerosol, smoke etc.
- ❖ Attenuation by absorption/scattering
  - ❖ Poor visibility on foggy and dusty days
  - ❖ scattering of sunshine by atmosphere: blue sky, rainbow



So, we understand that the particles, they tend to absorb and scatter radiation. As is clear from this image, we have bunch of photons coming. And when they enter the medium which is having particles that absorb and scatter radiation, some of the photons are attenuated or absorbed within the medium. And some of the photons are scattered in this direction. In fact, that scattering may actually depend on direction.

Different amounts of photons may be scattered in different directions. So, we have to consider both absorption and scattering from the particles. The particles may be gas molecules itself. So, we may have some applications where the gas molecules itself act as a scattering agent. But we may also have ash, char, soot, which are basically burnt or partially burnt particles, carbon particles.

We may also have various aerosols which absorb radiation, smoke, etcetera. So, these particles, they are may, they may be present in different amount and they may be present in the form of cloud and they are subject radiation to scattering and absorption. For example, if you have fog, you have dust in the atmosphere, then you have very poor visibility, because of the scattering from this fog and dust particles.

And similarly, in atmosphere, we have sky color which appears blue sometimes and it appears red sometimes. And it depends on scattering from the molecules of water molecules and other molecules in the atmosphere which scatter radiation from the sun and gives color to the sky. So, we have what we called blue sky color, we have rainbows. So, all this phenomenas are basically related to scattering by the particles.

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**Scattering**

- ❖ Amount and direction of redirected photon after hitting particle/particle cloud
  - ❖ Shape of particles (spherical or cylindrical)
  - ❖ Material of particles: complex index of refraction ( $\underline{m} = n - ik$ )
  - ❖ Relative size (compared to wavelength of radiation)
  - ❖ Clearance between particles

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So, how this scattering is basically depend on the properties of the particles, how scattering depends on what kind of particles we have, it can be basically broadly understand understood based on 3 major parameters. The first one is, what is the shape of the particles. If we have spherical or cylindrical particles, of course in real life the particles may not be actually spherical or cylindrical, they may have arbitrary shape.

But mostly, from theoretical point of view, we assume that the particles are either spherical or cylindrical. Then, we have to know what is the material of these particles, various aerosols. We may have material dust coming out of the desert. We may also have particles made of ash. So, different particles may have different compositions. So, we may be interested in finding the complex index of refraction  $m$ , which has real part  $n$  and imaginary part  $k$ .

So, the imaginary part is basically responsible for the absorption. And the real part is responsible for the scattering. Then, what we also interested in is whether the size of the particle is uniform or non-uniform. Whether the particles have the same diameter, if they are spherical or they have range of diameter. Some particles are small, some particles large. Small particles, they scatter or absorb radiation in a different way than the large particles.

We also are interested if we have more than 1 particle, we have a cloud of particles. Then we are interested in finding what is the distance between 2 particles. If the particles are very near, we may be looking for dependent scattering. If the particles are far from each other, we may

be interested in finding independent scattering. So, all these things basically effect the scattering.

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**Scattering**

- ❖ Common types
  - ❖ Rayleigh scattering, Mie scattering, Raman scattering
- ❖ Atmospheric Radiation
  - ❖ Blue color of the sky – Rayleigh scattering (small molecules)
- ❖ Combustion
  - ❖ Scattering of gas radiation by ash and soot particles (Mie scattering)
  - ❖ Reduces heat flux to boiler
  - ❖ More uniform temperature distribution
- ❖ Raman effect not important in heat transfer

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So, we have commonly 3 types of scattering. Although there are more than 3, but the general scattering that we encounter in normal applications are of 3 types. The first one is Rayleigh scattering. Rayleigh scattering is basically scattering by particles which are very small in size. So, very small in size compared to wavelength. If we have small particles with size less than, very less than wavelength of the light, we are basically looking for Rayleigh scattering.

And the blue color of the sky is basically attributed to this type of scattering what we call Rayleigh scattering. Then we have Mie scattering. So, Mie scattering is from particles which are relatively larger compared to wavelength. This type of scattering is what we found in most of the combustion applications where we have the presence of ash and soot particles which governs the govern, which are governed by Mie scattering.

So, reduction in heat transfer rates in boilers is attributed to Mie scattering by these particles. And the third one which we commonly hear is Raman scattering. Raman scattering as compared to the Rayleigh and Mie scattering is inelastic scattering. The first one, the first 2 are elastic scattering, while Raman scattering is inelastic scattering. In inelastic scattering, the energy of the photon is changed when it undergoes scattering.

So, photon may have different energy before scattering and photon may have different energy after the scattering. And as a result of change of energy of the photon, its wavelength may

change. So, Raman scattering is basically governed by change in wavelength. So, this type of scattering is not generally important for heat transfer applications, but there are certain applications where this may be important. So, we will not consider Raman scattering in this course. We will just limit ourselves to Rayleigh and Mie scattering.

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### Interaction from Single Spherical Particle

- ❖ Absorption and Scattering
  - ❖ Represented with cross-section
  - ❖  $C_{ext} = C_{abs} + C_{sca}$
- ❖ Cause of scattering:
  - ❖ Diffraction
  - ❖ reflection
  - ❖ refraction
- ❖ Elastic scattering ( $\lambda = \text{constant}$ )
- ❖ Inelastic or Raman scattering ( $\lambda \neq \text{constant}$ )

$\beta = k + \sigma$

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So, scattering is basically a phenomenon which is governed by 3 physical processes, the reflection, the refraction and diffraction. So, we have 3 physical phenomena basically that govern the process of scattering. And as you see here, reflection is basically a phenomena where light strikes a particle and it bounces back and it travels in a different direction. This is reflection.

In second phenomena the light wave penetrates the particle and then it basically is emitted out of the particle in a different direction. This is called refraction. And the third one, when we have more than 1 particles in vicinity, the light ray bends near the particle. That is the third one. So, actually in the third one, the light does not hit the particle. It basically bends around the particle. And it is called diffraction.

So, these 3 phenomena basically governs the physics of scattering. So, first we will consider scattering by a single particle. And then we will go for the, go by the scattering by the cloud. So, in the case of scattering by single particle, we do not talk about scattering coefficient, we do not talk about absorption coefficient. Rather we talk about scattering and absorption cross-section. So, for single particles, we talk about cross-section.

And we define this cross-section we will see the cross-section. The extinction cross-section is somehow basically the absorption cross-section and scattering cross-section. So, we have absorption of radiation as well as scattering of radiation. And the 2 cross-sections when added together basically gives you the extinction cross-section. This is similar to what we basically discussed earlier also.

We have extinction coefficient beta as = absorption coefficient kappa and scattering coefficient sigma. So, the sum of the 2 coefficient is the extinction coefficient. Similarly, we define the cross-sections. For elastic scattering, the wavelength is going to be constant. So, there is not going to be any change in the wavelength. While for inelastic scattering, which we are not going to discuss in this lecture, we have change of wavelength. So, lambda is not constant.

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### Interaction from Single Spherical Particle


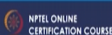
- ❖ Particle size:  $x = \frac{2\pi a}{\lambda}$
- ❖ Mie-scattering theory
  - ❖  $x \gg 1$
- ❖ Lord Rayleigh theory: small size particles
  - ❖  $x \ll 1$ 

$$Q_{\text{ext}} = Q_{\text{abs}} + Q_{\text{sca}}$$

absorption efficiency factor:  $Q_{\text{abs}} = \frac{C_{\text{abs}}}{\pi a^2}$  ✓

scattering efficiency factor:  $Q_{\text{sca}} = \frac{C_{\text{sca}}}{\pi a^2}$  ✓

extinction efficiency factor:  $Q_{\text{ext}} = \frac{C_{\text{ext}}}{\pi a^2}$  ✓



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So, to decide what type of scattering will take place, we have to first look at the size parameter. The size parameter is defined as; assuming that all the particles that we have are spherical in nature. So, this is just a size parameter defined by assuming that the particles are spherical in nature with radius a. So, we define a particle size parameter as  $2\pi a$  by lambda, where a is the radius of the particle, spherical particle.

And lambda is the wavelength of light. If x is very very large compared to the wavelength, that is  $x > 1$ , much  $> 1$ , then we have Mie scattering. So, large particles with particle size parameter  $x > 1$ , we have Mie scattering. On the other hand, if we have x much  $< 1$ , we have

Rayleigh scattering. So, for soot particles; so, we have soot in diesel engines for example, there is a formation of soot. Soot particles are very small in size.

So, we may be interested in Rayleigh scattering, while the ash particles may be significantly larger in size. So, we may be interested in Mie scattering. So, cross-section is 1 way of defining the absorption and scattering from particles, single particle. We also define absorption and scattering from particle, single particle in terms of efficiency, which is nothing but non-dimensional cross-section.

So, absorption efficiency factor is defined as  $Q$  is = the absorption cross-section divided by the cross-section area of the sphere,  $\pi a^2$ . Similarly, scattering efficiency is defined as scattering cross-section divided by the cross-sectional area of the sphere. And similarly, the extinction efficiency factor. So, this is just a way of representing the absorption and scattering from single particles.

So, this in principle can be calculated using theoretical approach as has been done by number of researchers. So, first we will take the Mie scattering. Mie scattering was solved using the theory proposed by Maxwell's. So, we assume that radiation is basically represented by an electric field and magnetic field.

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### MIE'S Theory

- ❖ Solution to Maxwell's equation of electromagnetic radiation  

$$\vec{S} = \vec{E} \times \vec{H}$$
- ❖ Oscillations in extinction efficiency factor with size parameter
- ❖ Large sphere
  - ❖ Efficiency factor equals 2

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So, it is an electromagnetic radiation, represented by electric field and magnetic field. And it was solved by Lorentz using Maxwell's theory. And they derived the relations for this one. So, we will not go into the mathematics, because the mathematics is very complicated for

this. They bring principal solved the transmission of electromagnetic radiation through this medium with index of refraction  $m = n - ik$ .

So, Lorentz and Mie, they basically solved for the electromagnetic radiation using the Maxwell's equation. And they obtain relation for the cross-section. What you see on this plot is basically the efficiency factor, extinction efficiency factor for some materials having complex index of refraction  $m$  is  $= 1.5, 1.33$  and so on. Normally, for dielectric materials; so, if we have dielectric materials, the value of  $k$  is 0. Okay.

So, for dielectric material, the  $k$  tends, the imaginary part of the complex index of refraction is 0, while for metallic particle, for metals  $k$  will not be  $= 0$ . For certain materials having certain dielectric materials  $k$  is  $= 0$ , the scattering, the extinction efficiency is plotted in this figure. What you should observe here in this plot is that, there is an oscillation in extinction efficiency versus size parameter.

So, on the x-axis we, is we have size parameter  $2x \text{ mod } m - 1$ . And on the y-axis we have extinction efficiency. And we see that there is an oscillation with respect to the size. We also should observe that for sufficiently large particles, the value is roughly  $= 2$ . So,  $q$  extinction for large particles is 2, irrespective of the value of  $m$ . Okay. So, this is 2. Okay. So, does not matter what is the value of  $m$ .

For sufficiently large particles, the extinction efficiency comes out to be 2. If we have sufficiently large particles, the extinction efficiency is 2. So, we basically are interested in finding the extinction efficiency, absorption and scattering efficiency factors. But also, we are interested in finding the scattering phase function. So, when we talk about scattering, we are also interested in finding scattering phase function.

How a particle basically scatters? Whether the particle scatters radiation in the forward direction; whether it basically scatters direction in the backward direction? And this is basically represented by scattering phase function. If the scattering is isotropic, then, equal amount of radiation is scattered in all the directions. We have discussed this.

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### MIE's Theory

- ❖ Small particles
  - ❖ Rayleigh scattering ✓
  - ❖ Isotropic scattering
- ❖ Large size parameter
  - ❖ Strong forward scattering

$\bar{P} = 1.0$  Isotropic  
x  
x

It is represented by the scattering phase function which is  $= 1$  for isotropic scattering. If the scattering is isotropic, the scattering phase function is 1. Now, scattering phase function can also be derived from the Lorentz Mie theory using Maxwell's equation. Same theory can be used to find out the scattering phase function. We are not going into the details, the mathematical details.

It turns out to be that for sufficiently small values of  $x$ . So, small  $x$ , if we have  $x$  very small, we get this type of scattering phase function. So, this is a phase function versus  $x$ . So, for the first figure, the figure A is for very small particles where  $x$  is very small. And we have what we called isotropic form of scattering. You see that the scattering phase function is very symmetric. It is almost isotropic.

And this is a case for Rayleigh scattering, where the particles are very, very small. If you have sufficiently large particles, you tend to see that the particles are forward scattering. So, in this, rest of this figures, we have Mie scattering taking place. The value of  $x$  is relatively large. And the size of the particle is relatively large. And you see that these particles tend to scatter in the forward direction.

So, if you have a particle, a spherical particle and the radiation is coming from this direction, then you assume that most of the radiation will be scattered in the forward direction. And very less amount of radiation will be back scattered. So, significant amount of radiation is scattered in the forward direction and very less amount of radiation is scattered in the backward direction. So, this is typical for Mie scattering.

We see Mie scattering, more forward scattering in the case of Mie scattering. While the Rayleigh scattering tends to have more isotropic phase function. Similar to the Mie scattering, we can also write down the scattering efficiency factor for Rayleigh scattering where the scattering, where the particle size is very small. When the particle size is very small, the solution can be obtained in closed form.

I did not try to give any mathematical relation for the Mie scattering, because the mathematical relation for Mie scattering turns out to be very complicated. While for Rayleigh scattering, the relations can be simplified under the assumption that the particles are extremely small.

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### Rayleigh Scattering

- ❖ Scattering particles are extremely small
  - ❖ Size parameter  $x = 2\pi a/\lambda$  (small)
  - ❖ Gas molecules and soot

$$Q_{\text{sca}} = \frac{8}{3} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 x^4 \quad \epsilon \quad \checkmark \quad Q_{\text{sca}} \ll Q_{\text{abs}}$$

$$Q_{\text{ext}} = -4\Im \left\{ \frac{m^2 - 1}{m^2 + 2} \right\} x \approx Q_{\text{abs}} \quad \checkmark$$

Imaginary

- ❖  $x^4 \ll x$  (negligible scattering compared to absorption)

We can simplify the mathematical relation. And the scattering efficiency and extinction efficiency is given by these relations. So, we have scattering efficiency is  $= \frac{8}{3} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 x^4$ ,  $x$  is a size parameter. And we have extinction efficiency which is  $= -4 \Im \left\{ \frac{m^2 - 1}{m^2 + 2} \right\} x$ , which is now  $=$  just absorption efficiency, because the this term is much larger than this term.

So, scattering efficiency factor for Rayleigh scattering is much smaller. That means,  $Q_{\text{sca}}$  is much smaller than  $Q_{\text{abs}}$ , for Rayleigh scattering. So, for particles which are very small in size, the scattering is negligible. So, small particles, they tend to scatter radiation in very small amount. And whatever they scatter, they scatter isotropically. So, it means, in most of the heat transfer applications, we can neglect scattering by small particles.

So, from heat transfer point of view, we can neglect scattering. We just consider absorption by small particles. But, from physics point of view, we cannot neglect it. Because, in atmosphere, we have Rayleigh scattering which gives blue color to our sky. So, from physical point of view, this is not negligible. It does contribute to the color of sky which is blue in nature due to scattering.

But, from heat transfer point of view, the Rayleigh scattering can be neglected. And all we have to interest, all we have to look for is absorption by small particles, which is given by this relation.


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### Rayleigh Scattering

- ❖ Color of the SKY
- $Q_{sca} \propto \frac{1}{\lambda^4} \propto v^4$
- ❖ Day: short path, only smallest wavelength radiation scattered
  - ❖ Blue color of the sky
- ❖ Sun-set: almost all wavelengths are able to undergo scattering
  - ❖ Longest wavelength least scattered: red color of the horizon
- ❖ Space: no scattering and sky appears black
- ❖ Absorption efficiency of small particles (soot)

$Q_{abs} \propto \frac{1}{\lambda} \propto v$ 

monotonic smooth function





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Now, looking at this relation, so from heat transfer point of view, the scattering efficiency factor is not important. But just for the sake of scattering from atmospheric physics point of view, we understand that the scattering efficiency is proportional to 1 upon lambda power 4 or frequency power 4. We have x power 4 and x depends on 1 upon lambda. So, the scattering efficiency factor is proportional to 1 upon lambda power 4.

So, what it means basically; when we have day, so during day, entire day, we have radiation travelling directly, shortest path. So, most of the radiation which is low wavelength will be scattered out. That means, low wavelength means high frequency. That means, the violet and blue radiation, it will be scattered out. And because scattering efficiency factor is largest for blue radiation, most of the sky appears blue in nature throughout the day.

Same thing, when we have evenings, the radiation is travelling not directly, but at an angle. So, in the day, let us say we are standing here. The radiation is travelling almost vertically in the day. While in the evening, the radiation is travelling at an angle and it is travelling much larger path. So, during this larger path, most of the radiation, not just blue; blue, green, except the red, most of the radiation is scattered out.

Only red radiation basically reaches us directly. And it gives us a red color of the sky during the evenings. Because red color, red radiation part of the spectrum of the sun is directly reaching us while rest of the wavelengths are scattered out and they do not reach us directly. While in the day, the blue radiation is scattered out and it gives blue color to the sky. Now, with the same logic; because, in the space, we do not have any scattering gases.

The space consist, comprises of vacuum. There is no scattering in space. So, the radiation travels directly from the sun and there is no sky. The space appears to be black. So, if we go into space, the space appears to be black, because there is no scattering in space. Thus, absorption efficiency on the other hand depends on  $1$  upon wavelength. And we see that it is monotonically smooth function.

So, the absorption by particles follows a smooth function. So, this is compared to gases relatively easy to handle. The gases absorbs in bands. They have lines. The spectrum is complicated. While the particles, the absorption efficient, absorption efficiency factor is inversely proportional to  $1$  upon wavelength. So, we get a basically a smooth function. And we can easily integrate for a spectral absorption coefficient for the particles.

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## Radiative Analysis of Cloud

❖ Clearance-to-wavelength ratio:  $c/\lambda$

❖ Independent scattering ( $c/\lambda \gg 1$ )

❖  $c/\lambda$  replaced by  $c/a$

$a$  = radius of spherical parameter

❖ Scattering and absorption coefficient

$$\sigma_{s\lambda} = N_T C_{sca} = \pi a^2 N_T Q_{sca}$$

$$\kappa_{s\lambda} = N_T C_{abs} = \pi a^2 N_T Q_{abs}$$

❖  $N_T$  = Total number of particles (all of same size  $a$ )

$x$   
 $a$   
 $c/\lambda$

0  
0

So, so far, we basically found out Rayleigh and Mie scattering for individual spherical particles. The same theory we can extend to a particle cloud. Now, particle cloud basically have 1 more additional parameter. So, the 2 parameters are size parameter  $x$ , as we discussed. And we also discussed the particle diameter  $a$ , if they are spherical in shape. Now, we have to talk about what we call clearance to wavelength ratio  $c$  by  $\lambda$ .

So,  $c$  by  $\lambda$  defines how much the 2 particles have proximity. If the 2 particles are very close to each other, then the diffraction will be affected. And we will call it dependent scattering. If the particles are relatively large,  $c$  by  $\lambda \gg 1$ , we call it independent scattering. And we just replaced  $c$  by  $\lambda$  by  $c$  by  $a$ . Okay. So, independent scattering means that the proximity of particles does not affect the scattering phenomena.

Each particle behaves independently. That is called independent scattering. So, we define scattering coefficient  $\sigma_{s\lambda}$  and absorption coefficient for particle cloud as = scattering cross-section and absorption cross-section multiplied by total number of particles. So, coefficient, scattering coefficient and absorption coefficient is nothing but cross-section of individual particles multiplied by total number of particles.

If we have all particles of same size, we can just write down this in terms of efficiency,  $\pi a^2 N_T$  scattering efficiency factor  $\pi a^2 N_T$  absorption efficiency factor. This basically assumes that all the particles are of the same size. So, for cloud which has same size, this is very easy to calculate. But if the particles are of not same size, they are distributed as per some distribution, some particles small, some particles large.

Then, we have to integrate over the particle size. We cannot directly calculate the absorption coefficient and the scattering coefficient. So, we have to integrate. Normally, the particles follow this what we call;

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### Clouds of Non-uniform Size Particle

- ❖ Described by number of particles as function of radius
  - ❖ Particle size distribution function
- ❖ Modified gamma distribution
 
$$n(a) = \underline{Aa^\gamma \exp(-\beta a^\delta)}, \quad 0 \leq a < \infty$$
- ❖ Parameters:  $A, \beta, \gamma$  and  $\delta$
- ❖ Total number of particles per unit volume
 
$$N_T = \int_0^\infty n(a) da = A \int_0^\infty a^\gamma \exp(-\beta a^\delta) da = \frac{A \Gamma\left(\frac{\gamma+1}{\delta}\right)}{\delta \beta^{(\gamma+1)/\delta}}$$

Modified gamma distribution, with parameters A, beta, gamma and delta. So, this is a distribution that is basically followed by the particles. This is beta here. So, we have what we called modified gamma distribution, generally defining the size distribution of the particles. What we do now is, total number of particles is basically = number of particles of size a all the particles here also are assumed spherical in shape.

But the spherical shape has different radius. And this distribution is given by small n. And total number of particle is  $n a da$  integrated from 0 to infinity, where n a is given by the modified gamma distribution. We can easily integrate this function. And water get is this value,  $A \Gamma(\gamma+1)/\delta$ ; this is gamma function and these are parameters. This is beta. Okay. So, this is basically the distribution, the gamma distribution. And based on this, we can calculate the scattering coefficient and absorption coefficient.

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## Clouds of Non-uniform Size Particle

❖ Where gamma function

$$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt$$

❖ Volume fraction of particles

$$f_v = \int_0^{\infty} \frac{4}{3} \pi a^3 n(a) da = \frac{4\pi A \Gamma\left(\frac{\gamma+1}{\delta}\right)}{3\delta B^{(\gamma+1)/\delta}}$$

$\sigma_{\lambda}, \kappa_{\lambda}$  } Size parameter  
distribution  
A,  $\beta, \gamma, \delta$



So, the gamma function is given by this relation. This is the gamma function. So, if we assume that a particle size follow modified gamma distribution, we get the total number of particles. We also define 1 more parameter for particle cloud, what we called volume fraction of particles, that is the total amount of space occupied by the particles. So, volume fraction is defined as volume of single particles,  $\frac{4}{3} \pi a^3$ .

Number of particles integrated over entire space, in 1 meter cube space; so, we have to find out total number of, total volume of the particles. And this is come, this comes out to be, this for the relation of modified gamma function, we have what we called volume fraction of particles. So, absorption coefficients  $\sigma_{\lambda}$  and scattering coefficient  $\kappa_{\lambda}$ , so absorption coefficient  $\kappa_{\lambda}$  and scattering coefficient  $\sigma_{\lambda}$ ; this depends on the size parameters and it also depends on distribution function.

That is, A, beta, gamma and delta. Okay. So, if we have to specify what kind of distribution this particles follow; and based on that we can calculate the scattering coefficient.

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## Clouds of Non-uniform Size Particle

### ❖ Scattering and extinction coefficient

$$\begin{aligned} \sigma_{s\lambda} &= \int_0^{\infty} C_{sca} n(a) da = \pi \int_0^{\infty} Q_{sca} a^2 n(a) da \\ \beta_{\lambda} &= \int_0^{\infty} C_{ext} n(a) da = \pi \int_0^{\infty} Q_{ext} a^2 n(a) da \end{aligned}$$

### ❖ Scattering Phase function not same as that of single particle

$$\Phi_{T\lambda} = \frac{1}{\sigma_{s\lambda}} \int_0^{\infty} C_{sca}(a) \Phi(a, \theta) n(a) da$$



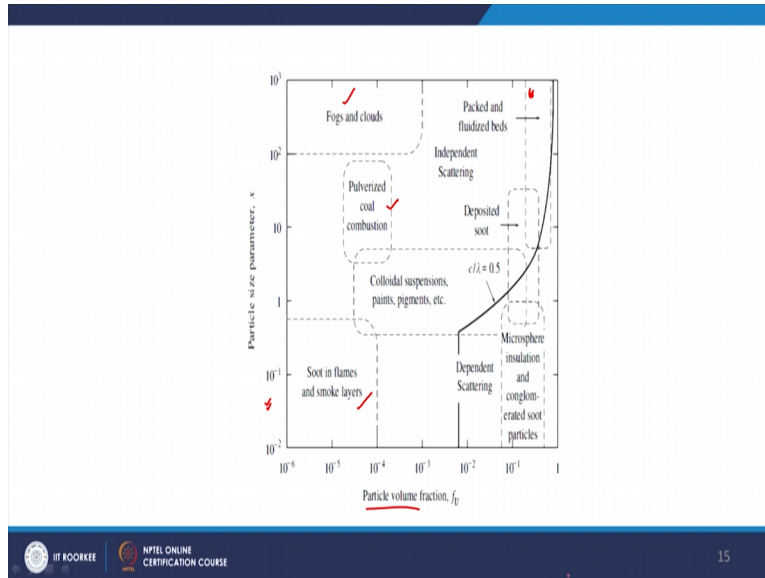
So, expression for scattering coefficient is, scattering cross-section multiplied by that number of particles of a given size. And when we integrate it, we have to basically find out this scattering coefficient and absorption coefficient in terms of the size parameter A, beta, gamma and delta. So, we will solve this equation for 1 example. But ultimately, the solution comes out to be in closed form, in the form of gamma function.

Now, scattering phase function. If all the particles are of the same size, the scattering phase function will be same as the scattering phase function for single particle. But, if the particles are of different size, then the scattering phase function may not be same. So, what we have to do is, total scattering phase function, we have to integrate on all the particles. And it is defined as scattering cross-section multiplied by scattering phase function and then the particles distribution.

So, this is how we have to calculate the total amount of scattering phase function for the given cloud. So, the total scattering phase function is not same as scattering phase function of any individual particles, because the particles are of different shape and size. So, we have to basically integrate over the entire cloud.

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Now, we have a different type of applications, where we have different type of clouds. We have fog and cloud. We have also have pulverized coal application, soot. Okay. So, depending on application, we have different volume fraction for this particles. For example, the soot in typical flames and smokes has very small volume fraction and the particle size is also very small. Okay.

So, soot especially comes under the category of Rayleigh scattering with very small volume fraction. Then we have fluidized bed, where we have Mie scattering, because the size is very large. And we also have very high volume fraction. So, in fluidized bed application, the particle size is large. We have to consider Mie scattering and we have to deal with very high loading, high volume fraction. Okay.

Now, in this pulverized coal combustion, we have moderate volume loading, moderate particle volume. And we also have particle size which comes under this category of Mie scattering. The fog, formation of fog and cloud, we have water particles which are present in very small volume fraction. So, volume fraction is small in the case of fog and cloud, but the size of the water particles will be large.

So, normally, the volume fraction we expect is very small in the case of fog and cloud, but the size is large. This is contrast to the Rayleigh scattering which gives blue color to the sky where the molecules are very small in size. But in fog and cloud, the formation of fog and cloud is governed by water particles which are relatively larger. And this comes in the

category of Mie scattering. So, different regimes, they are governed by different scattering phenomena. And we have to deal with different particle volume fraction.

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	Cloud#1	Cloud#2	Cloud#3	Cloud#4
	Const. Radius	Size Distr.	Const. Radius	Size Distr.
	$a = 5 \mu\text{m}$	$n(a)$	$a = 5 \mu\text{m}$	$n(a)$
	$m = 2 - i$	$m = 2 - i$	$m = 2$	$m = 2$
Absorption				
coefficient $\kappa$ [ $\text{cm}^{-1}$ ]	$8.307 \times 10^{-3}$	$1.524 \times 10^{-3}$	0	0
Scattering				
coefficient $\sigma_s$ [ $\text{cm}^{-1}$ ]	$1.073 \times 10^{-2}$	$1.674 \times 10^{-3}$	$6.420 \times 10^{-2}$	$3.363 \times 10^{-3}$
Extinction				
coefficient $\beta$ [ $\text{cm}^{-1}$ ]	$1.904 \times 10^{-2}$	$3.198 \times 10^{-3}$	$6.420 \times 10^{-2}$	$3.363 \times 10^{-3}$
Scattering albedo	0.5634	0.5235	1	1

$m = 2 - i$  is scattering & absorption  
 $m = 2$  is scattering

So, some of the properties of the cloud have been taken from the literature and are given on this slide. We have taken 4 different clouds. Cloud number 1 is uniform constant radius, cloud number 3 is constant radius. All particles of the same size. 1, we have complex index of refraction as  $2 - i$ . So, this is what we called absorbing and scattering particles. And the other one is purely scattering. Okay.

So, here the imaginary part is 0. And I told you that  $n - i k$ , this governs the absorption. And the real part governs the scattering. So, cloud 3 has 0 absorption coefficient because the imaginary part of the complex index of refraction is 0. So, this is giving you 0 absorption. The scattering coefficient comes out to be  $6.420 \times 10^{-2}$ . If you have uniform radius, while this comes out to be  $1.073 \times 10^{-2}$ . Okay. If we have complex index of refraction has  $2 - i$ .

So, for this case, we see that when the particles is scattering and absorbing simultaneously, we have higher extinction. Okay. So, in this case, we have what we called  $1.904 \times 10^{-2}$  as the extinction coefficient. While here we have  $6.420 \times 10^{-2}$  as the extinction coefficient. So, when we have scattering and the absorption, this comes out to be smaller and this comes out to be larger.

Now, same thing, when we go for size distribution, let us say we do not have a single size. We have size distribution governed by the modified function. Then we see the values to be smaller. The absorption coefficient is smaller, scattering coefficient is smaller here and extinction coefficient is also smaller. And why it is happening? Because some particles tend to; here we have uniform distribution 5 micron.

Here, some particles may be 5 micron, some particles may be larger than 5 micron, other particles may be very very small than 5 micron. So, the overall effect of the size distribution comes out to be smaller values of absorption and scattering coefficient. So, now, for the cloud, just like for the case of single particle, single spherical particle, we did not give any closed form formula for Mie scattering, but I gave you formula for the Rayleigh scattering. We can find close form formula for Rayleigh scattering also for cloud. For cloud we defined absorption coefficient Rayleigh absorption coefficient;

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### Rayleigh Scattering


- ❖ Absorption coefficient for cloud of nonuniform-size small particles

$$\kappa_\lambda = \pi \int_0^\infty \underline{Q_{\text{abs}}} a^2 \underline{n(a)} da = -4 \Im \left\{ \frac{m^2 - 1}{m^2 + 2} \right\} \int_0^\infty \left( \frac{2\pi a}{\lambda} \right) \pi a^2 n(a) da$$

- ❖ Where the volume fraction  $f_v$

$$f_v = \int_0^\infty \left( \frac{4}{3} \pi a^3 \right) n(a) da \quad | \quad m = n - ik$$

$$\kappa_\lambda = -\Im \left\{ \frac{m^2 - 1}{m^2 + 2} \right\} \frac{6\pi f_v}{\lambda} = \frac{36\pi n \kappa}{(n^2 - \kappa^2 + 2)^2 + 4n^2 \kappa^2} \frac{f_v}{\lambda}$$


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Kappa lambda is = absorption efficiency factor. Then multiplied by the size distribution. And we take modified gamma function as the size distribution. And this comes out to be  $-4$  imaginary part of  $m$  square  $- 1$   $m$  square  $+ 2$ . And then, this size distribution function. So, I am just leaving this as an exercise to derive from this here. Now, this time in the bracket is called volume fraction.

We discussed this earlier also. So,  $4$  by  $3$  pi a cube and a da is called volume fraction. So, in terms of volume fraction, this kappa lambda comes out to be imaginary part of  $m$  square  $- 1$  upon  $m$  square  $+ 2$   $6$  pi f v, where f v is now the volume fraction. Whatever distribution,

whether it is modified gamma function or what other function, you can calculate the volume fraction. So, in terms of volume fraction, the kappa lambda can be written by this formula.

Or, if you have  $m$ , substitute  $m = n - ik$ , where  $n$  is the real part of complex index of refraction and  $k$  is the imaginary power. You can find out the solution of absorption coefficient in closed form for Rayleigh scattering. So, for Rayleigh scattering, this formula can be used to calculate the absorption coefficient of a cloud of small particles. So, let us solve 1 problem. And then, how to apply this formula will be pretty clear. So, we have a combustion problem where we have propane.

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### Problem

**Problem:** During the burning of propane it is observed that the products contain a volume fraction of  $10^{-4}\%$  of soot with complex index of refraction  $m = 2.21 - 1.23ik$ . (measured at a wavelength of  $3 \mu\text{m}$ )

Assuming a mean particle diameter of  $0.05 \mu\text{m}$ , determine the absorption and scattering efficiency of this soot cloud as well as its absorption coefficient, all at wavelength of  $3 \mu\text{m}$ .

$Q_{\text{scat},\lambda}$   
 $Q_{\text{abs},\lambda}$   
 $K_{\lambda}$

$\lambda = 3 \mu\text{m}$

And we see that propane basically gives out soot. And this soot has complex index of refraction. This is basically, soot is form of carbon. So, the complex index of refraction is  $2.21 - 1.23 i$ , where is the under root – 1. And this complex index of refraction of course depends on wavelength. So, what we are basically dealing with is a single monochromatic wavelength at 3 micron.

So, we have to find out the solution for this problem at 3 micron. The volume fraction is given as 10 to the power of – 4. So, volume of soot in the entire volume of the combustion chamber is 10 point, 10 to the power of – 4. We assume a mean particle diameter of 0.05. And we have to find out the absorption and scattering efficiency of this particle cloud and scattering, sorry, absorption coefficient at this wavelength 3 micron.

So, we have to find out the Q scattering and Q absorption, as well as, we have to find out kappa lambda. Of course, this is also function of lambda. So, we have to find out all this at lambda is = 3 micron. So, let us solve this problem.

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### Solution

for single spherical particle

$$x = \frac{2\pi a}{\lambda} = \frac{2 \times \pi \times 0.05}{3 \mu\text{m}}$$

$$\frac{m^2 - 1}{m^2 + 1} = \frac{(2.21 - 1.23i)^2 - 1}{(2.21 + 1.23i)^2 + 1} = \frac{2.3712 - 5.4366i}{3.712 - 5.4366i}$$

$$m = n - ik = 2.21 - 1.23i$$

$$\frac{m^2 - 1}{m^2 + 1} = \frac{0.7241 - 0.2792i}{}$$

$$Q_{\text{scat}} = \frac{8}{3} \left| \frac{m^2 - 1}{m^2 + 1} \right|^4 = \frac{8}{3} |0.7241 - 0.2792i|^2 \times (0.0524)^4$$

$$Q_{\text{scat}} = 1.21 \times 10^{-5}$$

$$Q_{\text{abs}} = -4(-0.2792) \times 0.0524 = 5.85 \times 10^{-2}$$

$$Q_{\text{abs}} > 5.85 \times 10^{-2} > Q_{\text{scat}}$$

So, first of all we find the properties for a single particle. And then we will go for the cloud. For single particle, let us go to the Rayleigh scattering first. So the, for the single particle, this is a relation that we are going to use. The scattering efficiency factor is given by 8 by 3 m square – 1 upon m square + 2 whole square x power 4. And extinction efficiency factor is given by – 4 imaginary part of this function.

So, these formula, we will be using for the single particle. So, for single particle, single spherical particle. So, for single spherical particle, let us first find out the value of x. x is the size parameter. So, 2 pi a by lambda. So, it will be = 2 times pi. So, we have, the mean particle diameter as 0.05. And we have what we called 3 micron. Okay. So, this is 0.05. And we have 2 pi a by lambda as the size parameter.

So, we basically get this one as; so, we can just solve for this. Now, we fall find out the value of m square – 1 upon m square + 1. So, complex index of refraction m square – 1. m is = n – i k is = 2.21 – 1.23 i. So, this we will just write down as 2.21 – 1.23 i square – 1. And 2.21 + 1.23 i square + 1. So, we have to solve for this one. So, we just take the square and then we solve it.

So, we get this is  $2.3712 - 5.4366 i$  upon 2, this will be  $5.3712 - 5.4366 i$ . So, we are multiply by complex conjugate. So, we get  $m^2 - 1$  upon  $m^2 + 1$  as  $= 0.7241 - 0.2792 i$ . So, this is the value of the  $m^2 - 1$  upon  $m^2 + 1$ . Now, we get scattering efficiency factor as  $= 8$  by 3. So, this will be  $= \text{mod of } m^2 - 1 \text{ upon } m^2 + 1$ . And this will be  $= 8$  by 3.

And definitely, we have to multiply by this term also,  $x$  power 4. So, this will be 8 by 3  $0.7241 - 0.2792 i$  square, this will be mod square. So, this will be  $= 0.0524$  power 4. So, this will come out to be scattering efficiency of 1.21 into 10 to the power  $-5$ . This scattering efficiency factor comes out to be 1.21 into 10 to the power  $-5$ . Now, same thing we do for the absorption efficiency factor which is basically  $= -4$ , the imaginary part.

Imaginary part is  $= -0.2792$  into  $x 0.0524$ . And this comes out to be 1, sorry 5.85 into 10 to the power  $-2$ . So, we see that absorption efficiency factor is much larger than the scattering efficiency factor. So, this is much larger than scattering efficiency factor. So, in this application in the application of soot, we can better neglect scattering. So, we have to just consider the absorption by the soot particles.

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**Solution**

$$k_{\lambda} = -\text{Im}\left(\frac{m^2-1}{m^2+1}\right) \frac{6\pi f v}{\lambda}$$

$$= \frac{-(-0.2792) \times 6\pi \cdot 10^{-4} / 100}{3 \times 10^{-6}}$$

$$k_{\lambda} = 0.01756 \text{ cm}^{-1}$$

$f v = 10^4 \text{ Hz}$   
 $= \frac{10^4}{100}$   
 $\lambda = 3 \mu\text{m}$   
 $= 3 \times 10^{-6} \text{ cm}$



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Now, absorption coefficient  $kappa_{\lambda}$  is  $=$ ; so, we use this formula for the absorption coefficient as  $-$  the imaginary part of  $m^2 - 1$  upon  $m^2 + 1$  times  $6 \pi f v$  by  $\lambda$ . So,  $-$  imaginary part of  $m^2 - 1$  upon  $m^2 + 1$ . Okay.  $6 \pi f v$  by  $\lambda$ ,  $6 \pi f v$  by  $\lambda$ . So,  $f v$  value is given already. And imaginary part, we have already found. So,

this comes out to be  $-0.2792$ . The imaginary part of the complex index part  $m^2 - 1$  upon  $m^2 + 1$  times  $6\pi$ .

Now,  $f_v$  is  $10^{-4}$ , okay; by 100 and  $\lambda^3$  into  $10^{-4}$ . Okay. So, this basically gives you  $0.01754$  centimeter inverse. So, the absorption coefficient comes out to be  $k\lambda = 0.01754$  centimeter inverse, where  $f_v = 10^{-4}$  or  $10^{-4}$  by 100. Okay. So, the volume fraction was  $10^{-4}$ . So, the  $f_v$  is  $10^{-4}$  by 100.

And we have converted  $\lambda$  from 3 micron to 3 into  $10^{-4}$  centimeter. So, we wanted absorption coefficient units to be in centimeter inverse. That is why we have converted into centimeter. So, the absorption coefficient comes out to be  $0.01754$  centimeter inverse, for this particle cloud at this particular wavelength. So, this basically pretty much ends the theory on radiative heat transfer.

We are coming close to the end of this course. Next, we will take certain applications related to combustion industrial systems and atmospheric radiation. So, the next few lectures we will focus on the applications of what we have learnt so far to actual practical applications. So, thank you for your attention we will meet and discuss the applications in the next lecture. Thank you.