

Radiative Heat Transfer
Prof. Ankit Bansal
Department of Mechanical and Industrial Engineering
Indian Institute of Technology – Roorkee

Lecture - 03
Basic Laws of Thermal Radiation

Good morning friends. In the last lecture, we basically discussed some fundamentals of thermal radiation. We discussed about blackbody, blackbody emissive power, total blackbody emissive power, fractional blackbody emissive power, solid angle and radiative intensity. In this lecture, I will introduce you to some basic concepts, some basic laws of radiative heat transfer.

(Refer Slide Time: 00:46)

Problem

- Determine the solid angle with which the Sun is seen from Earth

$D_s = 1.496 \times 10^{11} \text{ m}$
 $R_s = 6.96 \times 10^8 \text{ m}$

$$\Omega_s = \frac{\pi R_s^2}{D_s^2} = \frac{\pi \times (6.96 \times 10^8)^2 \text{ m}^2}{(1.496 \times 10^{11})^2 \text{ m}^2}$$
$$= 6.80 \times 10^{-5} \text{ sr}$$

The slide also features a diagram of the Sun and Earth with a red line representing the radius of the Sun and a red line representing the distance from Earth to the Sun. The solid angle is indicated by a red arc.

ITR ROORKEE | NPTEL ONLINE CERTIFICATION COURSE | 2

From last time, let us do one problem first and then we will go to some other fundamental radiation laws. We discussed about solid angles in the last lecture. I will take one example on the determination of solid angles. So we have to find out in this example the solid angle with which the sun is seen from earth okay. So the sun is a distant object, the radiation coming from the sun is more or less coming from the same direction.

We want to see how much solid angle sun substance at earth okay. So let us say we have this is the sun okay and we have the radiation which is coming to earth okay. We want to find out the solid angle with which the sun is seen from earth okay. So some parameters we know that is the distance that is called the distance of sun as measured from earth and this is equal to 1.496×10^{11} meters okay.

So this is the sun distance from earth, now this distance may vary from day-to-day but this is an average value taken over a year okay. This value may change by some amount from day-to-day but this is an average value. Then, sun is seen just like a circular disc with radius equal to 6.96×10^8 meter okay. So the sun appears to be a circular disc with radius 6.96×10^8 meter.

So how do we calculate the solid angle? The solid angle is defined as the solid angle of the sun as seen from earth is defined as the projection area. So when we are talking about projection, basically we are talking about the circular disc. The projected area will be equal to πR_{sun}^2 . This is the projected area of the sun divided by the distance square, the distance of the sun square okay.

$$\Omega_s = \frac{\pi R_{sun}^2}{D_s^2} = \frac{\pi * (6.96 \times 10^8)^2}{(1.496 \times 10^{11})^2} = 6.80 \times 10^{-5}$$

So this will be the solid angle by which the sun will be seen from earth okay. So solid angle will be equal to, when we solve this, the solid angle will be equal to 6.80×10^{-5} steradian okay. So this is very small very small solid angle and that is why we can assume that all the radiation that is coming to earth from sun is basically coming in the form of parallel rays.

That is it is not having a range of solid angles, it is all basically coming from the same direction because the solid angle is small okay. Now we will discuss it elsewhere why we have radiation coming from all the directions. At earth surface, we receive radiation from all the directions but if we go into space, the radiation will be coming from only one direction.

It is because on surface of the earth, we have direct radiation coming from the sun which is basically parallel rays but we also have scattered radiation from the clouds which basically comes from all direction. So in the next example we will see how basically these components come into play in deciding how much radiation is received at earth surface but if you go into space basically there is only one component of solar radiation that is coming.

And that component is direct and direct radiation is coming in a small solid angle, the magnitude of that solid angle is approximately 6.8×10^{-5} steradian which is very small okay. So that is basically simple problem on solid angles.

(Refer Slide Time: 04:57)

Wien's Displacement law Lecture-3

$$\frac{E_{b\lambda}}{n^3 T^5} = \frac{C_1}{(n\lambda T)^5 [e^{C_2/(n\lambda T)} - 1]}$$

❖ Maximum spectral emissive power found by differentiating with respect to wavelength

$$\frac{d}{d(n\lambda T)} \left(\frac{E_{b\lambda}}{n^3 T^5} \right) = 0$$

$(n\lambda T)_{\max} = \frac{\text{Constant}}{n, T}$

$$(n\lambda T)_{\max} = 2898 \mu\text{m}\cdot\text{K}$$

IIT ROORKEE NPTEL ONLINE CERTIFICATION COURSE

Now let us go to the first fundamental law of radiation that is the Wien's displacement law. In our previous lecture, we developed the relation for a blackbody emissive power in a parameterized form. The parameter was $n\lambda T$ okay. So as a function of $n\lambda T$, we discussed this function, the Planck's blackbody emissive power okay $E_{b\lambda}$ okay which basically gives you the total amount of energy emitted by a blackbody in a given spectral range at a given wavelength okay.

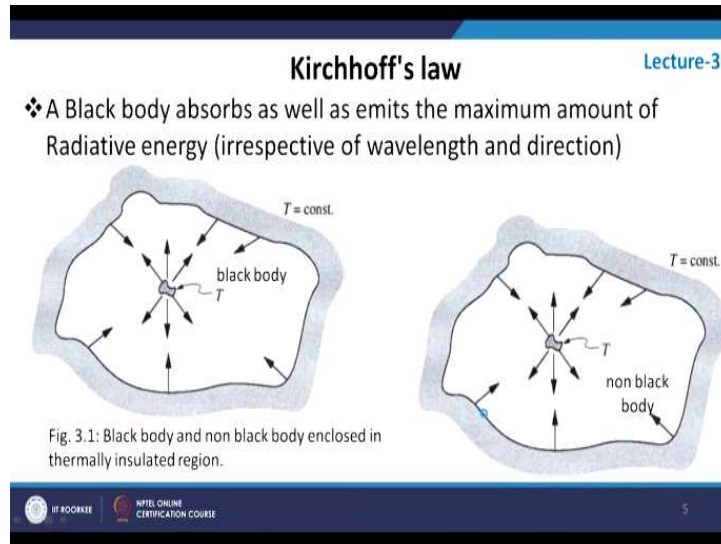
So C_1 and C_2 are first radiation constant and second radiation constant. Now we want to see what is the wavelength at which the maximum amount of energy is radiated? So what we need to do is we need to differentiate this relation with respect to this parameter $n\lambda T$ okay.

So if we differentiate this relation with respect to this parameter $n\lambda T$ and solve for this parameter for the maximum value okay what we get is $n\lambda T_{\max} = 2898$ micrometer Kelvin. So this is the constant okay. So what basically it says is that λT_{\max} , max means the maximum intensity, the wavelength of maximum intensity is basically constant okay and this constant depends on value of n and temperature okay.

So this constant depends on value of n and temperature okay. So $\lambda T_{\max} = \text{constant}$ and that is basically the Wien's displacement law. So what basically it says is when you increase the temperature, the maximum intensity appears at shorter wavelength region okay. So if you have sun which is at 5777 Kelvin, the maximum intensity of solar radiation is in the visible range and that is why our eyes are much more sensitive to visible radiation.

If we look at radiation coming from some other star which is at very high temperature, the radiation intensity will be shifted towards shorter wavelength and we see that at many stars basically they have radiation intensity peaking in gamma rays and other short wavelength range spectrum.

(Refer Slide Time: 07:11)



The second important law of radiation is Kirchhoff's law okay. Now Kirchhoff's law basically gives you a relation between energy emitted by a surface and energy absorbed by a surface. There are many variations of Kirchhoff's law but this one basically relate to the law of Kirchhoff's law to blackbody. What it basically says is a blackbody as we have already defined absorbs maximum amount of radiation.

In fact, the way it is defined it absorbs all the radiation that falls on it okay. So that is what we define that body. So blackbody absorbs maximum amount of radiation, other bodies they absorb radiation but less than the blackbody. What Kirchhoff's law says is that blackbody also emits the maximum amount of radiation. So blackbody maximum amount of radiation is absorbed as well as maximum amount of radiation is emitted.

Now how do we prove it using Kirchhoff's law? So let us say we have an isothermal enclosure. Isothermal enclosure means that this enclosure is not receiving any energy, not giving any energy. It is completely in thermal equilibrium that is why it is called isothermal enclosure. Inside this isothermal enclosure, we have a black surface. On the left hand side, there is an enclosure where we have a blackbody.

And on the right hand side, there is an enclosure, an isothermal enclosure where we have a non-blackbody okay. Now this blackbody and this non-blackbody when left for sufficient amount of time will be in thermal equilibrium with the enclosure. That means the temperature more or less will be equal to the temperature of the enclosure. So after sometime they will be in thermal equilibrium.

Now the amount of radiation that is coming onto the surface from this enclosure, so radiation is emitted from this enclosure and it travels towards the surface this body and it will be absorbed by that body okay. So blackbody absorbs maximum amount of radiation okay. Now if it does not emit the same amount of radiation or the maximum amount of radiation, its temperature will increase okay.

While the non-blackbody receives less amount of, it receives same amount of radiation but it absorbs less amount of radiation. So it definitely has to emit this less amount of radiation, otherwise this temperature will decrease okay. So by this analogy we can say that a blackbody receives maximum amount of radiation that is absorbs maximum amount of radiation, it simultaneously emits the maximum amount of radiation to keep this body in thermal equilibrium.

Similarly, a non-blackbody absorbs less amount of radiation compared to a blackbody and similarly it emits less amount of radiation compared to a blackbody. So this is basically the Kirchhoff's law okay. So it basically gives you the relation between emittance and absorptance of a surface okay. We will come to this in the next lecture. Emittance and absorptance of a surface are related.

The more a surface absorbs, the more a surface emits, so that is basically the corollary of Kirchhoff's law okay.

(Refer Slide Time: 10:19)

Intensity of a Black Surface Lecture-3

❖ The intensity leaving a black body is independent of direction (isotropic or diffuse)

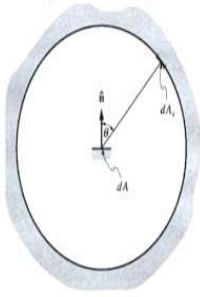
$I_b \neq I_b(\theta, \psi)$

$$E_b = \int_0^{2\pi} \int_0^{\pi/2} I(\theta, \psi) \cos \theta \sin \theta \, d\theta \, d\psi$$

$$= I \int_0^{2\pi} \int_0^{\pi/2} \cos \theta \sin \theta \, d\theta \, d\psi$$

$$= \pi I_b$$

$E_b = \pi I_b$



Source: Radiative Heat Transfer, Second Edition, Michael F. Modest, Academic Press, 2003

So now the intensity leaving a blackbody okay, we introduced this in the last lecture. So the important point now this can be proved using Kirchhoff's law. What basically we know about blackbody intensity is that it is isotropic or diffuse. That means the intensity leaving a blackbody is same in all direction okay. A black body emits and emits equally in all directions so that is what basically we called a blackbody okay.

Although, it is not practically true because no body can emit radiation in this direction that is parallel to the surface but still for theoretical purpose we assume that a blackbody is isotropic that it emits equal amount of radiation in all directions okay. So that means intensity of blackbody does not depend on two angles theta and psi that is polar angle and azimuthal angle because it is equal in all direction, it means it does not depend on theta and psi.

So what we can do is the total emissive power is=intensity okay, the total emissive power we defined is intensity, projected area cos theta and then solid angle d omega. So this is d omega, this is for the projected area of unit magnitude okay and the intensity and we have to integrate from 0 to pi and 0 to pi/2 okay. So 0 to pi and 0 to pi/2 basically they define the solid angle 2 pi over a flat surface.

$$E_b = \int_0^{2\pi} \int_0^{\pi/2} I(\theta, \psi) \cos \theta \sin \theta \, d\theta \, d\psi$$

So 0 to pi and 0 to pi/2 is the total solid angle over a flat surface okay. Now intensity is basically independent of theta and psi. So we can take this quantity out of the integral here and what we are left with cos theta, sin theta, d theta and d psi. So when we integrate this quantity, we get pi.

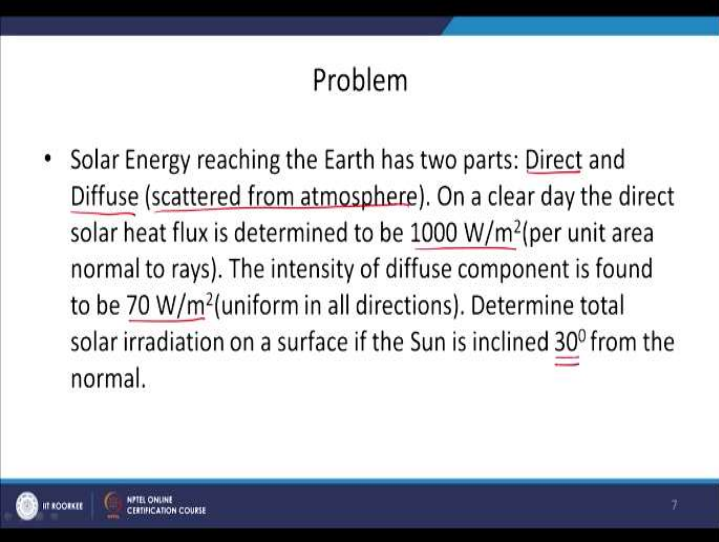
So that means the emissive power of blackbody is related to intensity of blackbody by this relation.

$$E_b = I \int_0^{2\pi} \int_0^{\pi/2} \cos \theta \sin \theta \, d\theta \, d\psi$$

So the factor is basically π okay. Many times you confused the total angle is 2π , why should not be the emissive power be equal to $2\pi I_b$ okay. So best you have to keep in mind that intensity is defined normal to the rays okay. The area, projected area is normal to the rays of travel of intensity and that is why this angle π appears here okay. So blackbody emissive power is $=\pi$ times the blackbody radiative intensity.

$$E_b = \pi I_b$$

(Refer Slide Time: 12:57)



Problem

- Solar Energy reaching the Earth has two parts: Direct and Diffuse (scattered from atmosphere). On a clear day the direct solar heat flux is determined to be 1000 W/m² (per unit area normal to rays). The intensity of diffuse component is found to be 70 W/m² (uniform in all directions). Determine total solar irradiation on a surface if the Sun is inclined 30° from the normal.

NPTEL ONLINE CERTIFICATION COURSE

So okay now we will do one more problem. So in this problem what will do is we will discuss solar energy reaching the earth okay. We have to find out how much energy is reaching the earth. Now what we have been given is as we have known that in extra-terrestrial space that is outside the atmosphere, we only have direct component of radiation, all coming in single direction.

But at the surface of the earth, we have diffuse radiation that is scattered from atmosphere. This radiation comes from all directions okay and this could be isotropic, it could be non-isotropic okay. Now on a clear day, the direct solar heat flux is determined to be 1000 watt per meter

square okay normal to the rays okay. The intensity of diffuse component is also found to be 70 watt/meter² and it is uniform in all direction.

That means it is diffuse, now we have to determine total solar irradiation on the surface if the sun is inclined 30 degree from the normal okay. Now we will see 30 degree, what is 30 degree from the normal okay? Let us solve this problem okay.

(Refer Slide Time: 14:07)

Solution

Direct solar irradiation
1000 W/m²

$dA = \sin\theta \rho d\omega$

$T_0 \text{ W/m}^2\text{-sr}$

$$q_{in} = q_{direct} + q_{diffuse}$$

$$= q_{sun} \cos\theta + \int_{2\pi} q_{diffuse} \cos\theta d\Omega$$

$$= 1000 \cos 30 + \int_0^{2\pi} \int_0^{\pi} \cos\theta \sin\theta d\theta d\phi \times q_{diffuse}$$

$$q_{in} = 1000 \times \cos 30 + \pi q_{diffuse}$$

$$= 1000 \cos 30 + \pi \times 70$$

$$q_{in} = 1056 \text{ W/m}^2$$

So we have this is a surface, the solar radiation, direct solar radiation is coming at an angle that makes an angle of 30 degree from the normal direction okay. So solar radiation is coming at an angle of 30 degree from the normal direction okay. Now this normal direction will definitely change over the duration of the day but at this duration the theta is given as 30 degree okay.

So this is a direct component, direct solar radiation and its value is given as 1000 watt/meter² okay. This value is given as 1000 watt/meter², that watt per meter square is normal to the area of rays okay that is it is normal to the rays that is coming in. Now there is another component of diffuse, so diffuse radiation comes from all sides, so diffuse radiation come from all sides okay and it has equal intensity okay.

And the value is 70 so this has equal intensity in all directions. Now we have to find out how much total energy is basically reaching the surface okay. So total energy reaching the surface

$$q = q_{direct} + q_{diffuse}$$

$$q_{direct} = q_{sun}$$

So q_{direct} will be q_{sun} okay, the solar radiation intensity $1000 \text{ watt/meter}^2$ and this angle $\cos \theta$ okay, this θ is the angle made by the solar radiation from the surface normal direction.

So $q_{\text{sun}} \cos \theta$ and q_{diffuse} now you will have to integrate it okay. So this will be equal to, we have to integrate it over all the angles. So this will be equal to, integrate over

$$q_{in} = q_{sun} \cos \theta + \int_{2\pi} q_{diffuse} \cos \theta d\Omega$$

. So we have to find out the normal components? So energy will be the normal component of the q_{diffuse} okay, it is coming from all directions.

Definitely, this component of q_{diffuse} will not contribute to any flux right. While this will contribute more right, so we have to find out all the components and we have to integrate over the solid angle. So this will be $1000 \cos 30^\circ$ now we have to, we write $d\Omega$ again as $\sin \theta d\theta d\psi$ okay. So 0 to 2π 0 to $\pi/2$ that basically defines the hemispherical solid angle above the flat surface $\cos \theta \sin \theta d\theta d\psi$ okay.

$$q_{in} = 1000 \cos 30^\circ + \int_0^{2\pi} \int_0^{\pi/2} \cos \theta \sin \theta d\theta d\psi \times q_{diffuse}$$

$$q_{in} = 1000 \cos 30^\circ + \pi \times 70$$

And we already know that this value will be equal to π , so we have to just multiply by q_{diffuse} here q_{diffuse} okay. So this q_{in} will be $1000 \cos 30^\circ + \pi q_{\text{diffuse}}$ okay. So this will be equal to now $1000 \cos 30^\circ + \pi$ and 70 , this will be equal to $70 \text{ watt per meter square}$. So this value will be equal to total $1086 \text{ watt/meter}^2$. So total amount of energy received from the sun will be $1086 \text{ watt/meter}^2$ okay.

So this is how basically you can use the direct and diffuse component of the radiation okay. Thank you very much.