

Radiative Heat Transfer
Prof. Ankit Bansal
Department of Mechanical and Industrial Engineering
Indian Institute of Technology - Roorkee

Module - 7
Lecture - 34
WSGG Model

Hello friends, we are discussing spectral models for radiative heat transfer. In last few lectures, we discussed narrow band and wide band models. In narrow band models, we averaged the absorption coefficient and emissivity of a homogeneous cell layer over a small wavelength range comprising of a few lines. In wide band model, we averaged over the entire row vibrational band.

Now, in today's lecture, we will focus on global models. So, global models are one of the most preferred models in radiative heat transfer. These models basically represents the entire spectrum in the form of gray gases. So, the properties, the spectral modelling is done over the entire spectrum. And we do not necessarily focus on narrow or wide bands. So, the advantage of these models is, they are very efficient.

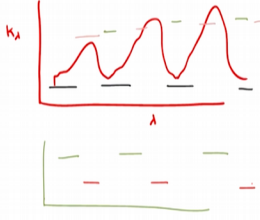
We need to solve radiative transfer equation only for few weights, few times and they can be easily integrated with any CFD model, because of their very high efficiency. So, in this category, we have number of models. Here in this lecture, we will focus on weighted-sum-of-gray-gases, WSGG model which has got very much popularity in radiative heat transfer after its development for the zone model.



So, there are number of global models that are employed in radiative heat transfer. Some of these models are weighted-sum-of-gray-gases, spectral-line-based weighted-sum-of-gray-gases, absorption distribution function and k-distribution model.

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Global Models

- ❖ Covers the entire spectrum
 - ❖ Weighted-Sum-of-Gray-Gases (WSGG) model
 - ❖ Spectral-Line-Based WSGG (SLW) model
 - ❖ Absorption Distribution Function model (ADF)
 - ❖ Full-spectrum k -Distribution model (FSK)
- ❖ Most computational efficient and accurate model
- ❖ Represent emitting/absorbing non-gray gas with few fictitious gray gases
- ❖ Need to solve RTE for only few times (4-8 times maximum)





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So, all these models basically have 1 or the other thing common. That we represent gas in the form of fictitious gray gases. So, we are basically representing the gas in the form of few fictitious gray gases. And we solve radiative transfer equation not for the real gas but rather for these fictitious gray gases. So, in the sense, all these models, the global models, they have very much common, that they represents a gray non-gray gas with gray gases.

And they are the most computational efficient models in radiative heat transfer calculation. Now, what we mean by fictitious gray gas? So, that point, let me make it very clear in the beginning itself, before we go into the details. So, let us say we have a spectrum of absorption coefficient say. And this spectrum basically varies; it may have some rotational band and so on.

So, what we do is basically, we represent this non-gray, of course, this absorption coefficient is varying. And we represent this as non-gray. So, non-gray absorption coefficient is represented in the form of few gray absorption coefficients. So, we represent it this like this. So, this is 1 gray absorption coefficient. So, some part of the absorption coefficient is represented by this non, this gray absorption coefficient.

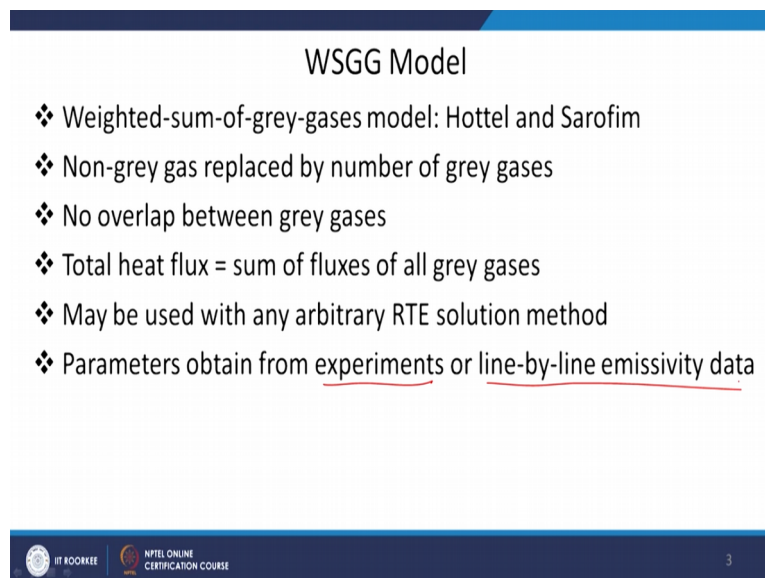
The other may be represented as this, like this. We may have a, we, what we called another gray gas. Then we may have another gas like this. So, what we basically are doing here is, the entire spectrum, we are dividing into a number of gray gases. Each gray gas has constant absorption coefficient over the spectrum. So, let us see how this basically can be represented. So, we have number of gases.

This may be 1 gas and this may be another gas. So, these are basically non-overlapping gray gases. And we are trying to solve the radiative transfer equation or radiation problem for constant absorption coefficient. And we have number of gray gases for which we want to solve this problem. And once we have solved the problem for these gray gases, we just add up the results to find out the total flux or total intensity.

Now, we go into the details of weighted-sum-of-gray-gases model. It was developed by Hottel and it was developed in the frame of framework of zone model that we have already discussed. So, for zone model, the weighted-sum-of-gray-gases model was developed. But later on, Modest extended this weighted-sum-of-gray-gas model to general solver for radiative transfer equation.

So, in the sense, this model weighted-sum-of-gray-gases can be applied to any RTE solver, be it exact solution, be it spherical harmonics, discrete ordinate method, zone model, any method can be applied along with weighted-sum-of-gray-gas model. So, as I said, we represents the non-gray absorption coefficient by a number of gray gases with constant absorption coefficient.



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The slide content is as follows:

WSGG Model

- ❖ Weighted-sum-of-grey-gases model: Hottel and Sarofim
- ❖ Non-grey gas replaced by number of grey gases
- ❖ No overlap between grey gases
- ❖ Total heat flux = sum of fluxes of all grey gases
- ❖ May be used with any arbitrary RTE solution method
- ❖ Parameters obtain from experiments or line-by-line emissivity data

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And the crux of this method is that, the parameters that we are going to use in weighted-sum-of-gray-gases model are either obtained from experiments as was done in the case of wide band and narrow band models also or they may be also obtained from line-by-line emissivity data. I will explain how the parameters are obtained and how the model is basically represented. But this method is very powerful and it is one of the most preferred method in

radiative transfer solution. And the data for this type of model is available in many commercial codes like Ansys and Star CCM. So, this model can be applied and effectively it gives very good results.

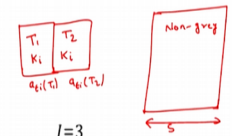
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WSGG Model

- ❖ Three grey gases and one clear gas (transparent).
- ❖ Transparent gas takes care of windows between lines/bands
- ❖ Emissivity of a homogeneous layer

a : emissivity weighting factor
 k_i : absorption coefficient (grey)
 p : sum of partial pressures of absorbing gases
 s : the path length



- ❖ Absorption coefficients assumed constant
- ❖ weights allowed to depend on temperatures.



$$\varepsilon = \sum_{i=0}^{I=3} a_{\varepsilon i}(T_g) [1 - e^{-k_i p s}]$$

$$a_{\varepsilon i}(T_g) = \sum_{j=1}^{J=4} b_{i,j} T_g^{j-1} \quad \left. \begin{matrix} \\ \\ \\ \end{matrix} \right\} \begin{matrix} \\ \\ \\ \end{matrix} \begin{matrix} \\ \\ \\ \end{matrix} \begin{matrix} \\ \\ \\ \end{matrix}$$

$$a_{\varepsilon 0}(T_g) = 1 - \sum_{i=1}^{I=3} a_{\varepsilon i}(T_g)$$



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So, how do we represent this model? So, in this model, normally, we take number of gray gases, typically 4 gray gases are used. And out of these 4 gray gases, 1 will be gas which is transparent. That means, absorption coefficient is 0. So, we will have 1 transparent gas which will represents the windows between the absorption bands, Ro-vibrational bands. As we have already seen, our gas may have number of row vibrational bands.

And the bands have some separation windows, which are basically there to transmit any radiation coming from the wall. So, we also in this model represents 1 transparent window, a transparent gas with absorption coefficient = 0. And other than this transparent gas, we have 3 more gases. So, total 4 gases are represented. And we write emissivity of a gas layer, a homogeneous gas layer.

So, we have a homogeneous gas layer. This is filled with some non-gray gas. But we are going to represent this with a gray gas. So, the emissivity of this layer is basically written as weighted sum of emissivity evaluated at gray absorption coefficient. So, kappa or k_i is basically the gray absorption coefficient, pressure based gray absorption coefficient. p is sum of partial pressures of absorbing gases. s is the path length.

And k_i is the absorption coefficient, gray absorption coefficient. So, we have represented basically this non-gray gas as a mixture of gray gases, total 4 in number. Each gray gas has absorption coefficient k_i . Each gray gas has weights a_{ϵ_i} , which depends on gas temperature. Now, the main assumption in this method is that absorption coefficient is independent of temperature and gas concentration.

So, that means, the value of this absorption coefficient is assumed to be constant, while the weights are allowed to vary with temperature. So, this is basically very restrictive in the sense. When we have 2 cells, for example. So, we have, let us say 2 cell, 2 cells. The conditions in 1 cell is let us say T_1 . Another cell has let us say temperature T_2 . Now, what we are basically assuming is that, whatever the temperature in the 2 cells is, but the absorption coefficient for the gray gas will be same k_i .

So, thus absorption coefficient will not change. But the weights can change. So, a_i will change. So, this will be based on T_1 . And this will be based on T_2 . So, these weights are allowed to change. And what do these weights represents? These weights basically represents the emissive power of the black body. So, they are basically, they are to represent how much the fraction of black body emissive power.

So, because the weights are correctly represented, the model will always go to correct optically thin limit, where the kappa value is very large or very small. So, for optically thin limit, it will always go to correct limit, because the weights are correctly represented. But the absorption coefficient is assumed to be constant. So, this is one of the restriction. Now, the entire emissivity data, entire coefficients of weight coefficients data is fitted in the form of curve fits.

So, the fraction of black body emissive power also called weights represented by a subscript ϵ_i , where ϵ is basically the weights for emissivity and i is a subscript for the gray gas which depends on gas temperature T_g is written as a polynomial fit $j = 1$ to $j = 4$ b_{ij} , where b_{ij} are now coefficients. They are tabulated coefficients and the temperature T_g . So, we have basically fitted a polynomial for these weights versus gas temperature.

So, weights are allowed to vary with gas temperature using a polynomial fit. Okay. Now there are total number of 4 gases. So, these fits are basically for $i = 1, 2$ and 3 . So, 3 gray gases.

For the transparent gas, now the sum of the weight should always be = 1. Because black body emissive power is constant and the total amount of emission should be conserved. So, the final weight for $i = 0$ is calculated based on the equality.

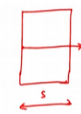
So, $i = 0$. The weight is simply = 1 - $i = 1$ to 3 a $\epsilon_i T_g$. So, that is a, this is how we basically conserve the total emission. So, for the transparent window, the weight is basically = 1 - the weights for the other gray gases.

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WSGG Model

- ❖ Treatment of wall emission (Absorptivity)
- ❖ Separate set of coefficients
 - ❖ Depends on wall as well as medium temperature

Wall emission T_s





$$\alpha = \sum_{i=0}^{l=3} a_{\alpha i}(T_g, T_s) [1 - e^{-k_i p s}]$$

$$a_{\alpha i}(T_g, T_s) = \sum_{j=1}^{J=4} \left[\sum_{k=1}^{K=4} c_{i,j,k} T_s^{k-1} \right] T_g^{j-1}$$

T_g^{j-1}

i, j, k

$$a_{\alpha 0}(T_g, T_s) = 1 - \sum_{i=1}^{l=3} a_{\alpha i}(T_g, T_s)$$



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Similarly, the method can accommodate cold black walls as well as, the method can accommodate gray non-black walls. So, if there is a wall emission; so, if we have wall emission, now the wall may be at temperature T_s . The temperature of the wall may be T_s . And the gas temperature is T_g . So, similar to emissivity, the absorptivity of the cell is also curve fitted.

So, we have a cell, the intensity is radiated from the wall. And we are looking for what is basically transmitted and absorbed in this cell. So, we write the absorptivity. The absorptivity of the cell is defined as the weight coefficient. Now which, the weight coefficients depends on wall temperature as well as the gas temperature. So, these coefficients are now fitted versus 2 parameters, the gas temperature T_g and surface temperature, wall temperature T_s .

And $1 - e^{-k_i p s}$, where k_i is the same absorption coefficient that we have found for the emissivity data. So, k_i is same. p is the partial pressure of absorbing gases. s is the path length. So, the absorptivity as well as emissivity data is fitted. And this is fitted from

experiments or through line-by-line calculations. We will see how these coefficients are calculated. So, the weight in the absorptivity are basically fitted versus T_g and T_s .

And the correlation is given by this relation. So, this is double polynomial fit. And the coefficient c_{ijk} . i is the gray gas, j is the temperature for gas temperature and k is the temperature for the wall temperature. So, 3 parameters here, i , j and k . i is the gray gas, j is the temperature for the surface, sorry the gas, and k is for the surface temperature. So, 3 parameters are there.

And for these 3 parameters, the coefficients c_{ijk} are basically given in the tabular form. And using these coefficients, we can calculate the absorptivity and emissivity of the gas. And we can basically solve any problem. Now, similar to emissivity data, the absorptivity of the transparent gas is simply $= 1 - \sum_{i=1}^3 \text{absorptivity of the } i \text{ gray gases}$.

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WSGG Model (Coefficients)

Parameters for Emissivity Fit
($p_{H_2O}/p_{CO_2} = 1.0$)

$\frac{p_{H_2O}}{p_{CO_2}} = 1.0$
 \uparrow
 \uparrow
 p_{CO_2}

i	K_i	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$	$b_{i,4}$
1	0.4303	5.150e-01	-2.303e-04	0.9779e-07	-1.494e-11
2	7.055	0.7749e-01	3.399e-04	-2.297e-07	3.770e-11
3	178.1	1.907e-01	-1.824e-04	0.5608e-07	-0.5122e-11

\uparrow
 $K_i > 0$
 \uparrow
 a_{E_i}

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So, these are, this is a table. Now, this table has been prepared for a gas mixture. So, we have a gas mixture. This gas mixture is such that the partial pressure of water vapor. So, we have a mixture of water vapor and carbon dioxide in this. And the partial pressure ratio is 1. So, we see that the method is very restrictive in the sense that the data is available for only few partial pressure ratios.

As is clear from this table, this table is made or constructed for a typical case where partial pressure of water vapor and partial pressure of carbon dioxide are equal. If these are not equal, as you may find in combustion problems, then you have to either generate new data or



if you use the same data, then your results will not be accurate. So, the most fundamental limitation of this method is that the data is available only for some restrictive gas conditions.

So, this is the data for emissivity fit. For emissivity fit, we have the absorption coefficient which is constant for 3 gray gases. The fourth transparent gas, $i = 0$ will have $k_i = 0$ as 0. So, the fourth gas is transparent or the 0 gas is transparent with 0 absorption coefficient. And the coefficients b_{i1} , b_{i2} , b_{i3} and b_{i4} are used to find out the weights for the emissivity. That means, a subscript epsilon i . So, the weights in the emissivity data are used to, are fitted basically based on the gas temperature. And the coefficients are given in this table.

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WSGG Model (Coefficients)
Parameters for Absorptivity Fit $b_{i,j,k}$

i	j	T_g	k			
			1	2	3	4 $\rightarrow T_s$
1	1	0.55657e00	-0.62824e-03	0.31876e-06	-0.52922e-10	
	2	0.16676e-01	0.15769e-03	-0.10937e-06	0.19588e-10	
	3	0.28689e-01	0.20697e-03	-0.17473e-06	0.37238e-10	
2	1	0.32964e-03	0.27744e-06	-0.26105e-09	0.37807e-13	
	2	0.50910e-03	-0.76773e-06	0.40784e-09	-0.69622e-13	
	3	0.24221e-03	-0.55686e-06	0.34884e-09	-0.67887e-13	
3	1	-0.53441e-06	0.33753e-09	-0.10348e-12	0.26027e-16	
	2	0.37620e-07	0.18729e-09	-0.15889e-12	0.30781e-16	
	3	-0.19492e-06	0.36102e-09	-0.21480e-12	0.41305e-16	
4	1	0.12381e-09	-0.90223e-13	0.38675e-16	-0.99306e-20	
	2	-0.32510e-10	-0.26171e-13	0.29848e-16	-0.58387e-20	
	3	0.41721e-10	-0.73000e-13	0.43100e-16	-0.83182e-20	


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Similarly, the coefficients are also given for absorptivity data. So, these are parameters for absorptivity fit for parameters $c_{i,j,k}$, where i is the gray gas. So, we have total 3 gray gases, 1, 2, 3. And we have what we called 3 coefficients, 4 coefficients for the gas temperature. And we have 4 for the surface temperature. So, based on these fits, we can calculate the absorptivity data and calculate heat transfer from a homogeneous cell.

So, so far we have limited our understanding of WSGG method to homogeneous cell having same gas conditions, uniform temperature. We will extend this method to non-homogeneous problems also.

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WSGG Model Limitation

- ❖ Spatial independent absorption coefficients (homogeneous media)
 - ❖ Weights allowed to depend on temperature } $a_{\epsilon_i}, a_{\alpha_i}$  k_i
 - ❖ Fractional blackbody emissive power
- ❖ Parameters obtained from total emissivity data
- ❖ Tabulated WSGG parameters only for CO₂-H₂O and CO₂-H₂O-soot mixtures
 - ❖ Fixed partial pressure ratios, $P_{H_2O}/P_{CO_2}=1$ or 2
 - ❖ Typical for combustion of methane.
 - ❖ For other partial pressures, WSGG data not available.



So, again I will summarize what are the limitations of this method. So, the limitations are that, weight is allowed to vary with temperature. So, a ϵ_i as well as the weights for absorptivity, the weights for the absorptivity α_i , they are allowed to vary with temperature. So, these are allowed to vary with temperature. The emissivity weight varies with gas temperature and absorptivity weights varies with gas as well as surface temperature.

However, the absorption coefficient is assumed to be constant. So, if we have change in gas condition, then this condition cannot be accommodated. If we have a different temperature in the gas, so, absorption coefficient is not going to change. So, whatever is the temperature in the gas cell, the absorption coefficient is same. So, this is one of the limitations. It is independent of temperature.

So, absorption coefficient k_i is independent of temperature. So, whatever the cell, the whatever the temperature of the gas, absorption coefficient is same. Which of course is not true. If the temperature changes, we understand, we know that the absorption coefficient is going to change. But, this model assumes that the absorption coefficient does not change with temperature. How the parameters are obtained?

The parameters are not obtained from any model as such. The parameters are not obtained from any line-by-line calculations. Although the new versions of weighted-sum-of-gray-gases do contain weights based on line-by-line calculations. But the original weighted-sum-of-gray-gas model contained parameters which were based on emissivity data. And I will explain this, what is basically the emissivity data.

So, the original weights a_{ϵ_i} and α_i were fitted to emissivity data. And they were not fitted to actual line-by-line calculations or absorption coefficient. The third limitation is that the data is available only for CO_2 and H_2O mixtures. CO_2 , H_2O and sometimes soot also for limited partial pressure. Either the partial pressure of H_2O versus CO_2 should be 1 or should be 2.


While in actual application, the partial pressure can be anything. It can be 1, it can be 2, it can be > 2 . So, for those applications where the partial pressure ratio is not 1 or 2, we do not have any data. So, that is the other limitation of this model.

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WSGG Parameters

$a_{\epsilon_i}, k_i, \alpha_i$



- ❖ Parameters evaluated by minimization of least square error in total emissivity data of homogeneous gas column
- ❖ Variety of length scale (~ 1 cm to ~ 100 cm).
- ❖ Range of temperatures and pressures



homogeneous cell

$$\epsilon(T, s) = \frac{1}{I_b(T)} \int_0^\infty (1 - e^{-k_\lambda s}) I_{b\lambda} d\lambda$$

$$\epsilon = \sum_{i=0}^{I=3} a_{\epsilon_i}(T_g) [1 - e^{-k_i p s}]$$



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Now, how do we evaluate this parameters. So, as I said, in the original weighted-sum-of-gray-gas models, the parameters, the weight parameters, the absorption coefficient parameters. So, a_{ϵ_i} , k_i and α_i . So, how these parameters were evaluated? So, lot of researchers have calculated this parameter. But, what approach did they use? If you understand that, we will be able to appreciate the limitation of this method very well.

So, what basically this researchers do, they took a homogeneous cell. They took this homogeneous cell. And this cell, the dimension of this cell, they varied from few centimeter to few 100 centimeter. So, they change the length of this cell. They change the temperature inside the cell. They change the pressure inside the cell. So, the gas condition, they changed. And they calculated the emissivity ϵ .

Now this emissivity, they might have obtained sometimes through experiments. So, original weighted-sum-of-gray-gases model fitted the experimental data. So, they obtained emissivity of gas mixtures. Now, this emissivity data is available in literature. Lot of researchers have done experiments and calculated the emissivity of a homogeneous gas layer under varying conditions.

So, many researchers fitted the emissivity data using minimization of least square error and evaluated these coefficients. Later on, the researchers calculated this emissivity from line-by-line data or narrow band data. And based on this emissivity calculated from line-by-line data, they evaluated this parameters a ϵ_i k_i and a α_i . So, the point is that it is basically a curve fit in such a form that we are minimizing the least square error.

We have calculated the emissivity based on line-by-line calculation or experiments. And we calculate the emissivity from our curve fit based on the coefficient that we want to determine a ϵ_i α_i and k_i . And when we have minimum error, minimum least square error, those parameters of the model we accept. And this form the parameters of weighted-sum-of-gray-gases model is obtained.

So, of course, there is a limitation, because the actual value of path length, actual value of absorption coefficient, actual value of pressure and temperature is not input to the problem. The only thing that is input to the problem is a curve fit which basically minimizes the least square error. So, actual problem may have some larger error as compared to these minimization of least square error. So, in actual practice, the weighted-sum-of-gray-gas model may not give you accurate results. And we will see the accuracy of this method when we solve 1 problem.

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WSGG Enhancement

- ❖ More accurate WSGG parameters from high-resolution databases.
- ❖ Applied to grey reflecting walls.
- ❖ Variable absorption coefficients (non-homogeneous path)
 - ❖ scaling approximation

$$\kappa^*(\varphi) = \kappa(\varphi_0)u(\varphi, \varphi_0)$$



$$u = \frac{p}{p_0}$$

$$u = 1.0$$

Number of enhancements, number of advancements have been done, because the method has enjoyed very popularity. So, number of modifications had been proposed in weighted-sum-of-gray-gases model. It can be applied to reflecting walls. Now the data or the parameters of weighted-sum-of-gray-gases model are calculated based on line-by-line spectral data, rather than experimental fitting.

Also, now the method can be applied to non-homogeneous gas also using scaling approximation. So, for example, if we have 2 gases, we know that the absorption coefficient in this method has been assumed to not depend on temperature. That is, an assumption in weighted-sum-of-gray-gases that absorption coefficient does not depend on temperature. But what will happen if the gas concentration is different.

Let us say the gas concentration is ϕ in this cell and gas concentration is ϕ_0 in this cell. Then, how do we relate absorption coefficient? So, for this, what we called scaling approximation has been proposed. What scaling approximation does is that, absorption coefficient in any gas condition ϕ is simply dependent on absorption coefficient at reference gas condition ϕ_0 times a factor u which is, where u is called scaling function.

So, based on this, we can scale the absorption coefficient. So, u can be just simply = the partial pressure also. It could be partial pressure p by p_0 . Or any function can be taken. If we take $u = 1$, then we are just saying that absorption coefficient is constant throughout. So, scaling approximation is used in large number of problems. And it enjoys a good success.

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Reduction of RTE

$$\frac{dI_k}{ds} = \kappa_k a_k I_b - \kappa_k I_k$$

$$I(s) = \sum_{k=0}^K I_k(s)$$

$\frac{dI_k}{ds} = \kappa_k (I_b - I_k) - 0$
 RTE
 ↓
 4 RTEs
 1 → 3 gray gases
 4 → transparent
 K = 4
 0, 1, 2, 3

- ❖ Need to solve RTE separately for each grey gas
- ❖ Four grey gases give sufficient accuracy
- ❖ Net intensity/flux is the sum of intensity/flux from individual grey gases (no-overlap between grey gases)



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So, before we solve any problem, let us see how the RTE basically is reduced for this method. So, we know that radiative transfer equation in spectral space is given by $\frac{dI_\lambda}{ds} = \kappa_\lambda I_b - I_\lambda$. Okay. So, when we applied weighted-sum-of-gray-gas model, the spectral model based on weighted-sum-of-gray-gases, what we are assuming is that absorption coefficient is replaced by a gray gas.

So, this is 1 RTE. Now, we have split this into 4 RTEs. Where we have 1 to 3 as gray gases and the fourth one is basically a transparent gas, where kappa is 0. Okay. So, for each gas, each gray gas, we can write the RTE as $\frac{dI_k}{ds} = \kappa_k I_b - \kappa_k I_k$. Kappa k is basically the gray gas absorption coefficient. a_k is the weight or weight of the weighted-sum-of-gray-gas model. I_b is integrated black body intensity – $\kappa_k I_k$.

So, in the sense, so what does weight represents here? So, weight basically represents fraction of black body emissive power or black body intensity. How much each gray gas carries energy with it. So, that is basically the use of weight in weighted-sum-of-gray-gases model. It basically defines how much energy is emitted by each gray gas. Once we have the intensity, once we have the solution for single RTE, the total intensity is just the sum of individual intensities $I = \sum_{k=0}^K I_k$.

And k in this case is nothing but 4 0. It will be rather 3; 0, 1, 2, 3, where 0 is the transparent gas and 1, 2, 3 are the gray gases. So, we need to solve RTEs individually for each gray gas. And then, we just add the results. And it gives reasonably good accuracy when we see this example.

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Problem

Consider an isothermal slab at temperature $T=1000$ K, and a total pressure of 1 atm. The slab consists of mixture of 70 % N_2 , 20 % H_2O and 10 % CO_2 by volume. The cell is bounded by cold black walls. Determine heat loss from the cell of thickness, $L=1$ m. Assume the following data:

$a_0 = 0.466$	$a_1 = 0.337$	$a_2 = 0.159$	$a_3 = 0.038$
$\kappa_0 = 0$	$\kappa_1 = 0.267 \text{ m}^{-1}$	$\kappa_2 = 4.65 \text{ m}^{-1}$	$\kappa_3 = 71.7 \text{ m}^{-1}$

↑
4 gray gas

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So, let us do 1 problem. In this problem, we are considering as isothermal slab. We have an isothermal slab at temperature is 1,000 kelvin. Okay. The total pressure is 1 atmosphere. Now, we have a mixture in this gas. A gas mixture 70% nitrogen, 20% water vapor and 10% carbon dioxide in this. The cell is bounded by cold and black walls. So, the walls are cold. So, they do not emit any radiation.

So, we do not have to worry about how much energy is emitted from the wall, because they are cold. And they are black, so they do not reflect. So, the walls do not emit, they do not reflect. So, the walls are basically as good as not there. So, we have to find out the heat loss from the cell assuming the thickness of the cell is 1 meter. The coefficients for this weighted-sum-of-gray-gas model has been given.

We have 4 gray gases. 1 transparent gas with $\kappa = 0$. So, 1 transparent gas is there. And then, 3 gray gases with coefficients κ_1 , 0.267 meter inverse, κ_2 , 4.65 meter inverse and κ_3 , 71.7 meter inverse. So, we have to find out the amount of heat flux from this cell.

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One dimensional Plane Parallel Slab

Exact Solution

$$\psi_i = \frac{q_i}{\sigma T^4} = \left[1 - 2 E_3(k_i L) \right] a_i$$

$$\psi = \sum_{i=0}^3 \frac{q_i}{\sigma T^4} = \sum_{i=0}^3 a_i \left[1 - 2 E_3(k_i L) \right]$$

$$\psi = 0.466 \left[1 - 2 E_3(0) \right] + 0.337 \left[1 - 2 E_3(0.267) \right]$$

$$+ 0.159 \left[1 - 2 E_3(4.65) \right] + 0.038 \left[1 - 2 E_3(71.7) \right]$$

$$E_3(0) = 0.5$$

$$E_3(71.7) \approx 0$$

$$\psi = 0.337 (1 - 2 \times 0.324) + 0.159 (1 - 2 \times 0.001) + 0.038$$

$$= 0.1184 + 0.1587 + 0.038$$

$$\psi = 0.3152$$

$$\psi = 0.3152 \times \sigma T^4$$

So, let us solve this problem. We will use, we can use any RTE solver for this 1-dimensional plane parallel slab. We have discussed many methods. I will use the exact solution. So, exact solution for this method; the exact solution for this method, for the heat flux, non-dimensional heat flux ψ is given by q upon σT^4 . Now, we can do it for a single gray gas, q_i where i is the gray gas coefficient.

And this will be multiplied by $1 - 2 E_3(k_i L)$. And this will be multiplied by a weight coefficient a_i . So, this is the non-dimensional radiative heat flux coming out of the cell using the exact method for a single gray gas. If you have number of gray gases, that is 4 in total, it will be just sum $i = 1$ to 4 or $i = 0$ to 3, whatever, just the notation thing. And this will be = summation $i = 1$ to 4 or take $i = 0$ to 3.

We can, just the notation thing. So, 0 to 3. And this will be = $a_i [1 - 2 E_3(k_i L)]$. E_3 is the exponential integral of order 3, we have discussed this; $k_i L$, so this is the non-dimensional heat flux. Now, all we have to do is, we have to evaluate this exponential integral and just add the terms together. So, ψ is =, the coefficients are already given in the table. So, we will just substitute these coefficients.

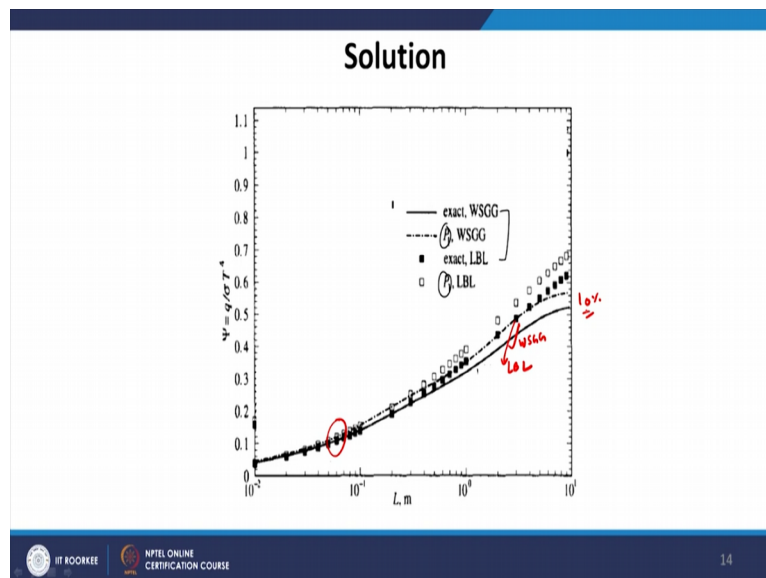
ψ is = 0.466, where 0.466 is the value of weight coefficient a_0 . $1 - 2 E_3(0)$. Because k is 0 for $i = 0$ + the second coefficient which is = 0 point 337 $1 - 2 E_3(0.267)$. The length is = 1 meters, + 0.159 $1 - 2 E_3(4.65)$ + 0.038 $1 - 2 E_3(71.7)$. So, we have substituted and expanded this summation and put it the weights of each gray gas. So, we have total 4 gray gases and we have added the contribution from all the gray gases.

Now, E_{30} is simply $= 0.5$. So, 2 times E_{30} is 1. So, $1 - 1$ will be 0. So, the first term basically turns out to be 0. And it makes sense, because for optically transparent window where absorption coefficient is 0, the gas will not emit any radiation. Because the absorption coefficient is 0, gas does not emit any radiation, it does not contribute to any heat flux. So, that is, that makes sense.

Similarly, exponential integral of 71.7, which is now very large optically thick case is approximately $= 0$. So, simplifying this, we get ψ is $= 0.337 \times 10^{-2}$ into; the exponential integral value can be looked into the tables given in the textbook. $0.324 + 0.159 \times 10^{-2}$ into 0.001. And this value comes out to be $0.1186 + 0.1587$. And definitely, third term I forgot to add here, $0.038 + 0.038$. So, the total value of the heat flux ψ is nothing but 0.3152.

So, this is the non-dimensional heat flux. Actual heat flux is nothing but q is $= 0.3152$ into σT^4 . Here T is 1,000 kelvin. So, you can calculate the heat flux coming out of this cell using weighted-sum-of-gray-gases. So, as is clear, this method is very easy to implement. All you have to do is, you apply whatever radiative transfer equation solver you are using to a gray gas. And then, just add the contribution of the gray gases together. This is a plot that demonstrates the accuracy of this method.

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We have a solution from spherical harmonics method P 1 also given here. And we are comparing the accuracy of the method using weighted-sum-of-gray-gases and line-by-line method. So, these 2 methods we need to compare. As we see here, these 2 methods, this is the

exact solution. And the square dots are basically the solution. So, this is the weighted-sum-of-gray-gases.

And the square dots are basically the exact line-by-line. And we see that, for small path lengths, that is optically thin case, weighted-sum-of-gray-gas model goes to correct limit. Because, there is very less self-absorption. The inaccuracies in the weighted-sum-of-gray-gases model arise from the assumption that absorption coefficient is constant. Absorption coefficient does not depend on temperature.

So, that is the limitation of the weighted-sum-of-gray-gas model. But since, for optically thin case, very small value of κ or L ; it does not matter what is the magnitude of absorption coefficient, because the absorption is small in any way. The weighted-sum-of-gray-gas model goes to correct limit. So, we see good accuracy in this range. However, when self-absorption is important, when the cell thickness is large, when κ is large, the weighted-sum-of-gray-gas model gives inaccuracy.

And we see that there is roughly 10% error here in this range. So, in this lecture, we focused on the global model, weighted-sum-of-gray-gas model, which is very powerful method and very popular in solving radiation problems in combustion. And lot of correlations have been developed. Although these correlations have been developed for typical combustion of methane and some organic fuel, but a few researchers have extended these correlations to some other non-conventional fuels as well.

So, the limitation with this method is that, we assume absorption coefficient to be uniform and the parameters are available only for certain cases. Other than that, the method gives good accuracy. In the next lecture we will focus on k-distribution model which is basically 1 of the most advanced model in this category of global models. So, Thank you.