Radiative Heat Transfer
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Module - 6
Lecture - 26
Exchange Areas

Hello friends, in the previous lecture we discussed the zone method. Zone method is basically an extension of net radiation method. Where, in net radiation method, we used the view factors and we applied the energy balance on surfaces. In zone method, we use direct exchange areas and total exchange areas. And we apply the energy balance to surface zones and gas zones. So, the domain of the problem is divided into a number of surfaces.

We call them surface zones. And the volume is divided into a number of sub volumes and we call them volume zones. The direct exchange area is analogous to view factor with an advantage that, here we have incorporated the participating media. That means, the effect of absorption and scattering is included in the definition of the direct exchange areas. In today's lecture, we will see how to calculate these direct exchange areas for some simplified geometry. For complicated geometry, we may have to go for statistical methods like Monte Carlo or some higher-order numerical integration scheme.

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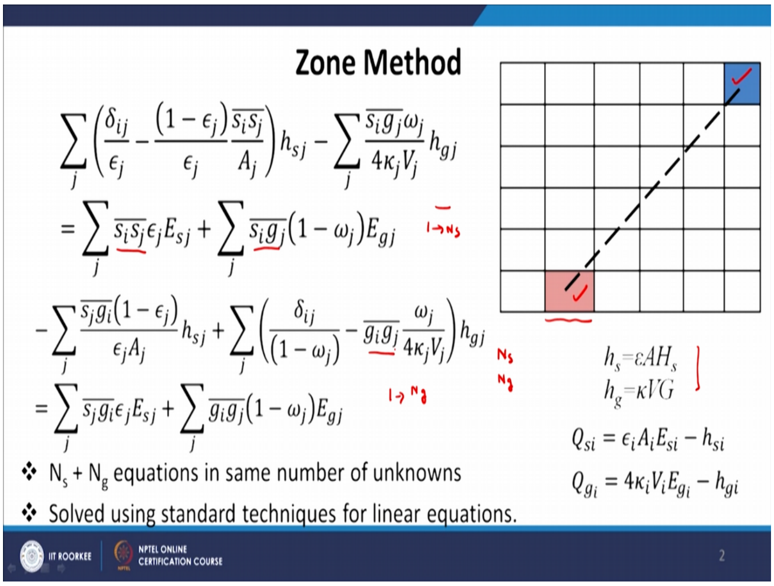

So, let us see, before we start, I will summarize the governing equation of the zone method. So, in the zone method, we have a domain of interest, which is divided into surface zones and volume zones. So, this is the volume zone. And we have number of surface zones. So, this is typically what we do in computational fluid dynamics. We divide the domain into finite volumes.

So, here the domain is divided into finite volumes. And we have surface zones and volume zones. Volume zones are also called gas zones. We have total N s number of surface zones and N subscript g number of gas zones. So, the total number of equations that we get is N s plus N g for total unknowns N s plus N g. So, N s plus N g is the number of equations. And N s plus N g is the number of unknowns.

And the variables here in the zone method are, the total absorption at the boundaries, that is, h underscore h subscript s. That is the total absorption at the boundary. And h g which is total absorption at the volume. So, g is the incident radiation and kappa times g basically gives you the absorption of radiative energy in a gas or volume zone. And h s is surface irradiation which multiplied by emittance or absorptance gives you total absorption at the surface.

So, with these variables, we have total N s plus N g number of equations. And the equations are basically given here. So, we have total 1 to N s number of equations. And total N g equations 1 to N g equations from here. So, total N s plus N g equations in h sj and h sg. So, this represents a system of linear equations which can be solved using standard techniques for solving linear equations. The unknowns here are h sj and h h g.

But, before we have to solve for this, we have to find out the matrix that basically governs this system of equation. And the matrix depends on number of variables like direct exchange areas from surface-to-surface s i s j. It depends on surface-to-volume direct exchange areas s i g j. And it also depends on volume-to-volume direct exchange areas g i j j. So, these variables these parameters, need to be evaluated, before we apply the system of a linear equation solver to find out unknown variable h s and h g. So, how do we define this variables.

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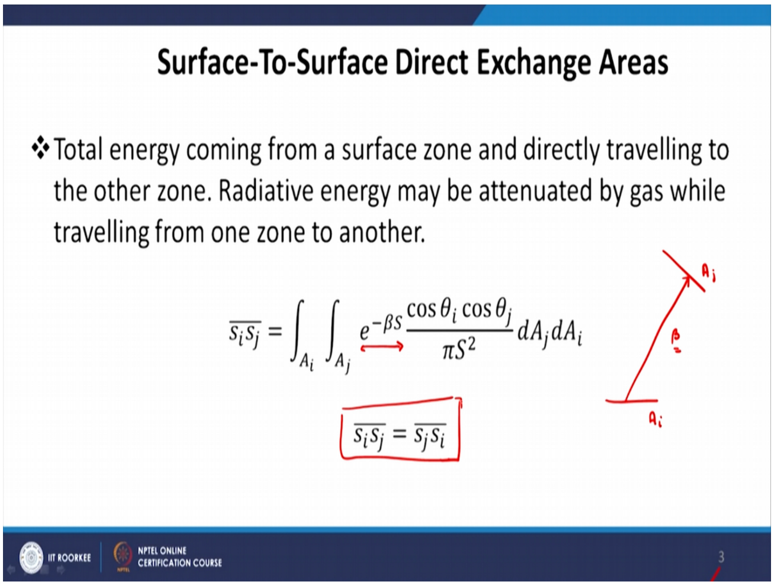

So, surface-to-surface direct exchange areas is defined basically as total energy coming from a surface zone and directly travelling to another surface zone. Now, this energy that is directly travelling without any reflection from the surface, without any scattering in the volume, it directly travels from 1 surface to another. But, when it travels from 1 surface to another, it may be subjected to absorption.

So, the view factor or the direct exchange area between the 2 surfaces i and j may be defined as double integration e power minus beta s cos theta i cos theta j divided by pi S square d A j d A i. This is the normal definition of a view factor. The thing that we have multiplied with is the transmittance or transmissivity of the gas. So, let us say we have 2 surfaces. This is A j and this surface is A i.

And we are interested in finding the direct exchange areas between these 2 surfaces. So, we have to double integrate over these 2 surfaces. The medium between these 2 surfaces have extinction coefficient beta. So, beta is the extinction coefficient between these 2 surfaces. So, when radiation travels from 1 surface to another, it will be subjected to attenuation. And there is an exponential term that basically comes into the definition of direct exchange area.

Of course, this evaluation of this exchange area is not trivial. We have seen that, even for medium or plates bounded by vacuum, when there is no medium, the evaluation of this double integral, weather with double integration method or contour integration method is very difficult. And when we have exponential term in this integral, the evaluation is going to be very very difficult.

We will see some simplified geometry and see how we can evaluate this. But, the expression that we have obtained for surface-to-surface direct exchange area is difficult to evaluate. And normally we apply Monte Carlo methods for the evaluation of this direct exchange area. The direct exchange areas evaluated in this fashion follow the reciprocity s i s j is equal to s j s i. That means, direct exchange area from surface i to surface j is same as direct exchange area from surface j to surface i.

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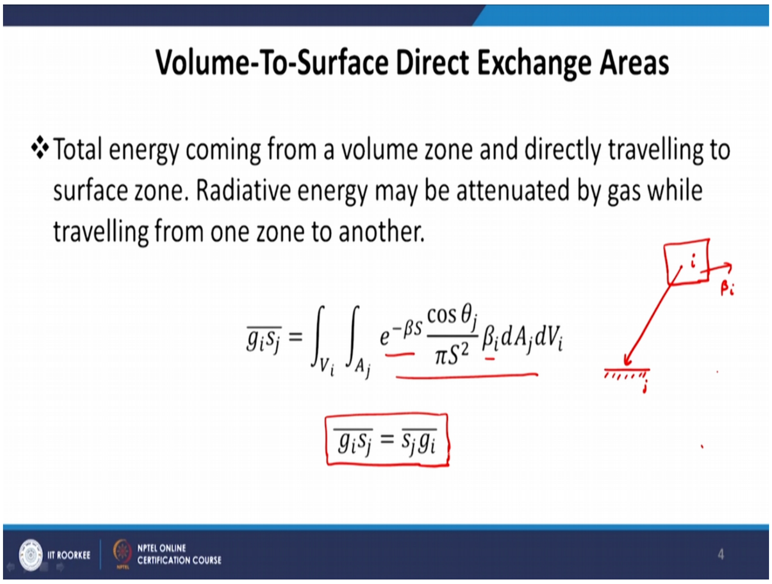

In a similar fashion, we define volume-to-surface direct exchange area or surface-to-volume direct exchange area. So, we have a surface and we have a volume zone. And let us call this is volume zone i and this is the surface zone j. And we are interested in finding the exchange area between these 2. So, g i s j, that is direct exchange area between surface volume i and surface j is defined as total energy coming from a volume and directly travelling to surface zone.

Now, this energy will be subjected to attenuation, as we discussed for surface-to-surface exchange. But, this energy will not be subjected to reflection and in-scattering. So, we are basically interested in direct travel. And in-scattering and reflection is not taken into this. The scattering will attenuate the radiation. So, that is accounted. Absorption by attenuation, attenuation by absorption and scattering is accounted, but augmentation is not accounted.

So, we are interested in direct travel. So, the definition of this exchange area by similar terminology is given by double integral. Now, 1 integration is over the entire surface j and 1 integral is over the entire volume i. So, we have this double integral e to the power minus beta s, where s is the path length beta is the extinction coefficient; cos theta j pi S square beta i d A j d V i. Where, beta i is now the extinction coefficient in the volume.

So, extinction coefficient in the volume appears in this integral because we are interested in how much energy is emitted from the volume. So, energy emitted within the volume is proportional to the extinction coefficient within the volume. So, we have now 2 variables, beta i and e to the power minus beta s appearing in this integral. Beta i is a local variable, that is inside the volume, while beta depends on the path.

So, beta will vary from the location of i to the location of j continuously over the path. Again, this is going to be very difficult to evaluate, but we will simplify this for simple geometries. The volume-to-surface exchange area also follows the reciprocity rule. That is, g i s j volume-to-surface direct exchange area is equal to surface-to-volume direct exchange area s j g i.

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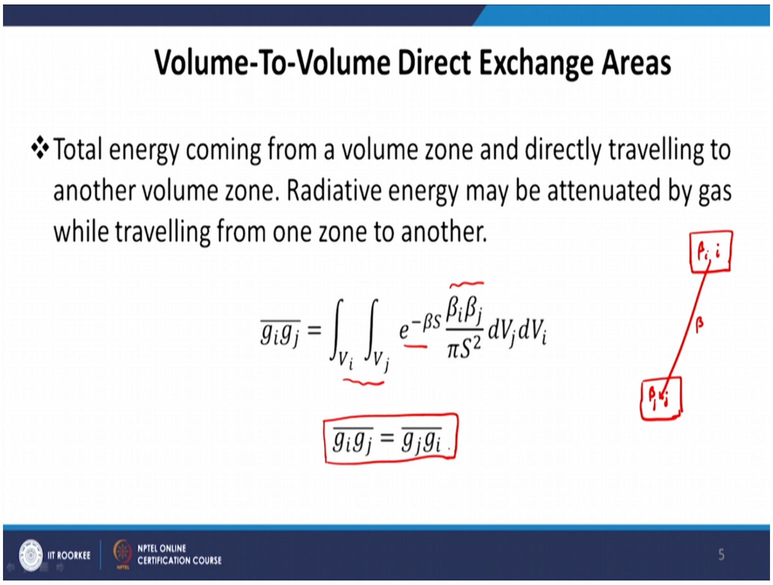

Finally, we also define volume-to-volume direct exchange area. So, we have 2 volumes. This is volume i and another volume in space, volume j. The extension coefficient here is beta i, extinction coefficient here is beta j. And we are interested in the direct exchange area between these 2 volumes. And over the path the extinction coefficient beta will vary. So, we define total direct exchange area between these 2 volumes as the energy coming from a volume zone and directly travelling without reflection and scattering to another volume zone.

This radiation may be subjected to attenuation by gas over the entire path length s. Okay. So, we define g i g j, where is g i g j is the direct exchange area between 2 volume zones i and j.

Its double integral over the 2 volumes V i and V j. And now, what we see here is, that we have 2 beta i and beta j appearing in this integral, where beta i beta j are local extinction coefficient. We have double integral over the volumes.

And e minus beta s is the attenuation over the entire path. Again, this is going to be evaluated in a difficult way. But, we have this reciprocity. And this reciprocity between 2 volume zones g i g j is equal to g j g i. Says that the direct exchange areas between 2 volumes is also following the reciprocity rule.

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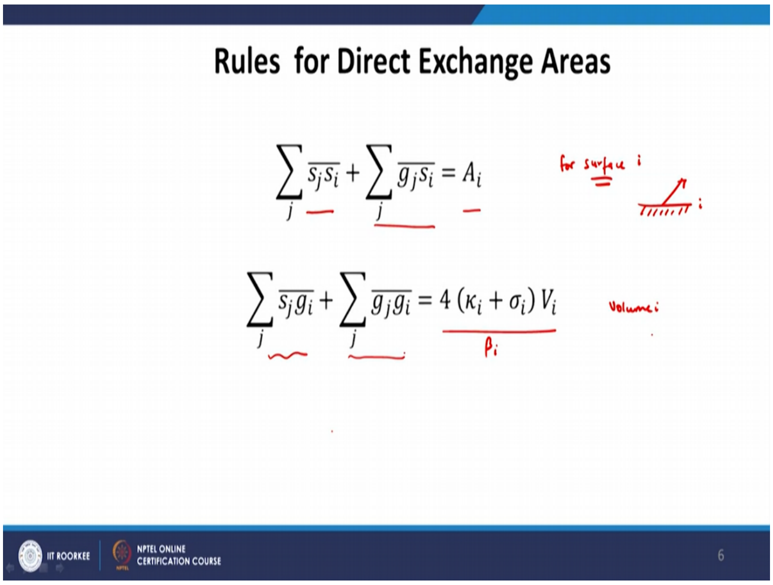

Now, we have some additional rules just like the view factor, we have rule of thumb summation, summation of all view factors from a surface is equal to 1. We also have a summation rule for direct exchange areas. So, we have from a surface. So, this is for surface i. So, for surface i total energy is conserved. So, total amount of energy emitted from this surface i is either going to other surfaces surrounding this surface i; so, summation over all the surface is j. So, summation j s j s i.

Or this energy will be absorbed by the gas. So, we have a summation j g j s i. And this should be equal to total amount of the exchange area which is equal to A i. So, this is same thing that we did for the view factors. So, total exchange area from the surface i should be equal to its area A i. Similarly, for volume, for volume i, the energy is conserved. Total amount of energy emitted from a volume is basically equal to 4 times kappa i plus sigma i, kappa i and sigma i is nothing but beta i, the extinction coefficient times its volume V i.

So, the total amount of energy emitted from the surface will be proportional to 4 times V times the extinction coefficient. And this should be equal to the sum of exchange areas between volume and surface. So, all the energy emitted from the volume will either be absorbed at the surfaces j or it will be absorbed by other volumes. So, we have to integrate or sum over all the volumes.

So, summation s j g i summation g j g i should be equal to 4 times beta i V i. So, these 2 rules will be applicable on exchange areas. Now, we will learn some more rules to find out exchange areas to ease the calculation of exchange areas in complicated geometries. We see that the evaluation of direct exchange area is going to be very very difficult. So, we will see how much simplification we can do, what kind of rules we can follow to evaluate the direct exchange areas.

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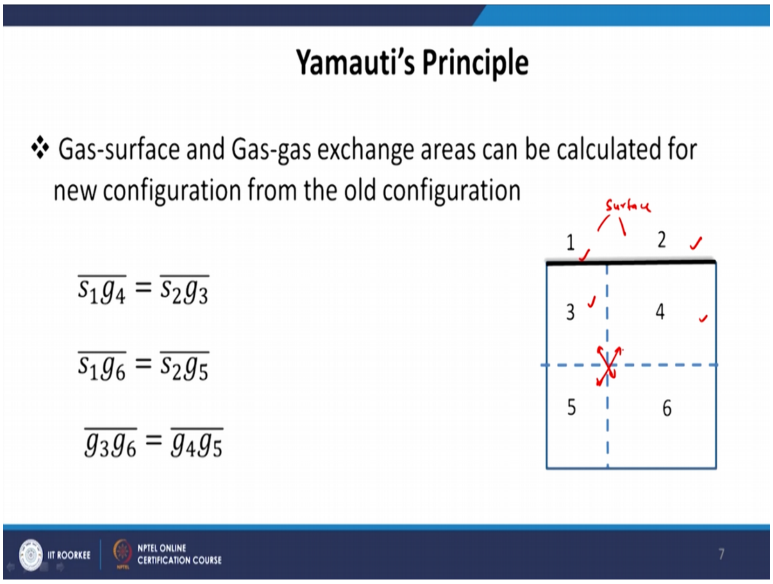

The first in this is basically the Yamauti's principle. Yamauti's principle says that gas-surface and gas-volume exchange areas can be calculated for a new geometry if the gas-surface and gas-volume exchange areas for some old configurations are known. So, from known configuration, we can calculate exchange areas for an unknown configuration. So, to demonstrate this, let us say we have a geometry, a rectangular enclosure.

We have divided this enclosure into 2 surface zones, 1 and 2. So, 1 and 2 are surface zones. And we have 3, 4 volumes: 3, 4, 5, 6. These volumes are of different dimensions. Volume 3, 4, 5, 6, they have different dimensions. Now, as per Yamauti's principle, these s 1 g 4, that means, direct exchange area between 1 and 4 is equal to direct exchange area between 2 and

3. Okay. Irrespective of the dimension of 3 and 4 not same, it is following the equality. That means s 1 g 4 is equal to s 2 g 3.

Similarly, s 1 g 6 is equal to s 2 g 5. And the principle is also applicable to volume-to-volume direct exchange areas. That means, s 3 g g 3 g 6 is equal to g 4 g 5. So, between these 2 volumes and these 2 volumes, the direct exchange areas are same g 3 g 6 and g 4 g 5 are same. So, this is Yamauti's principle. And this principle can be used to evaluate direct exchange areas in a typical geometry where some configuration exchange areas are known. Some other configuration exchange areas can be calculated easily.

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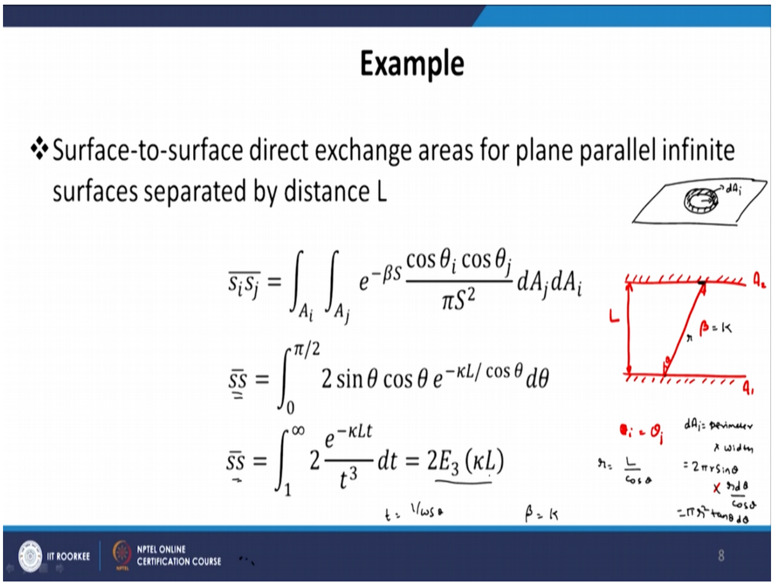

So, let us do 1 example to demonstrate how the exchange areas can be calculated based on the principles that we have learnt. We will assume a configuration of 2 infinite parallel plates, separated by a distance l. So, this is plate 1 with area A 1 and this is plate 2 with area A 2. And these plates are infinite in dimensions. So, in a sense, A 1 and A 2 are also infinite. We have a finite separation distance l between the 2 plates, but the x dimension and the dimension in the plane of the board is infinite.

Now, as per the definition, the exchange area s i s j is equal to double integral over surface A i and A j, where i is equal to 1 and j is equal to 2. e power minus beta s, where beta is the extinction coefficient between these 2 medium, between these 2 plates. And cos theta i and cos theta j are normal angles. So, from this configuration, we know that, theta i is equal to theta j, where from theta i is basically the; this is let us say some small element of the plate A i and at any element on A 2, the angle made from the normal is theta.

So, this theta is going to be same. Now, the way we will evaluate this; we will take an annulus of some thickness. So, let us, we will take an element of thickness. So, in a sense, if we look at the projected part of this plate, we are basically considering a annulus to evaluate this integral. So, what we have taken is, this is a plate, the annulus thickness. And we will integrate from r is equal to 0 to r is equal to infinity.

So, this is how we are going to evaluate this. Now, let us write down some simplified relations basically. So, we have a distance r here. So, r is basically nothing but L upon cos theta. Theta i and theta j are same. Now, the area of this annulus, this area, let us call this d A j. So, d A j is basically equal to the parameter of this annulus times the width of this annulus. So, this will be equal to, the parameter is going to be equal to 2 pi r sin theta.

And it will be multiplied by its width. So, width is basically rd theta by cos theta. Okay. So, this is going to be the d A j which is now coming out to be pi r square tan theta d theta. So, the area of this annulus is going to be pi r square tan theta d theta, where r is the separation distance from plate 1 to plate 2. Okay. So, we basically transform our integral. Okay. And after transformation, substituting for this, we get the exchange area between these 2 plates ss is equal to 0 to pi by 2 2 sin theta cos theta e power minus kappa L.

Now beta is equal to kappa. There is no scattering. So, the extinction coefficient is simply equal to absorption coefficient kappa. So, kappa is the absorption coefficient, L is the separation. So, we get 2 sin theta cos theta e powers minus kappa L upon cos theta d theta. So, this is the integral that we have to evaluate finally. And we can evaluate it by using the technique of change of variables.

So, we put this 1 upon cos theta as t. And our equation is transformed as ss is equal to 1 to infinity 2 e power minus kappa L t, where we have put t is equal to 1 upon cos theta, t upon t cube d t. And we have already learnt what this integral equals to. This is basically the exponential integral of third kind, of third order. So, this integral is basically nothing but E 3 kappa L.

So, the direct exchange area can be easily calculated using this approach. So, we have got the direct exchange area between these 2 plates as simply equal to 2 times third order exponential integral valuated at kappa times L. Now, we have to evaluate surface-to-gas direct exchange area as well. So, we will see how to evaluate this surface-to-gas direct exchange area. We will introduce 1 more concept.

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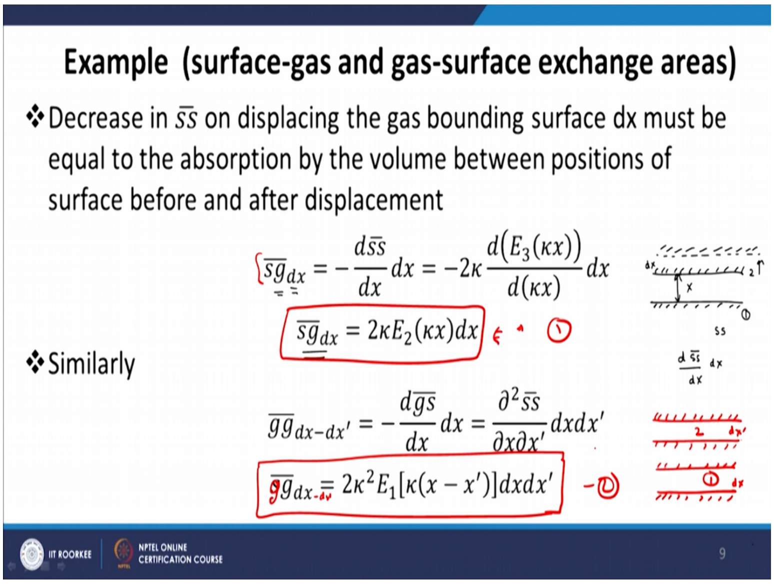

And this concept basically says that, if you have 2 plates; let us say we have 2 plates. Let us say this is plate 1 and this is plate 2. Okay. And the direct exchange area between these 2 plate is equal to ss. Now let us say we move this plate 2 little farther. This is the new position of the plate. Okay. We move it by, let us say amount dx. Okay. Then d, the surface-to-surface direct exchange area is going to change.

When we move the plate 2 farther away, the direct to, the direct surface-to-surface exchange area is going to change. And it says that, direct surface-to-surface exchange area change by a small amount dx. And this is equal to surface-to-gas exchange area. So, whatever change in the surface-to-surface exchange area happens, basically, that will be equal to the surface-to-gas exchange area.

So, what it means is, whatever energy earlier was absorbed by the surface, now after the movement, that energy is going to the volume. Earlier the energy was going from 1 surface to another. Now the surface has moved further. Then, the change in that energy going from 1 surface to another, will be actually going from surface to the gas. So, that is basically this principle.

So, surface-to-gas exchange area where dx is the thick, distance between the initial configuration and the final configuration. So, dx is a distance the plate has moved. So, this will be equal to minus the derivative of the direct surface-to-surface exchange area. Now, surface-to-surface exchange area, we have already calculated in the previous problem; 2 times exponential integral kappa times L.

Now, here L we replace by x. Let us say this distance is x. So, we get 2 times kappa d by d of kappa x E 3 kappa x dx. So, we have to differentiate this exponential integral. And the rules or the properties of exponential integral can be exploited now. So, surface-to-gas exchange area, direct exchange area sg dx is simply equal to 2 times kappa exponential integral of second order E 2 evaluated at kappa x and dx.

So, this is basically what we have got the new trick to find out surface-to-gas exchange area. So, surface-to-gas exchange area can be calculated in terms of surface-to-surface exchange area by just simply differentiating the result. Now, same thing we can do for a gas-to-gas exchange area. So, if we have 2 gas zones, let us say, separated by surfaces. This is 1 gas and this is second gas. Okay.

Now, what we do is we have a; this is dx and this is dx prime. So, what it says is that gas-to-gas direct exchange area; where 1 gas zone is of thickness dx and another gas zone is of thickness d x prime, is equal to minus derivative of surface-to-gas or gas-to-surface direct exchange area d by dx of gs dx. Okay. Now, substituting for this expression for gs from here, it becomes a double derivative, second order differential.

So, we get del square by del square ss upon del x del x prime dx d x prime. Now, using this double derivative, we can calculate gas-to-gas direct exchange area as well. And this basically simplifies to; when we substitute the value of sg dx from here, this simplifies to that surface-to-gas direct exchange area. This will be equal to gas-to-gas direct exchange area d x prime. This will be equal to 2 times kappa square first order.

Because, now we have differentiated twice, so first order exponential integral evaluated at x minus x prime times kappa. So, this is the expression for the volume-to-volume direct exchange area between 2 volume zones of thickness dx and d x prime. Now, let us apply this to a configuration.

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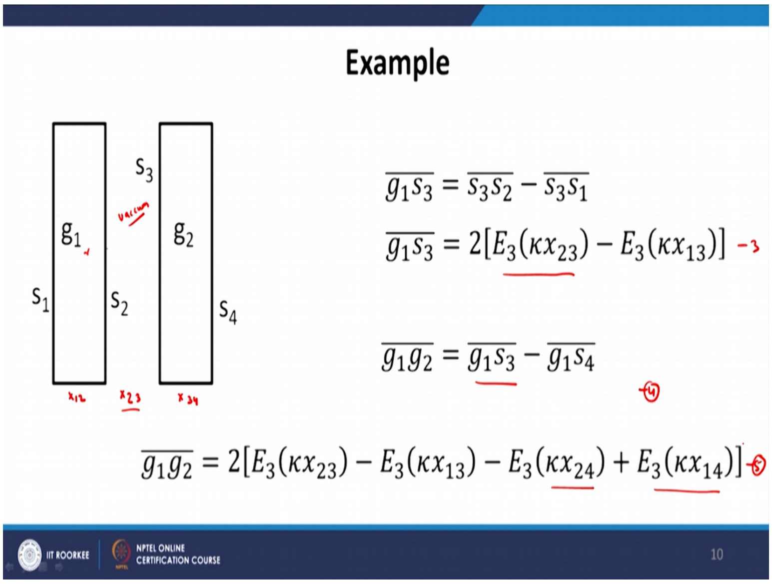

Let us see how we can use this to solve for the direct exchange area. So, I have taken this configuration. We have a 2 volume zones g 1 and g 2. And we have 4 surfaces s 1, s 2, s 3 and s 4. So, 4 surfaces bounding 2 volume zones and this is vacuum. So, there is no gas here. The distance, we can write down this distance as x 1 2, this distance as x 2 3 and this distance as x 3 4. So, g 1 s 3 as per the definition of this g 1 s 3 is equal to s 3.

So, g 1, this is the g 1; s 3 is equal to s 3 s 2 minus s 3 s 1. That is s 3 s 2 and s 3 s 1. So, g 1 s 3 is equal to s 3 s 2 minus s 3 s 1. So, summation rule on surface 3 we have applied. And this becomes g 1 s 3 is equal to s 3 s 2 minus s 3 s 1. Now, g 1 s 3 we can calculate from this relation. Let us call this equation 1 and this is equation 2. So, g 1 s 3 we can evaluate from equation 1.

And this becomes 2 times third order exponential integral kappa times x 2 3, where x 2 3 is the distance between plate 2 and 3 x 2 3 minus third order exponential integral kappa times x 1 3. Okay. So, this is how we can evaluate the volume-to-surface direct exchange area g 1 s 3. So, g 1 g 2 can be written as equal to g 1 s 3 minus g 1 s 4. And now what we can do is, we have to write g 1 s 3. So, g 1 s 3 already evaluated.

So, we substitute this value. Let us call this equation 3 and this is equation 4. So, substituting the value of g 1 s 3 from 3 into equation 4. And we have also need to evaluate g 1 s 4. g 1 s 4 is simply equal to minus E 3 kappa times x 2 4 and third order exponential integral kappa x times 1 4. So, we have basically calculated volume-to-volume direct exchange area as well, by simply using the rule of derivative.

So, all we need is basically to evaluate surface-to-surface direct exchange area and surface-to-volume and volume-to-volume direct exchange area can be calculated by just differentiating the expression for surface-to-surface exchange area. And this is how we have evaluated this. So, let us call this equation 5. Okay.

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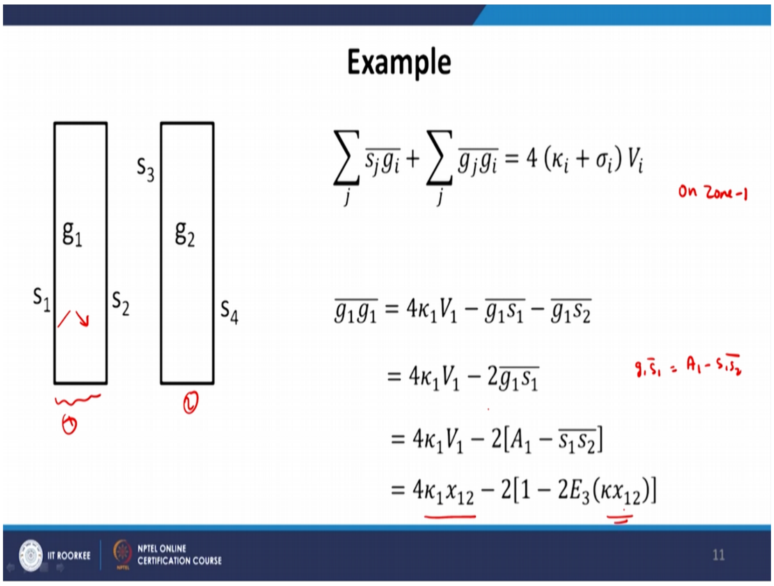

Now finally, we can simplify this again using the summation rule. So, from this one, now we apply the summation rule to gas zone 1. This is zone 1. This is gas zone 2. So, we apply on zone 1. So, on zone 1, we have this summation rule g 1 g 1 is equal to 4 kappa 1 V 1 minus g 1 s 1 minus g 1 s 2. So, with this summation rule, we get 4 kappa 1 V 1. Now, g 1 s 1 and g 1 s 2 are same. So, g 1 s 1 and g 1 s 2 are same because of symmetry.

So, we get 4 kappa 1 V 1 minus 2 g 1 s 1. Now g 1 s 1, again from the summation rule is basically equal to A 1 minus s 1 s 2. Okay. From this zone 1. g 1 s 1 is basically equal to A 1 minus s 1 s 2. That means, all the energy from surface 1 is either going to s 2 or going to volume zone 1. So, substituting this here, now our final expression becomes for g 1 g 1 as 4 kappa 1 x 1 2. V 1 is basically equal to x 1 2.

Assuming unit depth in the plane of the board, minus 2. We assume A 1 to be 1. Minus 2 times exponential integral kappa x 1 2. So, this is the expression for the direct exchange area from zone 1 to zone 1 itself. And from zone 1 to zone 2, we have calculated earlier, g 1 g 2. And the expression is from equation 5. So, all the exchange areas, direct exchange areas, we have calculated by either using the summation rule or using the differentiation method that we discussed in this lecture.

So, thank you for your kind attention. In the next lecture, we will discuss the Monte Carlo method for the solution of radiation problems. We will discuss how by emitting photons and tracking them in space can actually solve a problem in any complicated geometry.