Radiative Heat Transfer
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Lecture - 02
Fundamentals of Radiation

Good morning friends. In this lecture, I will introduce you to the basics of radiative heat transfer. So just like in conduction and convective heat transfer, we talked about heat flux. In this radiative heat transfer, we also talked about heat flux.

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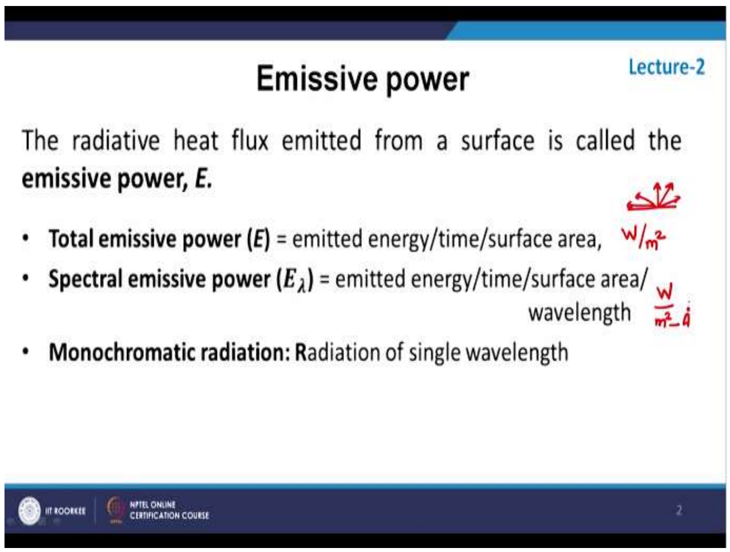

So emissive power of a surface is defined as emitted energy per unit time per unit surface area; however, unlike conduction and convection, the radiative energy is emitted in all the directions. So for example, if you have a surface, a flat surface, the energy will be emitted in all the directions. So in this case of flat surface, there are total 2π solid angles. So the energy will be emitted in 2π solid angles.

The units of emissive power will be watt/meter^2 okay. So this is the total energy that is emitted by a surface in all the directions. Now we know that the radiation, it is dependent on its color. Now color of radiation is decided by its wavelength, some people use frequency and some people use wavenumber but in this course we will be mostly dealing with the wavelength.

So we define spectral emissive power as emitted energy per unit area/time in a given wavelength okay. So the units of this will be watt/meter^2/angstrom okay so where angstrom is the selected unit for wavelength. So we will discuss this in detail in later lectures. Sometimes we will be using microns, sometimes we will be using angstrom but mostly in heat transfer community angstrom is the usual unit for wavelength.

So the total energy in a spectral wavelength lambda will be watt/meter^2/angstrom okay. So this is a monochromatic radiation, we call it monochromatic radiation.

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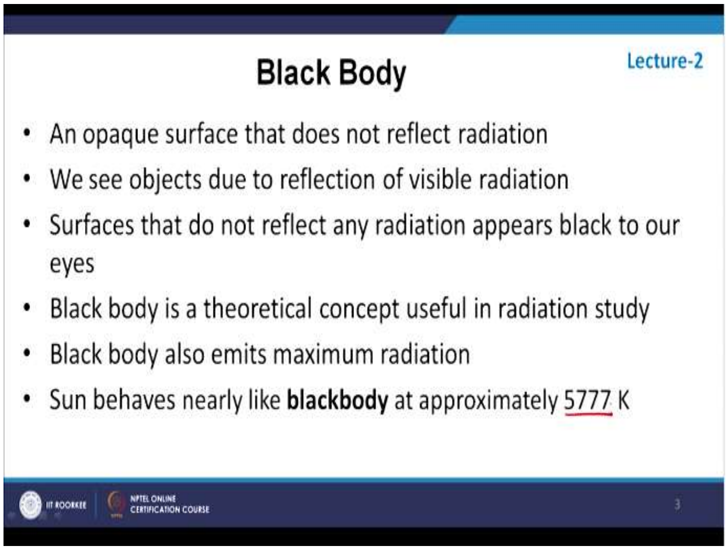

Now another concept of radiative heat transfer which is very important is the concept of blackbody. In daily life, we define a black surface as one which does not reflect radiation. So how do we see any body? So the sun radiation falls on a surface and then the reflected radiation from that surface reaches to our eye and if any surface by chance does not reflect the radiation, the solar radiation or radiation from any light source, then we say that the surface appears to be black.

Because the entire radiation is absorbed by the surface and this surface is called black. Now the interest in black surface is basically arose with the research in the area of solar radiation. So our sun which behaves pretty much like a blackbody at a temperature of 5777 K, it behaves like a blackbody. So later on researchers try to fit some mathematical relation to explain the radiation behavior of sun.

So Lord Rayleigh, Jeans, Wien lot of scientists tried to explain how the solar radiation behaves and then later on Max Planck basically came up with this mathematical relation which now known as Planck function or blackbody emissive power.

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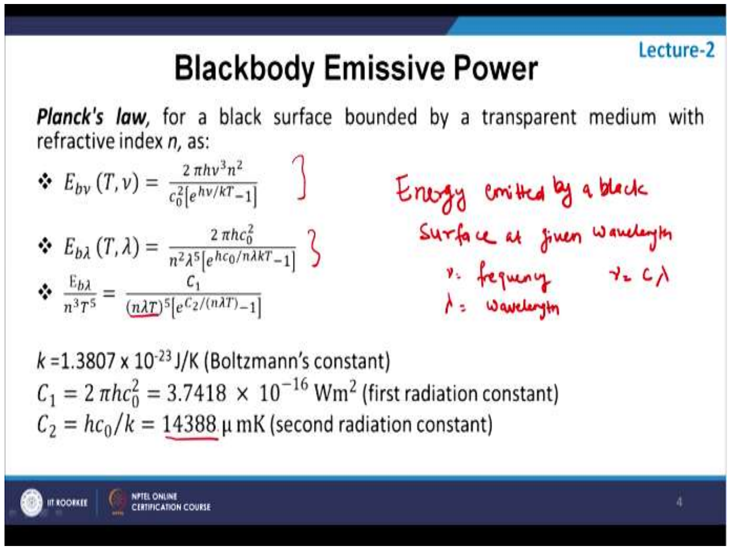

And this relation is given by this relation in frequency and also in the wavelength region okay. So it depends on number of parameters so what basically Max Planck did, he applied the concept of statistical mechanics and quantum mechanics and he came up with this mathematical relation based on the principles of quanta that the energy levels undergo transition at selective wavelengths and in quantized states.

So he came up with this expression of the Planck function and what basically this represents is energy emitted by a black surface at a given wavelength in a per unit time. So this is basically the Planck function or blackbody emissive power. Now as I told you some people, some researchers they prefer the units of frequency, so nu (ν) is the frequency here. While some other research would prefer lambda (λ) which is the wavelength.

And these two units are basically related with the speed of light so frequency is related to the wavelength

$$\nu = c \times \lambda$$

where c is the speed of light okay. So we can basically convert from different units that $E_{b\nu}$ in frequency units we can convert it into $E_{b\lambda}$ okay. So the various constants used in the explanation of this Planck's law or Boltzmann constant is k, so $k = 1.3807 \times 10^{-23}$ J/K, this is the Boltzmann constant.

Then, we have Planck constant h okay, so here we have defined number of constants,

$$C_1 = 2\pi \times h \times c_0^2$$

c_0 is the speed of light in vacuum. So this constant has value 3.7418×10^{-16} W/m^2 and it is called first radiation constant. Then, we have defined another constant

$$C_2 = h\times c_0/k$$

h is again Planck constant, k is Boltzmann constant, c_0 is speed of light.

The value of this is 14,388 μm-K; it is called second radiation constant. So there is quite advantage of non-dimensionalizing the blackbody emissive power. Now we will see how this blackbody emissive power varies with the spectrum with the wavelength okay.

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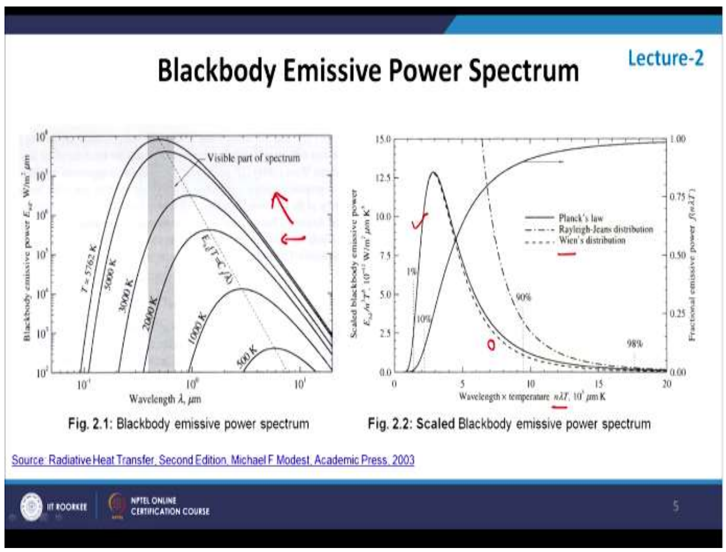

So on this slide, you see two curves, on the left curve what you see is basically the blackbody emissive power, variation of blackbody emissive power with wavelength. On the x-axis, we have wavelength and there are number of curves on this, each representing the temperature. So we have plotted these curves at the function of wavelength for different temperatures and what we see here is that the intensity, the peak value of the emissive power increases as we increase the temperature okay.

So the peak value is increasing in this direction as we increase the temperature. Furthermore, what we observe is that the peak is shifted towards left, so when we are increasing the temperature the peak is shifted towards the left and this is something we already know what we basically know by the Wien's displacement law, that is the maximum energy emitted by a black surface shifts towards smaller wavelengths as we increase the temperature.

On the right side of this slide, what you see is basically the non-dimensional emissive power as a function of the parameter and lambda T where n is the refractive index of the medium, lambda is the wavelength in micrometer and T is the temperature in K. So on the x-axis is this parameter and lambda T and on the y-axis we have the parameterized blackbody emissive power $E_{b\lambda}/n^3\ T^5$.

So when we do this, what we observe is that all the curves on the left hand side for different temperatures, they basically merge together in a single curve. So we have a single curve for all the temperatures. Basically, all the curves for different temperature they collapse into a single curve. Now this curve has been basically studied by many researchers in respect to solar spectrum okay.

And solar spectrum we know, it behaves like a blackbody emissive power and it is at 5770 K. On this curve, I have also shown you some approximate relation. So the blackbody emissive power was first given by Max Planck but researchers have been trying to fit solar spectrum even before Max Planck.

So there has been work by Wien's and Rayleigh-Jeans. As we see that they tried to predict the spectrum versus wavelength but the accuracy is very limited. Although, the agreement by Wien's law is pretty much in good agreement but the Rayleigh-Jeans spectrum is significantly in error as compared to the actual Max Planck spectrum. So we will basically use this Max Planck blackbody emissive power for solving our calculations.

To solve the problems, we need to calculate total blackbody emissive power that is how much energy basically a body emits in its entire spectrum. Spectrum may have different ranges, visible spectrum, ultraviolet spectrum, infrared spectrum. So we are interested in how much energy totally energy is emitted by a body in all the wavelengths and we sometimes are also interested in finding how much energy is emitted by a body in some part of the spectrum.

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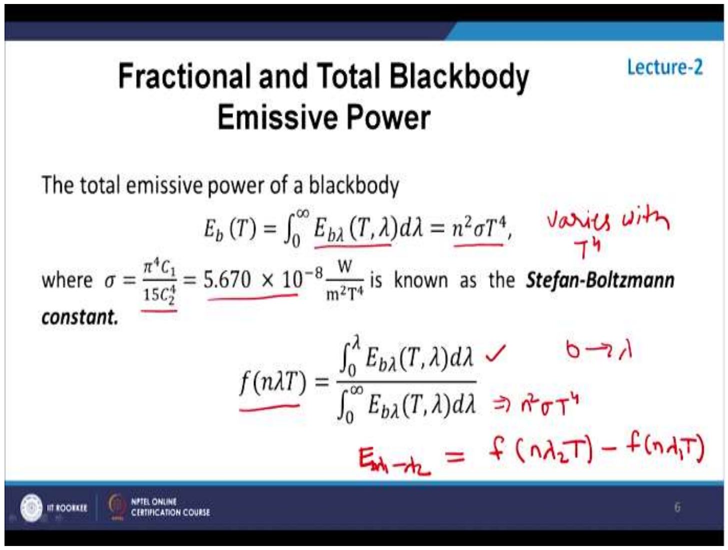

Let us say only the visible part of the spectrum or let us say just the infrared part of the spectrum. So total emissive power of a blackbody, it can easily be calculated by integrating the spectral emissive power, blackbody emissive power integrated over all the wavelengths. So here I have used the integration from 0 to infinity just to represent that we are considering the smallest wavelength possible and the largest wavelength possible.

So we are integrating over all the wavelength and when we integrate this blackbody emissive power that is a Planck function over all the wavelengths, what we get is basically

$$E_b\,(T) = n^2\sigma T^4 n^2$$

and this is something you are all familiar with this Stefan-Boltzmann law and the total blackbody emissive power is basically proportional to T^4 so varies with T^4.

Unlike conduction and convection which are basically linear phenomena, the radiation varies non-linearly with temperature and the variation is power 4 with temperature and that is why the variation at high temperature, we need to study radiation because at high temperature the radiation energy significantly dominates over conduction and convection because of this power 4 effect.

So radiation will be very important phenomena at high temperature. The Stephen constant, the Stefan-Boltzmann constant sigma is defined in terms of radiation constant C_1 and C_2 by this relation

$$\sigma = \frac{\pi^4 C_1}{15C_2^4} = 5.670\ \times\ 10^{-8}\frac{W}{m^2T^4}$$

. So this is the total blackbody emissive power integrated over all the spectrum.

Now as I said we sometimes need only the energy emitted within a given spectral range okay either between 0 to λ as I have done in this part fractional blackbody emissive power or sometimes we may need emissive power in the range λ_1 to λ_2. So fractional emissive power

$$f(n\lambda T) = \frac{\int_0^{\lambda} E_{b\lambda}(T,\lambda)d\lambda}{\int_0^{\infty} E_{b\lambda}(T,\lambda)d\lambda}$$

And then divided by the total emissive power which is basically sigma T^4, so the denominator is nothing but equal to $n^2\sigma T^4$. So this ratio is called fractional blackbody emissive power. That is the amount of energy emitted by a blackbody okay in a given wavelength range varying from 0 to λ. So this is the fractional emissive power of a blackbody.

Now if you are interested in finding what is the amount of energy emitted between λ_1 and λ_2 in two different wavelength regions, all you have to do is just find out $E_{\lambda 1-\lambda 2}$ that is the amount of energy emitted by a blackbody between these two spectral range

$$E_{\lambda 1-\lambda 2} = f(n\lambda_2 T) - f(n\lambda_1 T).$$

will be equal to $f(n\lambda_2 T)$. That is the amount of energy emitted between 0 to λ_2 $T f(n\lambda_1 T)$. That is the amount of energy emitted within the range 0 to λ_1 . So difference between these two will be the amount of energy emitted between λ_1 and λ_2 . So we do not need to calculate these integrals again and again, these having calculated by researchers, there is a simple program available. One can use that simple program and tabulate these values in a tabular form.

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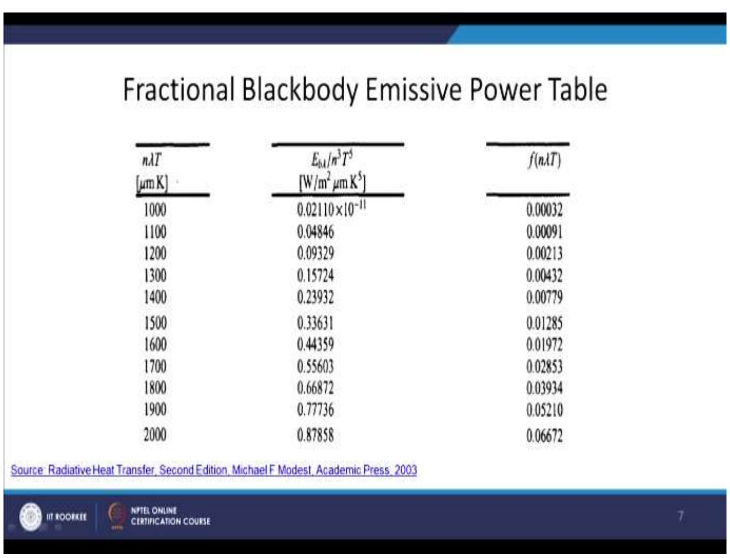
Fractional Blackbody Emissive Power Table

$n\lambda T$ [μm K]	$E_{b\lambda}/n^3T^5$ [W/m² μm K⁵]	$f(n\lambda T)$
1000	0.02110×10^{-11}	0.00032
1100	0.04846	0.00091
1200	0.09329	0.00213
1300	0.15724	0.00432
1400	0.23932	0.00779
1500	0.33631	0.01285
1600	0.44359	0.01972
1700	0.55603	0.02853
1800	0.66872	0.03934
1900	0.77736	0.05210
2000	0.87858	0.06672

Source: Radiative Heat Transfer, Second Edition, Michael F Modest, Academic Press, 2003

So for example this particular table taken from the book by Michael Modest gives you the table of fractional blackbody emissive power as well as the spectral emissive power. So the first column gives you the value of $n\lambda T$ in μm K. The second column give you the spectral emissive power at that particular wavelength in of course parameterized form and the third column gives you the fractional blackbody emissive power.

So this is the emissive power between a given wavelength range 0 to λ divided by total blackbody emissive power. So we will use these tables to solve different problems.

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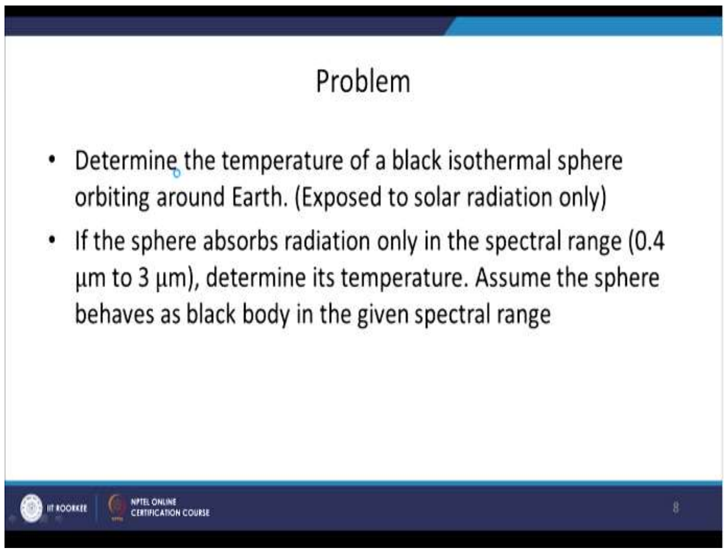

So let us solve one problem on the concepts that we have learnt so far. So in this problem, we have to find out the temperature of a black isothermal sphere. So there is a isothermal sphere whose temperature is constant and it is suspended in orbit around the earth okay. So the sphere is exposed to solar radiation and we have to find its temperature. So the temperature of this sphere will be found based on the radiative equilibrium.

Because the only energy this sphere is receiving is from the sun, so it is receiving the energy from the sun and it is radiating energy back to the atmosphere that is a vacuum in the space. We will do the same problem again by assuming that the sphere absorbs radiation only in the spectral range 0.4 micron to 3 micron. So we will solve these two problems and see how the temperature of the sphere is affected when the sphere does not absorb all the radiation, only a part of the radiation is absorbed by the sphere.

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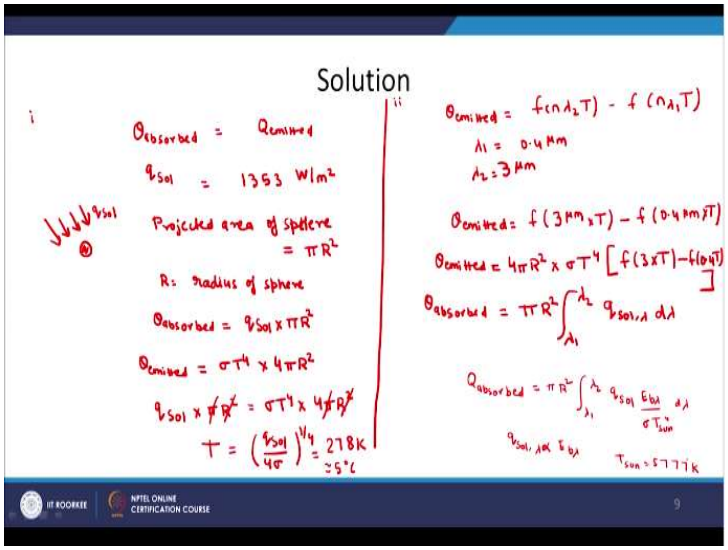

So let us solve this problem. So the first part of the problem is for the black sphere where the sphere absorbs all the radiation. So we will do an energy balance, first of all we write the absorbed energy by the sphere is equal to emitted energy by this sphere. So we have some energy absorbed and some energy emitted. Now solar is known that is the amount of radiation coming from sun is known and it is called q_{sol}.

This is equal to 1353 W/m^2 okay, this is the heat flux the radiative energy that is coming from sun okay and we can assume that the radiation energy is coming from single direction. So we have this radiation energy, this is q_{sol} and this is the sphere that we have basically receiving the energy. Now the projected area of this sphere is given as

$$\text{The projected area of sphere} = \pi R^2$$

where R is the radius of sphere.

$$\text{Total energy absorbed by the sphere is} = q_{sol} \times \pi R^2$$

okay. The radius can be anything okay. So the total energy absorbed by the sphere is $q_{sol} \times \pi R^2$. Now Q emitted so just keep in mind that this sphere is black okay. It is black so it will emit radiation as per this law σT^4 and then its total area that is $4 \pi R^2$. So total sphere area, surface area of the sphere is $4 \pi R^2$.

And total energy emitted by a blackbody is σT^4 as per Stefan-Boltzmann law. So when we equate so we get

$$q_{sol} * \times \pi R^2 = \sigma T^4 \times 4 \pi R^2$$

okay. So actually we do not need the radius values, so radius value basically cancels out okay and we do not even need the value of pi which also cancels out. So what we get is

$$T = \left(\frac{q_{sol}}{4\sigma}\right)^{1/4}$$

So this is the temperature of the sphere just by radiative equilibrium between the solar radiation and this sphere, the temperature of the sphere can be calculated and when we do this this value comes out to be 278 Kelvin okay which is around 5˚C okay. So this is the first part of the problem where we have to find out the temperature of the sphere assuming that the sphere behaves like a blackbody.

And it absorbs the entire radiation over the entire spectrum. It emits radiation over the entire spectrum. Now in the second part of the problem, what we know is that it absorbs radiation only in the 0.4 micron range to 3 micron range. So we will do the same thing again. Let us do this again. So

$$Q_{emitted} = f(n\lambda_2 T) - f(n\lambda_1 T)$$

where λ_1 is=0.4 micrometer and λ_2 is=3 micrometer.

So we have to calculate this f function and then we will be able to calculate the emitted energy. So total energy emitted will be Q_{emitted} will

$$Q_{emitted} = f(3\mu mT) - f(0.4\mu mT)$$

be equal to f now we are talking in terms of vacuum, so we can take refractive index n to be equal to 1. Now λ_2 is 3 micron and we have to multiply by temperature.

So this is the relation and Q emitted will be

$$Q_{emitted} = 4\,\pi\,R^2 \times \sigma T^4\ [f(3\mu mT) - f(0.4\mu mT)]$$

basically we have to multiply this relation by the surface area $4\,\pi\,R^2$ and we have to multiply by the σT^4. So this is the surface area of the sphere multiplied by σT^4. okay and then this fraction $f(3\mu mT) - f(0.4\mu mT)$ okay. So this is the amount of energy that is emitted by the sphere. Now let us see how much energy is absorbed.

So energy absorbed by this sphere will be also in the same range, so we multiply by area πR^2 okay and then we multiplied from λ_1 to λ_2 okay q solar. Now we have to take solar spectrum okay lambda okay and then we have to multiply by d lambda.

$$Q_{absorbed} = \pi R^2 \int_{\lambda_1}^{\lambda_2} q_{sol} \frac{E_{b\lambda}}{\sigma T_{sun}^4} d\lambda$$

where we have assumed that the spectral variation of solar flux is proportional to the blackbody emissive power of the sun.

That means $q_{sol,\lambda}$ is proportional to $E_{b\lambda}$ because sum behaves like a blackbody at a temperature of 5777 K. So we can replace the spectral flux with the blackbody emissive power of the sun at the temperature of the sun equal to 5777 K.

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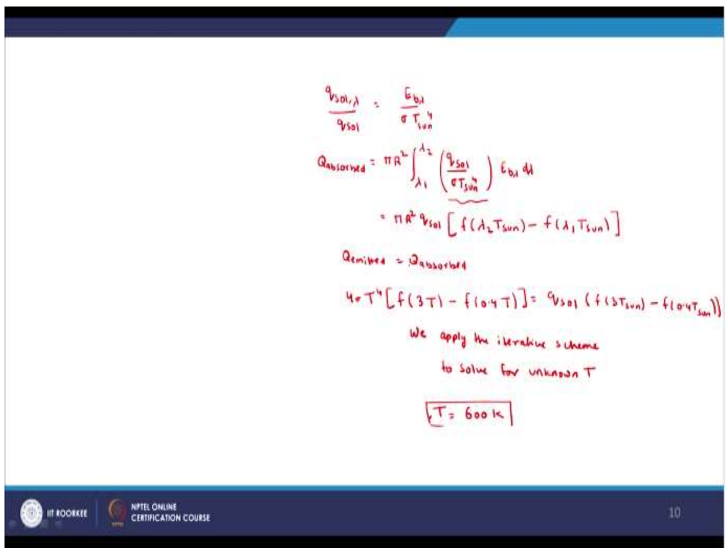

So in other words we have written

$$\frac{q_{sol,\lambda}}{q_{sol}} = \frac{E_{b\lambda}}{\sigma T_{sun}^4}$$

where T sun is 5777 Kelvin. Now our q_{absorb} becomes

$$Q_{absorbed} = \pi R^2 \int_{\lambda_1}^{\lambda_2} \left(\frac{q_{sol}}{\sigma T_{sun}^4}\right) E_{b\lambda} d\lambda$$

This quantity is independent of wavelength $E_{b\lambda} d\lambda$. So this quantity is independent of wavelength and we can take it out and all we are basically left with

$$Q_{absorbed} = \pi R^2 q_{sol} [f(\lambda_2 T_{sun}) - f(\lambda_1 T_{sun})]$$

where f is basically the normalized blackbody function for wavelengths λ_1 and λ_2.

Now we do an energy balance on the sphere that means

$$Q_{emitted} = Q_{absorbed}$$

So if you look at this equation, this is basically a nonlinear equation in unknown temperature T.

$$4\sigma T^4[f(3T) - f(0.4T)] = Q_{sol}[f(3T_{sun}) - f(0.4T_{sun})]$$

Why it is a nonlinear equation? Because this function f depends on power 4 of the temperature and this equation cannot be solved directly. We can solve this equation either iteratively. So we will apply an iterative method to solve for unknown T and when we do that we get the temperature of this sphere as around 600 Kelvin.

$$T = 600 \text{ K}$$

So next we move to the important concept of solid angle. Just a thought experiment when we stand in front of fire, we see a lot of heat but when we are standing at the same distance but at an angle to the fire, the same amount of radiation does not reach us okay. Similarly, hot surface, a hot metal plate appears of a different color when seen from an angle while it appears of different color when seen normally. So basically the idea that comes to our mind is that radiation probably does not behave in a same way in all the directions.

In some direction, it behaves in a different way okay, so that is what basically it is. So solid angle basically defines the directional dependence of radiation. So what we basically mean by solid angle is, it is basically in spherical coordinates.

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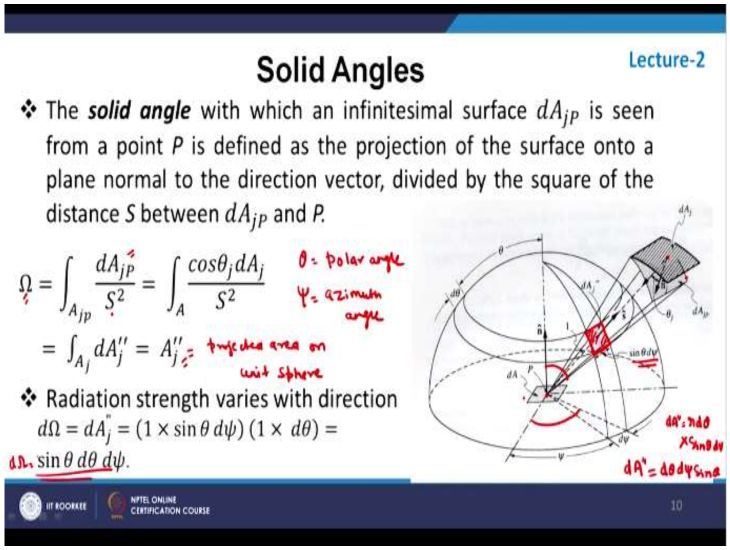

In spherical coordinates, we have basically two angles, one is the polar angle taken from the normal okay, this is the polar angle and one is the azimuthal angle taken from some arbitrary direction in the given plane. So the so theta (θ) is the polar angle and psi (ψ) is the azimuth angle okay. So we define basically the solid angle as the so if we have any plane of area let us say dA_j and we have some energy leaving this plane dA okay.

So there is a plane dA and there is a plane in space dA_j, so solid angle is basically defined as the ratio of this plane, projected area of this plane in the direction joining the plane P to the plane dA_j. So the line joining this is given represented by the vector $\hat{s}$ okay. So $\hat{s}$ is the direction of this plane dA_j okay, so we define solid angle as the projected area dA_{jP} okay, this is the projected area onto this direction okay.

Divided by the distance of this plane, projected plane from the point we are basically looking this plane okay, so we are trying to find out the solid angle of this plane dA_j from point P which is on the plane dA okay. So the solid angle by which this plane dA_j is seen from P is equal to dA_{jP} where dA_{jP} is the projected area divided by distance S square. So S is the distance of this projected plane from the point P.

$$\Omega = \int_{A_{jp}} \frac{dA_{jP}}{S^2} = \int_A \frac{cos\theta_j dA_j}{S^2}$$

$$= \int_{A_j} dA_j'' = A_j''$$

So this is the solid angle, so solid angle basically gives you the idea of the directional dependence of radiation. Now this projected area can be of unit magnitude so what I mean by that is this projected area need not be here, it can be on the sphere unit sphere. So if this projected area is taken at the units sphere okay that is the sphere of radius 1, then this solid angle omega will simply be equal to projected area.

So A_j" prime is basically the projected area on unit sphere that is the sphere of radius 1. So when we take this projected area on unit sphere, then the solid angle will be equal to the projected area itself and this solid angle is basically this area. This is the projected area that I have marked given by dA_j double prime okay and this magnitude

$$d\Omega = dA_j^{"} = (1 \times \sin\theta \, d\psi)\,(1 \times \, d\theta) = \sin\theta \, d\theta \, d\psi.$$

So this is equal to $d\theta$ and this magnitude is $\sin\theta \, d\theta \, d\psi$. So in that sense, $dA_j^{"}$ is equal to r $d\theta$ times $\sin\theta \, d\psi$. Now r is=1 so $dA_j^{"}$ =$\sin\theta \, d\theta \, d\psi$ and that is basically the small solid angle. So $d\Omega$ is the small solid angle by which this small area dA_j is seen from point P okay. So this is a small solid angle d $d\Omega$.

And we can represent the solid angle in terms of azimuthal angle ψ and polar angle θ. So as I told you that radiation depends on angle okay. So emissive power does not give us any idea how the intensity or radiative intensity emitted from a surface behaves in a given direction okay. So it gives you total amount of energy emitted per unit area but it does not give you how much energy is emitted in a given solid angle.

So to basically include the effect of solid angle, we define what we call radiative intensity okay.

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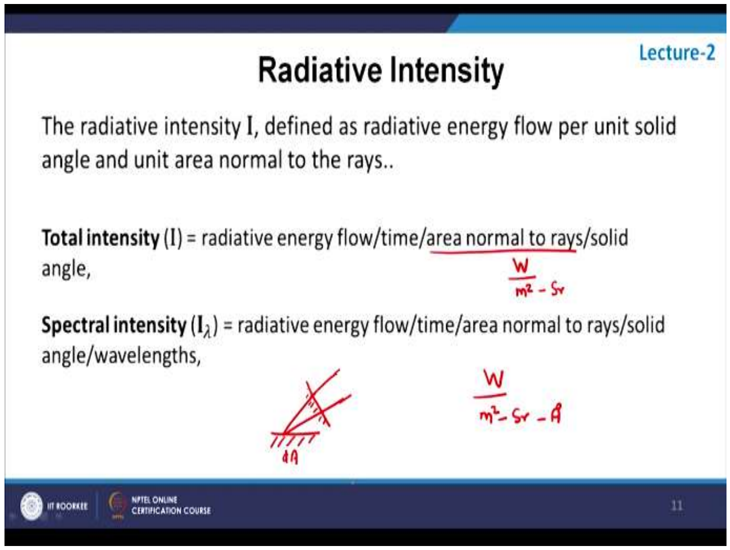

So radiative intensity is basically the energy emitted per unit time per unit area in a given solid angle that is the energy emitted per unit time that is watt per unit area meter square and per unit solid angle steradian. So the units of solid angle I need to mention is steradian okay. So intensity will have units watt per meter square per steradian. We also defined spectral intensity just like we defined spectral emissive power.

So spectral emissive power is radiative energy emitted in a given solid angle per unit area in a given wavelength okay. So the units will be here watt per meter square steradian angstrom where angstrom is the unit for wavelength okay. Now one thing you should be basically remember is when we talk area normal to rays in the intensity what we are basically talking is not the area from which the radiation is emitted, we are talking about area which is normal to the intensity.

So if you have this surface and it is emitting radiation in this solid angle okay, then we are talking about this area which is normal to the rays okay. We are not talking about the area from which the energy is emitted so we are talking about area normal to the rays. So intensity is always defined in terms of area normal to the direction of propagation of radiative intensity okay. So this thing sometimes is confusing.

You should keep in mind that radiative intensity is defined as the energy emitted per unit area normal to the direction of propagation of the radiation. Now definitely the two are related, radiative intensity and emissive power are related. Emissive power gives you total energy emitted from a surface while radiative intensity gives you energy emitted in a given solid angle.

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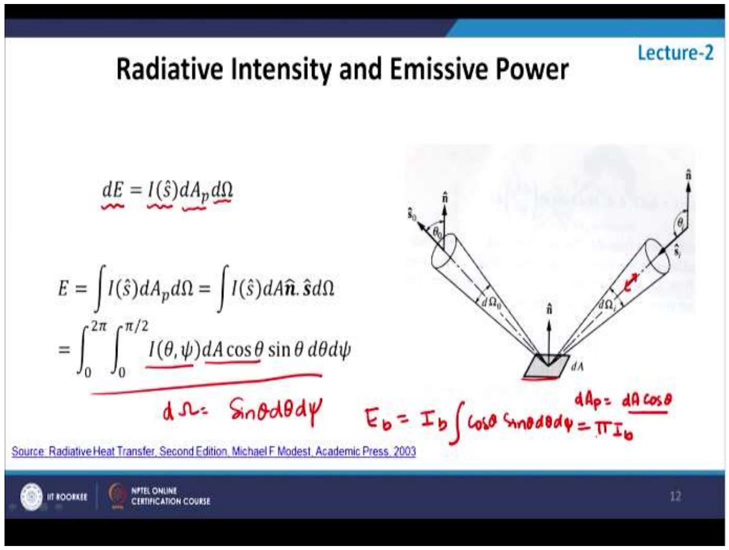

So we can relate, so an amount of energy emitted in a given solid angle is related to as per the definition of intensity, area projected that is area normal to the rays and multiplied by small solid angle. So this relation on the right hand side intensity multiplied by projected area multiplied by small solid angle gives you amount of energy emitted in a given solid angle. When we integrate it over all the solid angles, it should give us the emissive power.

$$dE = I(\hat{s})dA_p d\Omega$$

So we can write intensity Is which is in a given direction dA_p which is the projected area, $d\Omega$ the small solid angle and integrate over all the solid angle that is 2π. So when we do this, we get this relation. So we have written d omega in terms of theta and π, so we have $\sin\theta\, d\theta d\psi$. So $d\Omega$ we can write $\sin\theta\, d\theta d\psi$ that is what we have done. Intensity depends on polar angle and azimuthal angle.

$$E = \int I(\hat{s})dA_p d\Omega = \int I(\hat{s})dA\hat{\boldsymbol{n}}.\hat{\boldsymbol{s}}d\Omega \quad = \int_0^{2\pi}\int_0^{\pi/2} I(\theta,\psi)dA\cos\theta\sin\theta\, d\theta d\psi$$

So instead of writing intensity as a function of cap, $\hat{s}$ is the propagation as we see in this picture also, intensity in the particular direction so this is the direction $\hat{s}$. So this direction can be written in terms of polar angle and azimuthal angle. So I is a function of theta and psi and then $dA\cos\theta$ is basically the projected area. So we are not taking dA, we are taking the projected area.

So projected area dA_p is equal to $dA\cos\theta$. So this is the projected area okay. So when we do this, we get the emissive power okay and for a blackbody as you will see later, the intensities does not depend on θ and ψ that is it is independent of direction and we can take it out and this will be equal to, so blackbody emissive power, we will prove this in next lecture also will be equal to intensity taken out.

Because it is not a function of theta and psi and then we just integrate over cos theta sin theta d theta and d psi and this will be equal to pi times Ib. So blackbody emissive power and blackbody intensity are related with this relation

$$E_b = \pi I_b$$

okay. So we will go into this detail again okay. Thank you. In the next lecture, we will cover the fundamental laws of radiation like Kirchhoff’s law, Wien’s displacement law and Planck's law. I thank you all for giving your time to this lecture.