Radiative Heat Transfer

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Module - 7
Lecture - 33
Wide Band Models

Hello friends, we are discussing spectral models. In the last lecture, we discussed narrow band models, where we took an average over a small spectral range stretching over few lines 10 to 15 lines, few wavenumbers. The argument in favor of narrow band model was that when we are taking average over small wavelength range, the Planck black body function does not change significantly.

And also, while taking average, there is no point in exactly finding where the lines are located. We can simply assume lines to be randomly located or uniformly spaced. When these simplifications are done, we are able to find out mean value of absorption coefficient and emissivity. And to match the results better, the parameters of the models were fitted with experimental data.

Now, this simplification of the spectrum can be taken a step further using the wide band models that we are going to discuss in this lecture. Now, wide band models, they also rely on averaging over lines. But this time, we are going to take an average over the entire Ro-vibrational band. So, unlike in narrow band models, where the averaging was done only over few lines, in wide band model, the averaging is done over the entire Ro-vibrational band.

So, of course, the accuracy of this approach is going to be < the accuracy of narrow band models. So, typically you can expect 40 to 50 percent accuracy for this wide band model as compared to reported 20 to 30 percent in the case of narrow band models.

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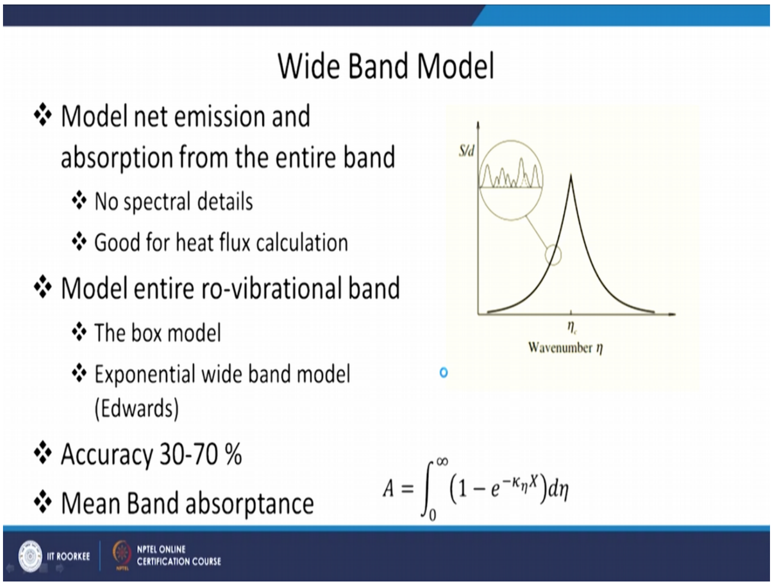

So, we start this discussion by looking at the shape of a typical Ro-vibrational band. A typical Ro-vibrational band may be symmetric or it may be non-symmetric as we will see. What we try to do is, we take an average over the lines and fit with a smooth function, as is shown in this particular image. For heat transfer calculation, this turns out to be reasonably good approximation.

The detailed spectral details are not needed. Reason being, most of the heat transfer takes place from line centers, where the strength is highest. And very less amount of heat transfer takes place from line wings, where the absorption coefficient is small. So, we can take an average. We can smoothen the function, the way the absorption coefficient varies in the Ro-vibrational band, we can smoothen it.

And general accuracy for this model is 30 to 70 percent. So, in this category, we will discuss 2 models, the box model and exponential wide band model. Box model is a very crude approximation of gray absorption coefficient. Where in gray absorption coefficient, we just take an average value over the entire spectrum, in box model, we take an average value over the entire band.

While exponential wide band model tries to fit an exponential function, as is also visible here over the rotational band. The quantity just like in the narrow band model, we were interested in mean emissivity of the narrow band. The quantity here we will be interested in mean absorptance A. Which is defined as 1 e to the power – kappa eta X d eta. So, this quantity, we will be interested in.

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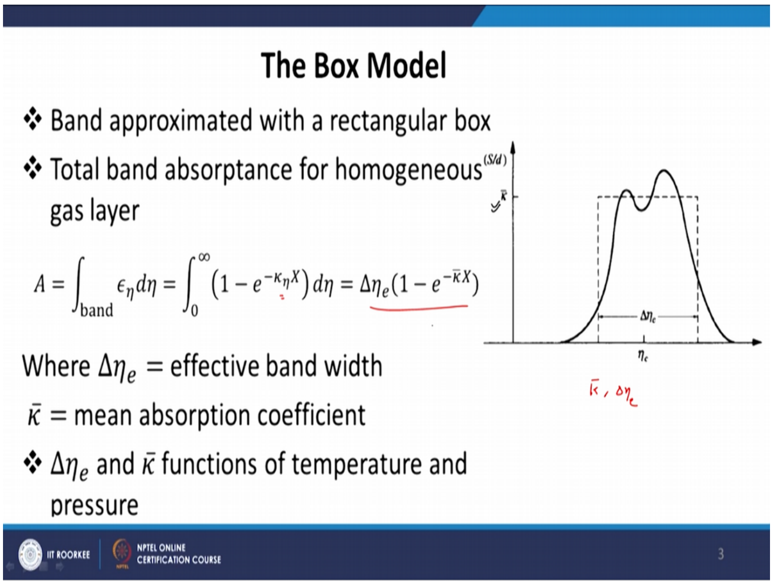

So, as I said, the box model basically it fits an average value of absorption coefficient over the entire band. Now, this delta, this box will have certain dimensions. The dimensions will be given by the magnitude of mean absorption coefficient kappa bar. The dimensions of this box will be the mean absorption coefficient kappa bar. And the width of this box delta eta e. So, the 2 parameters in this are mean coefficient and box width delta eta e.

So, we substitute when these values in the expression of band absorptance, we assume kappa eta does not change with eta. And we take a path of length X. So, we reduce this band absorptance formula delta eta e 1 – e power – mean kappa X. Where mean kappa is the mean value of the average absorption coefficient. Now, delta eta e and kappa, they will be function of temperature and pressure.

And normally, the evaluation of these parameters is not trivial. It may depend on number of factors like pressure, temperature and sometimes these values are determined from experiments. So, this is a very crude way of approximating a band. But this is very much preferred especially when we are trying to solve complicated problems using one of the standard techniques like spherical harmonics or discrete ordinate method.

The box model together with these type of models has gained very popularity. And it gives reasonably good results. We define 1 more parameter. That is, mean or integrated absorption coefficient, alpha. Which is nothing but, absorption coefficient integrated over the entire spectrum. Now, the way the wide band models have been developed; so, the wide band models have been developed from narrow band models.

So, people developed narrow band models first. And then, from narrow band models, they developed the wide band models. The parameters of wide band models are generally derived from narrow band parameters. We discussed in the previous lecture that the narrow band models are basically defined by 2 parameters, the line strength, mean line strength and mean line spacing s and d. So, we can derive the integrated absorption coefficient alpha from narrow band parameters by using this relation.

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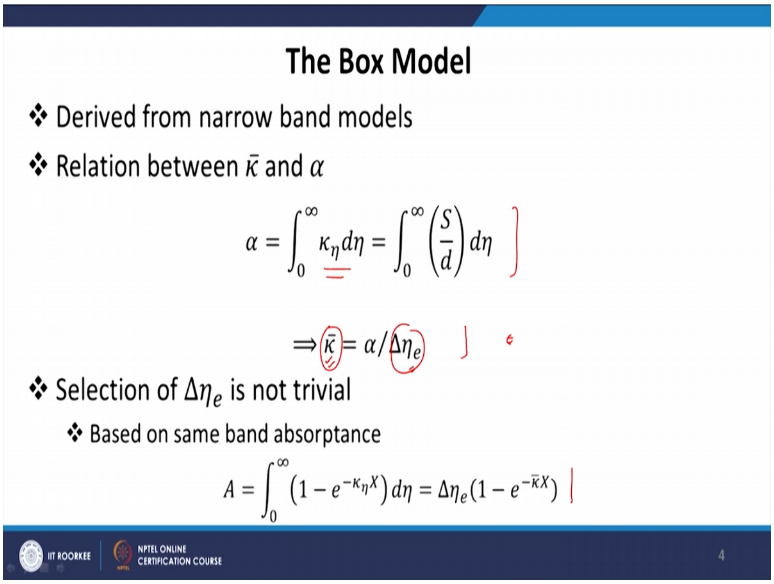

And then, we can relate this to the average absorption coefficient kappa bar using the relation here. We discuss this here also. The band absorptance is given by delta eta e. And average absorption coefficient. So, we can relate the absorption coefficient kappa with integrated absorption coefficient alpha. So, in this way, narrow band parameters can be used to calculate the parameters for the wide band model.

And narrow band parameters as we discussed are already experimentally fitted. So, in a sense, the parameters for the wide band model are indirectly derived from experiments only. Now, once we have determine kappa, we can calculate delta eta e from this expression. Now, delta eta e is based on the argument that the total band absorptance should remain same. So, based on this argument the delta eta e will be determined.

The 2 parameters for the box model are kappa bar and delta eta e. So, kappa bar is calculated based on the narrow band parameters and delta eta is calculated based on the assumption that the total band absorptance should remain same. So, this will be much more clear when we do 1 example. And then, we will explain how these parameters are basically evaluated. The second model in this category is exponential wide band model. And you will be able to appreciate this model by just looking at the structure of a Ro-vibrational band.

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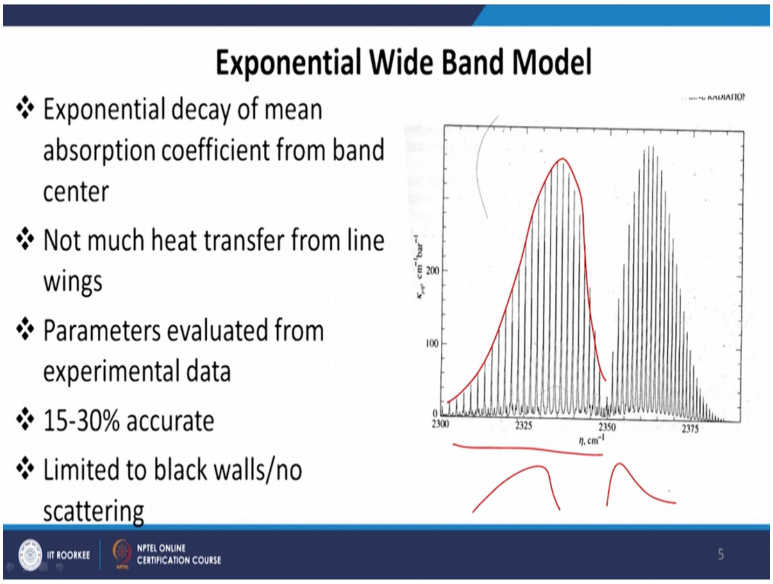

So, if you look at a Ro-vibrational band, you see that the intensity varies just like an exponential function. And it turns out to be that, we can fit some exponential function to these type of bands. This is one's band which is symmetric in nature. Some bands may look like this and some bands may look like this. Okay. So, there may be bands, bands with a head. And bands with symmetric nature, symmetric bands. And there are 3 types of bands for which exponential function has been given. The method was basically developed by Edwards.

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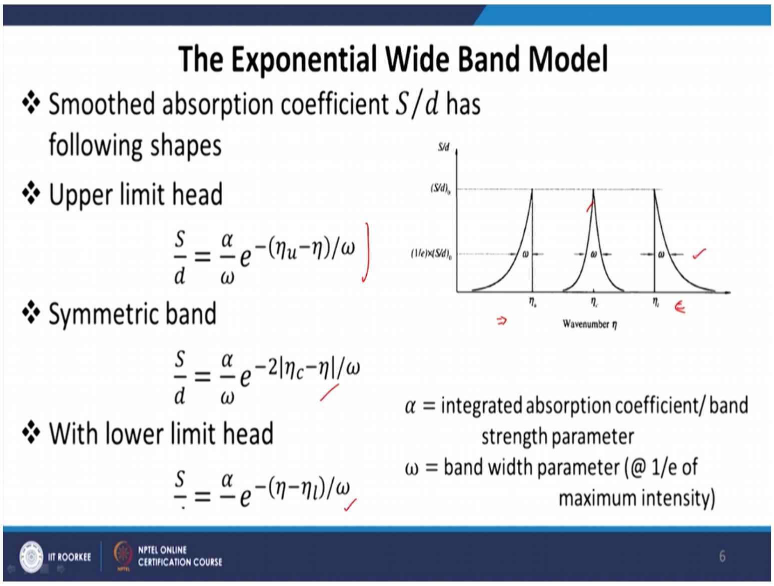

So, we have what we call band with head. There may be upper limit, that is band head is in the forward direction. Or band head in the backward direction, it is called band with lower limit head. And for this, we can write function S by d which is basically smoothened exponential function. S by d is = alpha by omega. Where omega is the bandwidth parameter, which is defined as the width of the band at 1 by e.

That means, 1 by e of the maximum intensity. So, this is basically a parameter that defines how wide the band is. And the function S by d is given by this. So, what it gives you is the strength of the band at a given distance away from the band head or band center. Similarly, for the symmetric band given by this function and the band with the lower limit head given by this function.

So, what Edwards basically did is, he identified the rotational bands to have these 3 formulation. He identified that some bands tend to be symmetric in nature, some bands tend to have a head. And based on that, he fitted an exponential function to the Ro-vibrational band. And based on that, he calculated the mean absorptance. And he gave a correlations. So, he found the correlations for mean absorptance,

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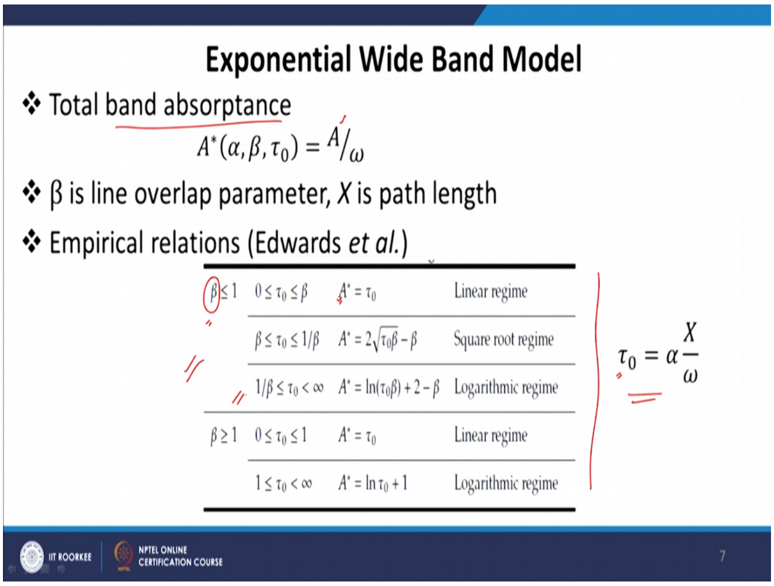

That is total band absorption, he found some correlations. And he gave correlations in this table. This table, the correlations given here will be best explained, when we do 1 example. But the parameters here in this table are line overlap parameter beta, which we discussed in the previous lecture. And optical thickness tau nought given by alpha X by omega. Where omega is the width parameter, X is the path length and alpha is the integrated absorption coefficient.

So, based on these parameters, we will be able to solve the problem of total brand absorption. And it will be best explained when we do a problem. So, let us do a problem on this and see how we can apply the wide band model to find out total band absorptance.

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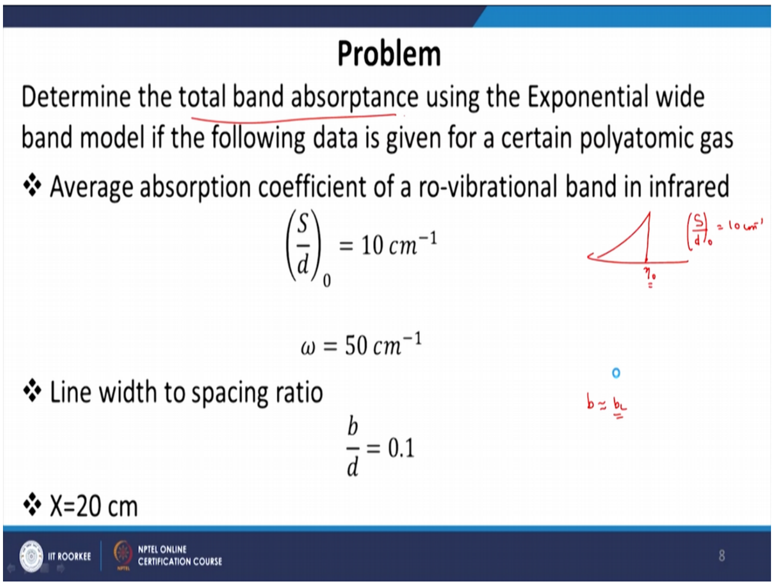

So, we have been given a problem, where we have been given the value of S by d at the center, which is 10 centimeter inverse. So, let us say we have a band, let us say this is the band. So, this is the band head or center. In case of symmetric band, this will be called band center. In case with the brand with the head, we just can denote it by eta nought. So, this is the eta nought. And S by d, that is line strength divided by line separation distance d at the center 0, that means at the band head is given as 10 centimeter inverse.

So, this value is given. The omega value is also given at 50 centimeter inverse. And line width to spacing ratio b by d is given as 0.1. So, b is same as b L. It basically denotes the line spacing, spacing between rotational lines. And the path length is given as 20 centimeter, X is 20 centimeter. So, we have to find out total band absorptance. Now, we will be using this table. Okay.

This table gives you A star values. And A star and A are related by this relation. So, first we found A star, and then we multiplied by omega to get total band absorptance A. Now, A star values is based on the value of beta and tau nought. So, these 2 parameters, we have to find out. So, first of all, let us find out the value of beta and tau nought. Okay.

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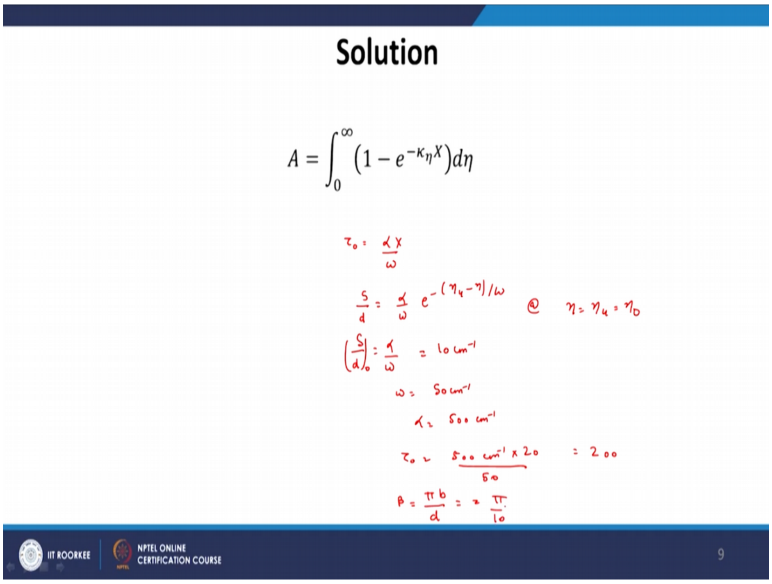

So, tau nought is based on the value of, the expression is given; tau nought is alpha X by w omega alpha X by omega. This is the expression for the tau nought. Now, we have S by d is = this is the expression for band with a head. So, this expression we will take S by d is = alpha by omega e power – eta u – eta by omega. Alpha by omega e power – eta u – eta by omega. Okay.

Now, at eta is = eta u or is = eta nought, that is at the brand head, this term will be simply = 1. And we get S by d nought is = alpha by omega. Now, S by d nought is already given to you. The value is given as 10 centimeter inverse. So, this will be = 10 centimeter inverse. And omega value is also given as 50 centimeter inverse. So, you get alpha is = 500 centimeter inverse.

So, the integrated absorption coefficient you have calculated. Now, we will basically find out the value of tau nought. So, tau nought is = 500 centimeter inverse. The path length is given as 20 and omega is basically given as 50. So, this will come out to be 200. So, tau nought will value comes out to be 200. Now, beta value; the line overlap parameter, the relation is pi b by d. So, b by d value you already know. b by d is given as 0.1.

So, this becomes = pi by 10. So, we have already calculated the 2 parameters needed to look at the correlations, that is tau nought and beta. Now, we go to this table again. This is the table. So, in this category, we have beta, which is value pi by 10. So, we come under this category. Beta is < 1. And we have to look for the relations. So, our tau nought is much > 1 upon beta. So, this correlation will be satisfied. So, A star will be ln tau nought beta + 2 – beta.

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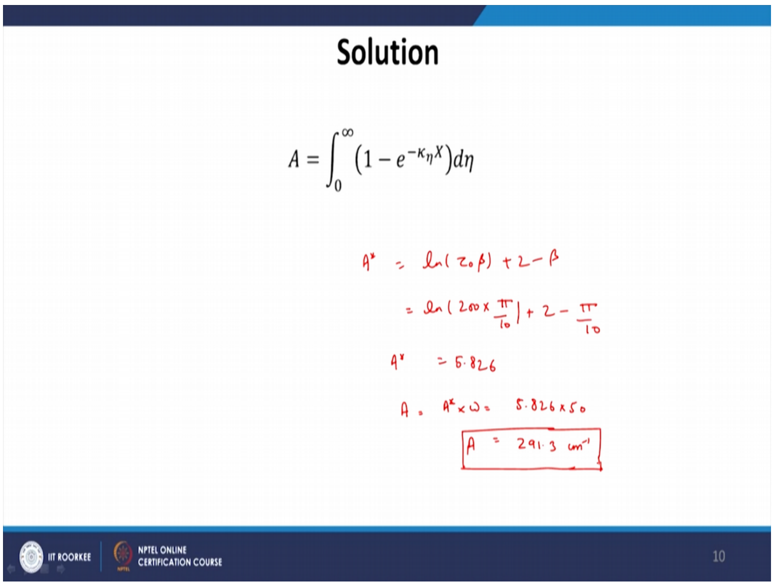

So, A star is = ln tau nought beta + 2 – beta. So, this will be the relation. And we substitute the value of tau nought here. It, we get ln 200 into pi by 10 + 2 – pi by 10. So, this value comes out to be 5.826. And the value of A is simply A star times omega, is = 5.826 into 50. And this value comes out to be 291.3 centimeter inverse. So, total band absorptance is 291.3

in this case. So, this is a value of total band absorptance using the method of wide band model. Okay.

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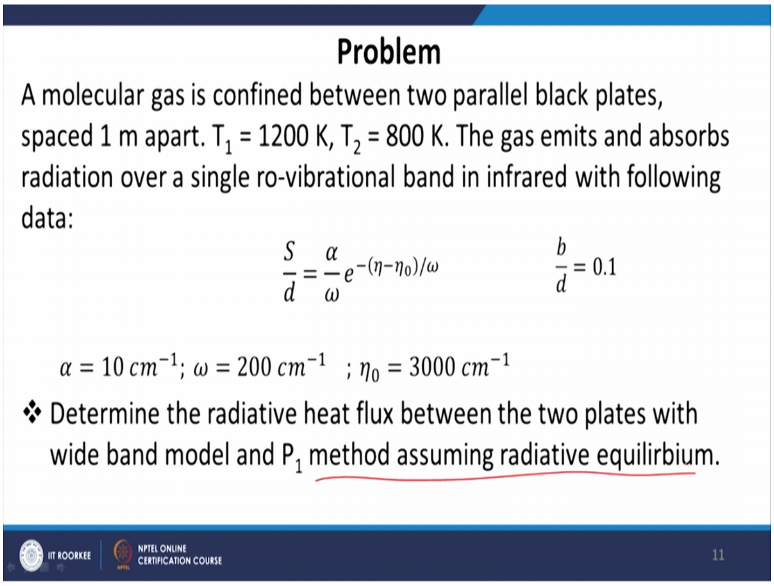

Now, we will do 1 more example to relate how we can solve this problem to actually find out heat flux. So, we have a molecular gas confined between 2 parallel black plates 1 meter apart. 1 plate is at 1200 kelvin, the second plate is at 800 kelvin. The gas emits and absorbs radiation over a single Ro-vibrational band. So, we have a gas which basically emits and absorbs radiation over a single Ro-vibrational band.

And the data is given S by d is = alpha by omega e – eta – eta nought by omega. So, exponential wide band model. b by d parameter is given as 0.1. Value of alpha is 10 centimeter inverse. Omega is 200 centimeter inverse. And eta nought is 300 centimeter inverse. So, we have to find out, determine the radiative heat flux between the 2 plates with wide band model and P 1 method, assuming radiative equilibrium. So, P 1 method with radiative equilibrium, we have discussed while discussing the method of spherical harmonics. And let me give you the result derived from there.

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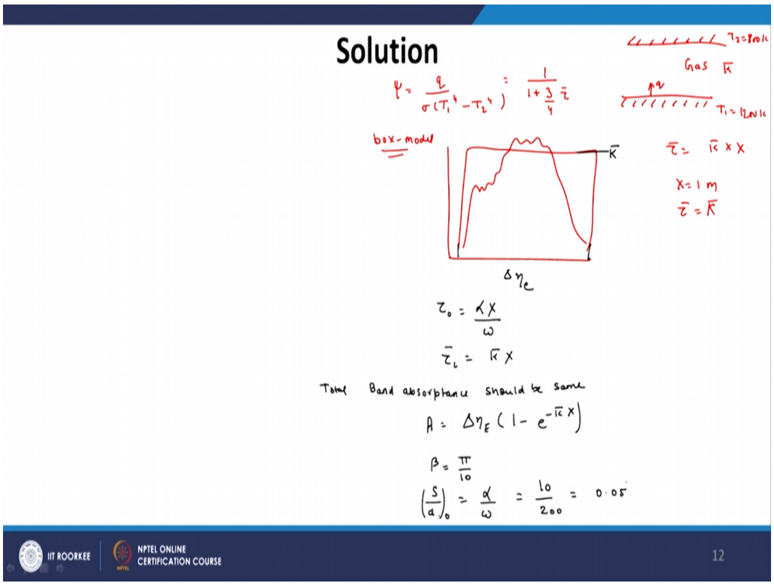

So, the heat flux, non-dimensional heat flux psi is q by sigma T 1 power 4 – T 2 power 4. Okay. For this configuration where we have 2 plates. There is a gas. Now, the gas is not gray, it is a non-gray gas which emits radiation over a rotational band. And we are interested in heat flux from these plates. The plate 1 is at 1200 kelvin and plate 2 is at 800 kelvin. So, for this non-dimensional heat flux is given by this relation.

We can write as 1 + 3 by 4 tau prime. Okay. So, for this case, we are going to apply the box model. So, for box model, the value of gas, mean absorption coefficient is kappa prime. So, tau prime is nothing but kappa prime times x. x is given as 1 meter. So, tau prime is nothing but = kappa prime. Okay. Kappa prime we need to find out. So, the relation for the heat flux is very simple, 1 upon 1 + 3 by 4 tau.

So, the relation we have, for the box model, we have, let me just draw the box model. So, we have, let us say this is the band. And we have fitted with a box. This is our box. So, let me just change the color. So, we have changed the, we have, there is a box here. The value of absorption coefficient is kappa prime and the width is delta eta e of the box. Okay. Now, we have to find out these parameters kappa prime and delta eta e. Okay.

Now, it turns out to be that tau l is alpha X by omega. Okay. And tau l bar is kappa prime into X. Okay. So, we basically use that mean band absorptance should remain same. So, band total band absorptance should be same. So, we, with that logic, we basically try to find out the brand absorptance. Let me just show you the relation that we are going to use. So, total band absorptance is given by delta eta e 1 – e power – kappa X.

So, we have to find out delta eta e and kappa prime from this relation. So, total band absorptance is = delta eta E 1 – e power – kappa prime X. Okay. So, this is basically our formula. Now, we have to first find out the kappa prime. And then we can find out delta eta e in this relation. So, first of all, let us see how we can basically relate this. So, our beta, the line parameter is given as pi by 10.

And we already have been given S by d nought as = alpha by omega. This we did in the previous problem. So, S by d nought is = alpha by omega. We did in the previous problem. The value of alpha is given as 10. And omega is given as 200. So, this value comes out to be 0.05. Okay.

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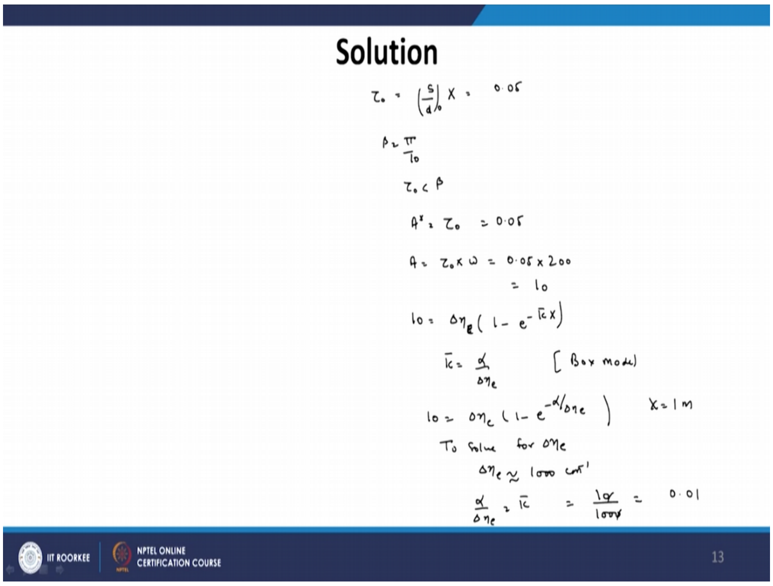

Next, we find tau nought. So, this calculation is what we did in the previous example also, same calculation. So, tau nought is =; now, S by d nought, that is tau nought times X is =, X is = 1, so 0.05. So, tau nought is = 0.5 and beta value is pi by 10. So, tau nought is < beta. So, from the table, we use the same table as was used by the exponential wide band model. It turns out to be because the relation, this same, the, why, the model, the band is represented by the same relation as given in this problem.

The same relation as used in exponential wide band model, we can use the same table. So, for the, from the table, we basically get A star is = tau nought for the given range. And this value is = 0.05. And from this, A is = tau nought times omega, is = 0.05 into, omega value is 200. And this value comes out to be 10. So, value of A comes out to be 10. We will put this value here to solve for delta eta e.

So, we get 10 is = delta eta e 1 – e power – kappa X kappa bar X. Okay. Now kappa bar is =, the mean absorption coefficient is simply = alpha by delta eta e. From the definition of the box model, kappa bar is = alpha by delta e, box-model. So, we substitute it here. 10 is = delta eta e 1 – e power alpha by delta eta e, X value I have taken 1. X is = 1 meter. So, this becomes a equation now to solve for delta eta e.

So, we need to solve for delta eta e. And it turns out to be, again solution to this equation is not trivial. But you can solve it. Delta eta e roughly comes out to be 1,000 centimeter inverse. By just solving this equation you can solve for delta eta e; that means, the width of the box comes out to be 1,000 centimeter inverse. And from this, you can calculate the alpha by delta eta e is = the mean value of kappa which is now =; alpha value is given in the problem as 10 centimeter inverse. And delta eta e is 1,000. So, this comes out to be 0.01. So, the value of kappa bar comes out to be 0.01.

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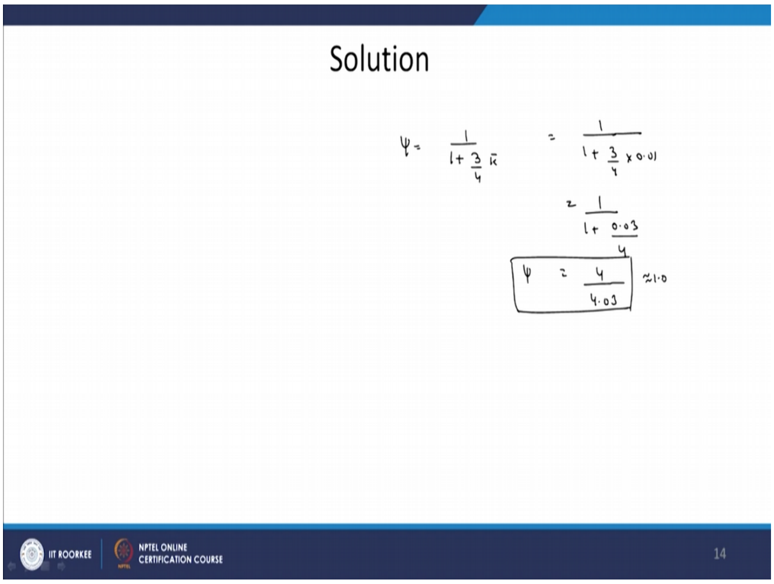

Now, our flux, non-dimensional flux is simply = 1 upon 1 + 3 by 4 kappa bar is = 1 upon 1 + 3 by 4 into 0.01. So, this will be = 1 + 0.03 by 4. Okay. And this will be = 4.03. So, this will be very close to 1, because the medium is optical thin. Okay. So, this value is very close to 1, because the medium is coming out to be optically thin.

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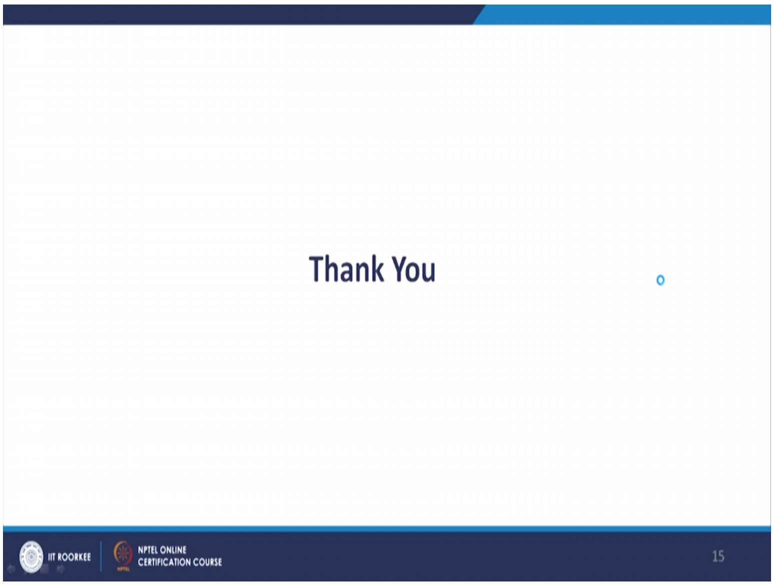

So, thank you. In this lecture, we discussed the 2 wide band models, the box model and the exponential wide band model. And we see how this models can be applied to solve problems between actual cases. let us say flat plates. And we applied this spherical harmonics method together with the box model to solve for the radiative heat flux. In the next lecture, we will discuss about the global models which cover the entire spectrum.

So, the narrow band model and wide band model, they cover only certain part of the spectrum. 1 contains only few lines and another contains only the entire Ro-vibrational band. But the global models, they cover the entire spectrum. And they tend to be much more efficient. So, we will discuss this method in the next lecture. Thank you.