Radiative Heat Transfer
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Lecture- 17
Equation of Radiative Transfer

Hello friends, so we will continue our derivation of radiative transfer equation. So in the previous lecture we discussed what are the basic mechanisms by which radiative intensity can increase or decrease so we will continue by adding all the components.

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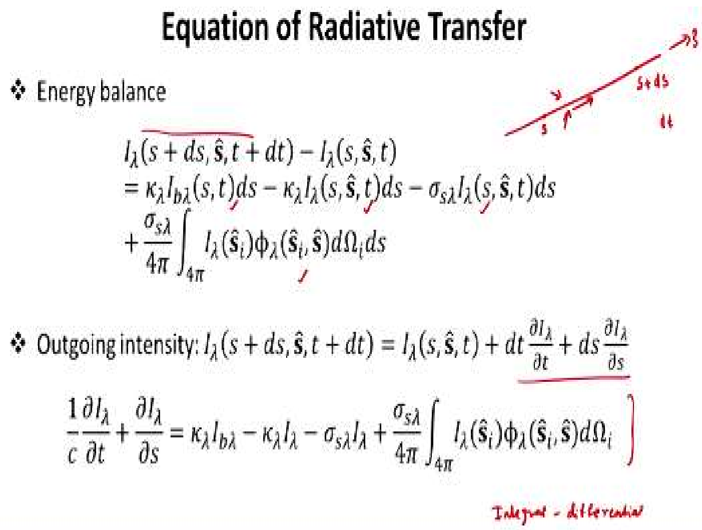

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So we have ray or photons and this location is let us say s and this location is s+ ds. The time taken by the photon to reach from s to s + ds is dt. This is the time required although this time is going to be very, very less because the speed of radiation travel is speed of light. When the radiation travels from s to s + ds its intensity will increase because of In-scattering. So In-scattering is the amount of energy scattered into this direction.

So intensity will come from other directions and it will In-scatter. There will be some emission then intensity will decrease because of absorption and Out-scattering. We discussed all this facts. So we all we add all the components so we write that intensity at s + ds at time t + dt in a given direction s cap. So this direction is s cap-intensity at point s at time t is basically the component from emission augmentation by emission, absorption attenuation by absorption, attenuation by Out-scattering and augmentation by In-scattering.

So we see that the integration in this equation appears in the In-scattering terms and $\phi_\lambda$is the scattering phase function. Now the difference between the intensity at s + ds and at s. we can write down using the differential calculus as $dt\frac{\partial I_\lambda}{\partial t} + dt\frac{\partial I_\lambda}{\partial s}$. Now substituting this for I_λ (s+ ds) -I_λ(s) in this equation we get the following equation okay.

Now this equation appears to be a partial differential equation because of the partial derivatives appearing on the left hand side; but this equation also appears to be an integral equation because of the integral appearing on the right hand side. So this equation is basically called integral differential equation. So in radiative heat exchange between plain surfaces you studied integral equation this is one step further and more complicated integral differential equation okay.

So it has 7 independent variables as we discussed in the previous lecture also 3 space coordinates, 2 direction coordinates θ and ψ, time and wave length and this is an integral differential equation. Now unless and until we are dealing with lasers of the order of picoseconds and femtosecond where we may be interested in modeling the transient radiative heat transfer.

So in lasers with such a short duration pulsed laser there may be of interest to deal with transient radiative heat transfer because the time scale is so small that the radiation travel between two parts s+ ds in this short duration may actually be of concentration. But for other applications this transient term is not required and we can neglect it and once we do that.

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Quasi Steady Form

$$\frac{dI_\lambda}{ds} = \hat{\mathbf{s}}.\nabla I_\lambda = \underbrace{\kappa_\lambda I_{b\lambda}}_{\text{Emission}} - \underbrace{\beta_\lambda I_\lambda}_{\text{Absorption by scattering}} + \frac{\sigma_{s\lambda}}{4\pi}\underbrace{\int_{4\pi} I_\lambda(\hat{\mathbf{s}}_i)\phi_\lambda(\hat{\mathbf{s}}_i, \hat{\mathbf{s}})d\Omega_i}_{\text{In scattering}} \quad \text{R.T.E}$$

❖ Nondimensional optical coordinates

$$\tau_\lambda = \int_0^s (\kappa_\lambda + \sigma_{s\lambda})\,ds = \int_0^s \beta_\lambda ds$$

$$\omega_\lambda \equiv \frac{\sigma_{s\lambda}}{\kappa_\lambda + \sigma_{s\lambda}} = \frac{\sigma_{s\lambda}}{\beta_\lambda} \qquad \textit{(single scattering albedo)}$$

$$\frac{dI_\lambda}{d\tau_\lambda} = (1-\omega_\lambda)I_{b\lambda} - I_\lambda + \frac{\omega_\lambda}{4\pi}\int_{4\pi} I_\lambda(\hat{\mathbf{s}}_i)\phi_\lambda(\hat{\mathbf{s}}_i, \hat{\mathbf{s}})d\Omega_i$$

The equation what we called quasi-steady form can be written as this equation where partial derivative has now replaced with direct total derivative $\frac{dI_\lambda}{ds}$ because the dependence on time has been removed and this equation again this is an integral differential equation. On the right hand side, we have the emission, absorption. We have introduced the extinction coefficient beta lambda which includes absorption by or attenuation by absorption.

And scattering also the out scattering and the last term is In-Scattering or augmentation by scattering. Often we non-dimensionalized this equation this is called RTE or Radiative Transfer Equation. So this equation is called Radiative Transfer Equation. We non-dimensionalized the radiative transfer equation by introducing the optical depth τ_λ we have already defined it.

We defined optical depth in terms of extinction coefficient. So ω_λ is defined one more non-dimensional parameter omega lambda which is basically the ratio of scattering coefficient and extinction coefficient. So $\sigma_{s\lambda}$ is scattering coefficient and β_λ is extinction coefficient and the ratio is called single scattering albedo okay. This is basically a measure of scattering as a function of the extension and this parameter is called single scattering albedo.

So by introducing this parameter the non-dimensional optical depth and non-dimensional single scattering albedo. We get the equation $\frac{dI_\lambda}{d\tau_\lambda} = (1-\omega_\lambda)I_{b\lambda}$ which is again the emission terms $-I_\lambda$ (the absorption term) + $\frac{\omega_\lambda}{4\pi}$ and the integral ($\int_{4\pi} I_\lambda(\hat{\mathbf{s}}_i)\phi_\lambda(\hat{\mathbf{s}}_i, \hat{\mathbf{s}})d\Omega_i$)which is In-scattering term. So this is non-dimensional form of the RTE.

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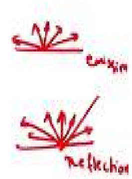
Boundary Conditions

- **Diffusely Emitting/Reflecting Opaque Surfaces**
  - Intensity independent of direction.
- Intensity equation

$$I(\mathbf{r}_\omega) = J(\mathbf{r}_\omega)/\pi = \epsilon(\mathbf{r}_\omega)I_b(\mathbf{r}_\omega) + \rho(\mathbf{r}_\omega)H(\mathbf{r}_\omega)/\pi$$



$$I(\mathbf{r}_\omega) = \epsilon(\mathbf{r}_\omega)I_b(\mathbf{r}_\omega) + \frac{\rho(\mathbf{r}_\omega)}{4\pi}\int_{\hat{\mathbf{n}}.\hat{\mathbf{s}}'<0} I(\mathbf{r}_\omega)\,|\hat{\mathbf{n}}.\,\hat{\mathbf{s}}'|\,d\Omega'$$

- Integration over all outgoing direction $\quad \hat{\mathbf{n}}.\,\hat{\mathbf{s}}' = \cos\theta'$
- Boundary condition also follows integral equation
- Black surface (with $\rho = 0$), $\quad I(\mathbf{r}_\omega) = I_b(\mathbf{r}_\omega)$

Now boundary conditions for this equation. So this is first order differential equation so we need to specify one boundary conditions and this boundary condition we can define for various types of surfaces we have already discussed, but we will just take a special case that is diffusely emitting reflecting surfaces. So the emittance of the surface maybe epsilon. So for this case the intensity leaving the surface I_{λ}, I it could be a function of wavelength, but here I am just writing it for the total. So intensity leaving the wall it basically diffuse and can be written in terms of radiosity.

So this is the intensity leaving the wall. It has two components the emission from the surface and reflection for the radiation coming on to this. Okay so this is emission and the diffuse reflection so this is reflection okay. Now this H is irradiation we have already discussed this. Irradiation is integrated on all the solid angles so we have used the vector notation on all the solid angles we have to integrate.

Now this radiation is coming from the gas or it may be coming from other surfaces. So we have to include the fact of radiation coming from gas as well as other surfaces. So again the boundary conditions is going to be integral equation. This is same as we have done for the heat transfer between surfaces the boundary condition for this case is integral equation. The governing equation is integral differential equation and the boundary condition is an integral equation.

For a black surface the analysis simple and most of the problems that we will solve we will assume that the surface is black that means a medium is bounded by black surfaces and the intensity the boundary condition the intensity is simply= the black body intensity. Here we are allowing that the intensity may vary over a surface RW means the vector of any point on the wall.

So intensity will vary over a wall, but it does not vary in the direction. So we are assuming diffuse surface. Now once we have solved for intensity. Once we have the solution for radiative transfer equation for any problem we can find out the quantities of our interest and two quantities of interest in heat transfer community are the heat flux and divergence of heat flux. Heat flux is used to design surfaces for heat transfer equipment's.

Be at boiler surface, be it the combustor surface and divergence of heat flux is needed to know how much energy is absorbed at any location. This may be of interest in combustion application.

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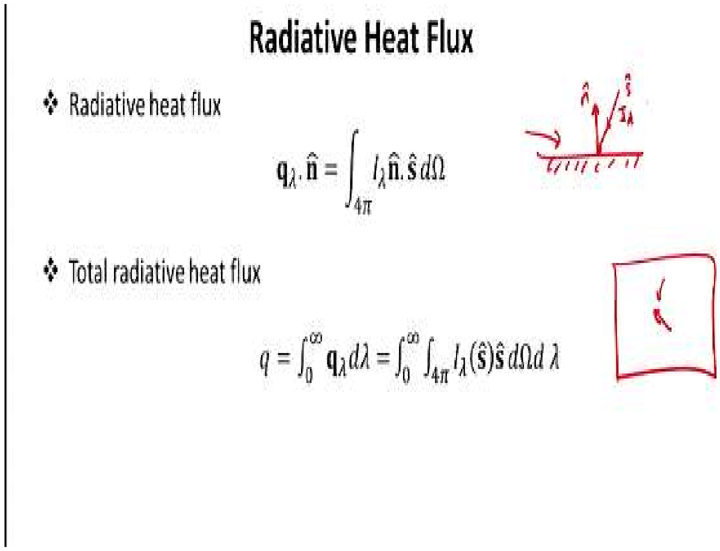

So we define heat flux as basically intensity dotted with the normal vector. So we have a surface on which we want to find out heat flux so this is the intensity I_λ coming from certain direction $\hat{\mathbf{s}}$ okay and this is the normal vector $\hat{\mathbf{n}}$. So heat flux is basically the normal component of the intensity. So we take a dot product of intensity and $\hat{\mathbf{n}}.\hat{\mathbf{s}}$ basically appears.

And we have to integrate because appears and we have to integrate because intensity may come from all directions so we have to integrate all the solid angles. So the heat flux is basically defined as $\hat{\mathbf{n}}.\hat{\mathbf{s}}$ where n is the unit vector normal to the surface. s is the direction from which the intensity is falling on the surface and we have to integrate our all the solid angles.

And solid angles is going to be 2π if we are dealing with real surface and it is going to be 4π if we are just trying to find out heat flux in any point suspended in space. If we have a space and we are interested in finding the heat flux at this point then we have to integrate over the solid angle 4π because intensity will come from all the directions, but if you want to find out heat flux on a flat surface then definitely the solid angle will be just 2π.

The total radiative heat flux will then be calculated by integrating this spectral radiative heat flux over all the wavelength. So we integrate over the entire spectrum 0 to infinity and this will give us the total heat flux. Now divergence of radiative heat flux is needed in knowing how much total energy is emitted or absorb at a given location in the medium.

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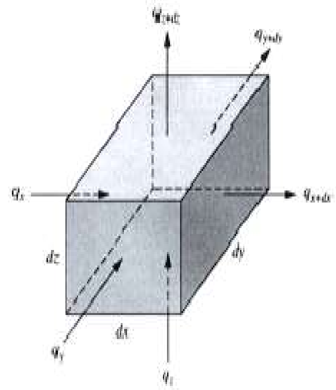
Divergence of the Radiative Heat Flux

❖ Radiative energy balance on infinitesimal volume ($dV = dx\,dy\,dz$)



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So we take a small volume of element of volume dV. The size of this volume is dx dy dz. So the total volume dV=dx*dy*dz. The radiative energy enters this volume from 6 faces. some amount of energy is absorbed within this volume and some amount of energy is emitted within this volume. So we write an energy balance for this volume.

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Divergence of the Radiative Heat Flux

$$\begin{pmatrix}\text{radiative energy}\\ \text{stored in } dV\\ \text{per unit time}\end{pmatrix} - \begin{pmatrix}\text{rad. enrgy generated}\\ \text{(emitted) by } dV\\ \text{per unit time}\end{pmatrix} + \begin{pmatrix}\text{rad. energy destroyed}\\ \text{(absorbed) by } dV\\ \text{per unit time}\end{pmatrix}$$

$$= \begin{pmatrix}\text{flux in at } x \; - \text{flux out at } x+dx\\ +\text{flux in at } y \; - \text{flux out at } y+dy\\ +\text{flux in at } z \; - \text{flux out at } z+dz\end{pmatrix}$$

- Right-hand side of the equation

$$\left.\begin{matrix} q(x)dydz - q(x+dx)dydz\\ +q(y)dxdz - q(y+dy)dxdz\\ +q(z)dxdy - q(z+dz)dxdy\end{matrix}\right\} = -\underline{\left(\frac{\partial q}{\partial x}+\frac{\partial q}{\partial y}+\frac{\partial q}{\partial z}\right)}dx\,dy\,dz = -\underline{\underline{\nabla.\mathbf{q}}}dV$$

Divergence of Rad

Rad. Source term

So radiative energy is stored in dV per unit time that is amount of radiative energy change in this volume per unit time - radiative energy emitted+ radiative energy destroyed or absorbed by this volume should be= so the left hand side is the total change in volume per unit time and the right hand side is the flux. So flux of radiative energy at face x-flux of radiative energy at phase x + dx and the same thing we have to do on the 6 faces.

The right hand side can be simplified by solving for q (x+ dx) and we get

$-\left(\frac{\partial q}{\partial x}+\frac{\partial q}{\partial y}+\frac{\partial z}{\partial z}\right) dx\ dy\ dz$ and this will be= simply $-\nabla.\mathbf{q}$dV. So $\nabla.\mathbf{q}$dV is the divergence of radiative heat flux. This is the divergence of radiative flux we also call it radiative source term. So this radiative source term often appears in the overall energy balance equation.

So if you have other modes of heat transfer this radiative source term will appear as a source term in the overall energy balance equation. Now to solve for the left hand side we take the radiative transfer equation and we integrate over all the solid angles.

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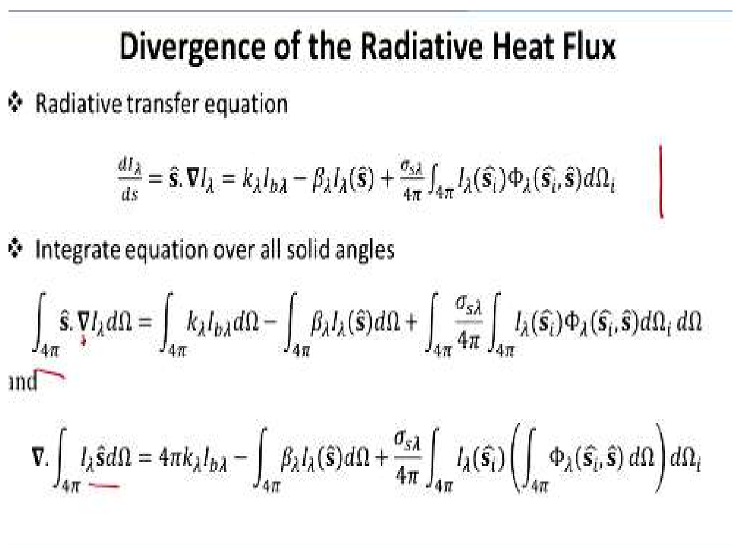

So this is the radiative transfer equation. We integrate over all solid angles 4π both left hand side and right hand side we do. Then we do little mathematical manipulation. We take the divergence operator outside this integral and we define this integral was $4\pi I_\lambda \hat{\mathbf{s}} d\Omega$ and same thing we do on the other terms. So without going into the much of the mathematical manipulation.

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Divergence of the Radiative Heat Flux

- On integration

$$\nabla.\mathbf{q}_\lambda = \underline{4\pi k_\lambda I_{b\lambda}} - \beta_\lambda \underline{\int_{4\pi} I_\lambda(\hat{\mathbf{s}})d\Omega} + \sigma_{s\lambda} \underline{\int_{4\pi} I_\lambda(\hat{\mathbf{s}}_i)d\Omega_i}$$

$$\nabla.\mathbf{q}_\lambda = k_\lambda\left(4\pi I_{b\lambda} - \int_{4\pi} I_\lambda d\Omega\right) = k_\lambda(4\pi I_{b\lambda} - G_\lambda) \Leftarrow$$

$$\nabla\cdot q = k(4\pi I_b - G) \Leftarrow \qquad G_\lambda = \int \underline{I_\lambda(\hat{s}_i)\,d\Omega_i}$$

Here we will introduce one quantity which we call incident radiation G. So incident radiation G is defined as basically the intensity coming from certain direction integrated over omega. So this G is basically different from flux. The flux is always measured normal to a surface. So if we have to calculate the heat flux we will take I_λ and the dot product between the surface normal and the direction $\hat{\mathbf{s}_\iota}$, but incident radiation is defined as the total intensity coming from 4π direction.

So it is basically a measure of total intensity coming from all directions. So we integrate the intensity from all the directions to get the incidence radiation. So simplifying this equation in terms of G we get the first term the emission term is $4\pi\kappa_\lambda I_{b\lambda}$ and this is the absorption term ($\beta_\lambda \int_{4\pi} I_\lambda(\hat{\mathbf{s}})d\Omega$) and this is the scattering term ($\sigma_{s\lambda} \int_{4\pi} I_\lambda(\hat{\mathbf{s}_\iota})d\Omega_i$). So we can simplify this and we get basically the divergence of heat flux ($\nabla.\boldsymbol{q}$)=$\kappa(4\pi I_b - G)$.

So this is valid for spectral basis also and this is valid for total basis also. So this is a equation for radiative heat flux for on a spectral basis and this is the equation for the radiative heat source term on overall basis, it could be valid on a gray basis also. So we will do one problem and this problem will clarify most of the concepts that we have learned so far on the equation of radiative transfer.

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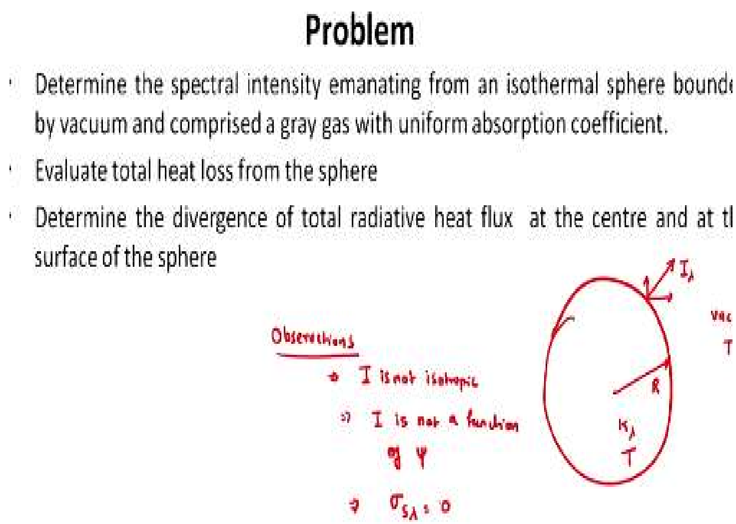

So what we have in this problem is basically a sphere. So there is a sphere the radius of this sphere is R. The sphere is filled with some gas with absorption coefficient κ_λ okay outside the sphere is vacuum okay and the temperature is almost closer to 0. So vacuum and 0 temperature outside this sphere is specified because we want to exclude any external irradiation.

So we have this sphere which is filled with the gas the absorption coefficient is kappa lambda the temperature of the gas is uniform κ_λ a is uniform the temperature is T the radius is R. Now we have to find out the intensity of radiation coming out of this sphere I_λ. We have to find out total heat loss from this sphere and we have to find out the divegence the radiative heat source term at the centre of this is sphere as well as at the surface of this sphere.

Now this problem looks pretty simple as such, but as you will see that the mathematic even for this simple case is not that easy. So now what we basically have to observe some observation we have to basically do first. So the first observation here is that the intensity is not isotropic okay that means intensity on this surface is not going to be same. It is going to depend on θ.

So intensity is going to be different at different angles and that will depend on the polar angle that is the first observation. The second observation is I is not a function of azimuthal angle okay. So it will depend on θ but it will be uniform in azimuthal angle okay and then we have to basically solve for this problem. So let us see how to solve this problem we will the third assumption is that $\sigma_{s\lambda}$ is 0 there is no scattering.

The gas is purely absorbing and emitting there is no scattering. So let us solve this problem. So first of all we write down the governing equation of radiative heat transfer.

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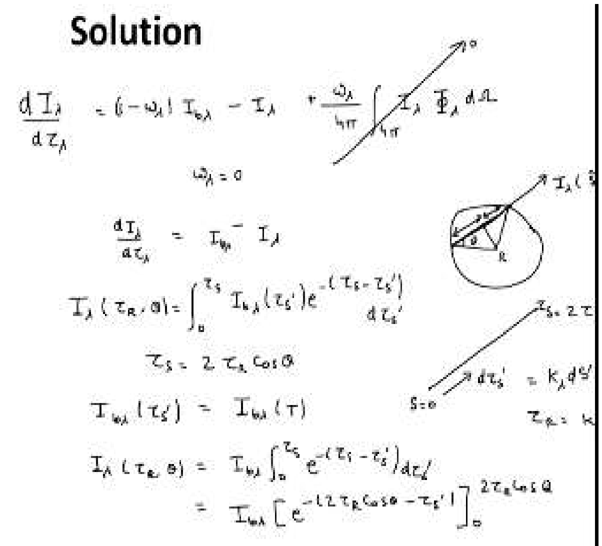

So dI_λ in non-dimensional form, $d\tau_\lambda = (1-\omega_\lambda)I_{b\lambda} - I_\lambda + \frac{\omega_\lambda}{4\pi}\int_{4\pi} I_\lambda\, \phi_\lambda d\Omega$. So this is the governing equation the integral differential equation. Now based on the assumptions that scattering is not there. We have ω_λ=0 okay. So you just observe how we are simplifying the problem and still the problem the mathematics of the problem is involved.

So this term $(\frac{\omega_\lambda}{4\pi}\int_{4\pi} I_\lambda\, \phi_\lambda d\Omega)$ will be = 0 okay. so our equation now reduces to $\frac{dI_\lambda}{d\tau_\lambda} = -$ or simply $I_{b\lambda} - I_\lambda$. So very simple differential equation. Okay now let us look at it this is in a given direction. So we draw our sphere again so we are looking at the intensity at this point. So this is our intensity I_λ in a given direction let us call this $\hat{\mathbf{s}}$. Now where this intensity coming from. This intensity is coming from this part of the sphere this is the center of the sphere.

So radiation along this path we have to calculate this is the path okay. So it may start at this point and it may go all the way up to this point on the sphere surface okay. So this angle may be specified as θ okay. Now what we will do is we introduce the coordinates along this direction. So let me draw this one this is s=0 and this is s = 2 times $\tau_R \cos\theta$. This path non-dimensional path I am talking so this is going to be 2 times $\tau_R \cos\theta$ okay.

This is R this distance is R$\cos\theta$ so this will become 2 R$\cos\theta$ and we have multiplied by absorption coefficient to define the path this is τ_S that means the optical path at the surface is=$2\tau_R \cos\theta$. So we write this now I_λ which is function of now τ_R and θ = we integrate it 0 to

τ_S, $I_{b\lambda}(\tau_s')e^{-(\tau_S-\tau_s')}d\tau_s'$ where τ_s' is a variable along this path.

So τ_s' is simply dS' times the absorption coefficient κ_λ okay. So we have to integrate this so this is the solution for this path I_λ which is function of τ_R and θ and = 0 to τ_S $I_{b\lambda}(\tau_s')e^{-(\tau_S-\tau_s')}$ where τ_S is nothing but $2\tau_R\cos\theta$. So τ_S is measured in this direction while τ_R is simply, τ_R is nothing but κ_λ times R okay.

So now we will write this as$I_{b\lambda}$, $I_{b\lambda}$ as a function of τ_s'. Now $I_{b\lambda}$ does not depend on path okay. Black body intensity is isotropic so $I_{b\lambda}$ is simply a function of temperature only okay it does not depend on path. So we can take it out of the integral. So our I_λ then which is function of now τ_R and θ = $I_{b\lambda}$ taken out 0 to τ_S, $e^{-(\tau_S-\tau_s')}d\tau_s'$ and this will be= $I_{b\lambda}[e^{-2(\tau_R\cos\theta-\ _s')}]$. And then this will be 0 to $2\tau_R\cos\theta$, where we have substitute $\tau_S = 2\tau_R\cos\theta$ okay.

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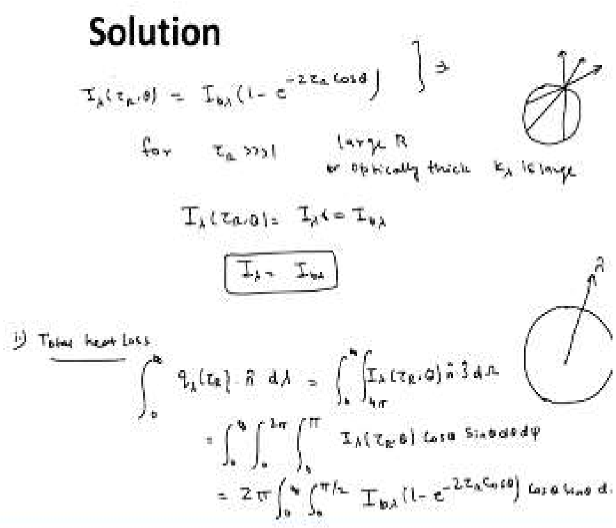

Now we just finalize the solution by putting the limits so I_λ (τ_R,θ). So I_λ is a function of τ_R and θ = $I_{b\lambda}$ which is a function of temperature only, $\left(1 - e^{-2\tau_R\cos\theta}\right)$ okay. So this is the equation this is the solution for intensity okay as it is clear intensity is a function of θ. So for a sphere the intensity will be different in different direction okay. It will be different in different directions okay.

So at any point on the surface of the sphere the intensity is going to be different in different direction is a function of θ. For τ_R>1 that means where you have a large sphere, large sphere or optically thick. Optically thick means we have κ_λ as large that means we have strong absorption. κ_λ large means we have significant absorption κ_λ or τ_λ> very, very large than 1 is optically thick in that case the exponential terms will be=0 and I_λ (τ_R, θ) will be simply = I_λ. It will not depend on τ_R and theta and it will be simply = $I_{b\lambda}$ okay. So for optically thick spheres or any medium it applies to any medium it could be sphere, it could be rectangle, it could be cube okay. So for any medium optically thick medium the intensity does not depend on direction and it does not depend on the absorption coefficient okay. So it is independent of κ as well as θ and the intensity is simply = the black body intensity and that is what basically we have in case of sun.

So sun is a very large sphere and the intensity coming out of the sun is black-body intensity= black-body intensity. And we have basically proved why sun behaves like a black-body. Now the second part of the problem the total heat loss. So total heat loss is basically the total amount of heat flux over the surface at any point okay. So total heat loss= 0 to infinity q_λ at any point.

So q_λ (τ_R) where we dot with the normal vector. So we have this sphere this is the local normal $\hat{\mathbf{n}}$. So we have to fund out the heat flux q_λ dotted with a local normal and integrating over the entire wavelength region so we have 0 to infinity I_λ and again integrating over 4π. So I_λ which is a function of τ_R and θ, $\hat{\mathbf{n}}$. and $\hat{\mathbf{s}} d\Omega$ okay.

So intensity is a function of θ. So we have to take a product with a normal vector to calculate the heat flux and that heat flux it will be spectral heat flux we have to integrate over all the wavelength. So this will be= 0 to infinity 0 to 2 π $I_\lambda(\tau_R, \theta)$ $\cos\theta$ for , $\hat{\mathbf{n}}$. $\hat{\mathbf{s}} d\Omega$ become $\sin\theta$ $d\theta$ $d\psi$. Now there is no dependence on ψ so we can solve for the ψ first and this will become 2π, 0 to infinity 0 to $\pi/2$. So we get $I_{b\lambda}$ so substituting the expression for I_λ this expression we put. $I_{b\lambda} \left(1 - e^{-2\tau_R \cos\theta}\right) \cos\theta \sin\theta \; d\theta \; and \; d\lambda.$

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Solution

Total heat loss $= 2\pi\int_0^\infty\int_0^{\pi/2} I_{b\lambda}\left[1-e^{-2\tau_R\cos\theta}\right]\cos\theta\sin\theta\,d\theta\,d\lambda$

$= \pi I_b\left\{1-\frac{1}{2\tau_R^2}\left[1-(1+2\tau_R)e^{-2\tau_R}\right]\right\}$

$= \sigma T^4\left\{1-\frac{1}{2\tau_R^2}\left[1-(1+2\tau_R)e^{-2\tau_R}\right]\right\}$

$\nabla\cdot q_\lambda = 4\pi\kappa_\lambda I_{b\lambda} - \kappa_\lambda G_\lambda$ | $\kappa_\lambda = \kappa \Rightarrow$ Gray

$G_\lambda = \int_{4\pi} I_\lambda(\hat{s})\,d\Omega = 2\pi\int_0^{\pi/2} I_{b\lambda}(1-e^{-2\tau_R\cos\theta})\sin\theta\,d\theta$

$= \frac{\pi I_{b\lambda}}{\tau_R}\left[2\tau_R-1+e^{-2\tau_R}\right]$

$\int_0^\infty \nabla\cdot q_\lambda\,d\lambda = \int_0^\infty (4\pi\kappa_\lambda I_{b\lambda}-\kappa_\lambda G_\lambda)\,d\lambda = \frac{\sigma T^4}{\tau_R}\left[2\tau_R+1-e^{-2\tau_R}\right]$

@ centre

$I_\lambda(0) = I_{b\lambda}(1-e^{-2\tau_R})$

$G_\lambda(0) = 4\pi I_{b\lambda}(1-e^{-2\tau_R})$

$\boxed{\nabla\cdot q = 4\kappa\sigma T^4 e^{-2\tau_R}}$

So total heat loss that is basically the heat flux per unit area at any location on the surface of the sphere can be written then as 2π integration from 0 to infinity, integration from 0 to $\pi/2$. So we have splitted this solid angle into azimuthal angle and polar angle and this will be = $I_{b\lambda}$ $\left(1-e^{-2\tau_R\cos}\right)\cos\theta\sin\theta$. This is the variation of intensity with respect to θ. So we have substituted the value of intensity $\cos\theta\sin\theta\ d\theta\ and\ d\lambda$.

Now in this expression the plan black body $I_{b\lambda}$ function is independent of θ. So we can just pull it out and this will be= π times I_b $\left\{1-\frac{1}{2\tau_R^2}\left[1-(1+2\tau_R)e^{-2\tau_R}\right]\right\}$okay. So this is a expression for the total heat loss πI_b is simply σT^4 the black-body emissive power. So $\left\{1-\frac{1}{2\tau_R^2}\left[1-(1+2\tau_R)e^{-2\tau_R}\right]\right\}$. So this is a total heat loss from the surface of the sphere at any location on the sphere.

This is per unit area if you want to apply the total heat loss we have to multiply by the surface of the sphere. Now the other quantity of interest is basically the divergence of heat flux. We have to find out divergence of heat flux at the two locations. So the divergence of heat flux is defined as $\nabla.q$=$4\pi\kappa_\lambda$ now this could be spectral or it could be total $4\pi\kappa_\lambda I_{b\lambda}-\kappa_\lambda G_\lambda$ lambda okay.

Now here we just assume that $\kappa_\lambda=\kappa$ that means the gas is gray. The absorption coefficient does not depend on the wavelength okay. So with this first we evaluate G_λ which is= intensity at a given direction integrated over all the solid angles okay. So we can write $d\Omega$ as $\sin\theta$

$d\theta d\psi$ and then integrate over the solid angle so this will be=$2\pi \int I_{b\lambda} \left(1 - e^{-2\tau_R \cos\theta}\right)$ that is the intensity $\sin\theta \; d\theta$.

And we have to integrate from 0 to $\pi/2$. So this will once we simply this we get π times $I_{b\lambda}/\tau_R$. Okay so τ_R although is a function of wavelength, but we can assumed it to be gray so I will just write it as τ_R and this will be=$2\tau_R - 1 + e^{-2\tau_R}$. So this is a radiative intensity so $\nabla.\boldsymbol{q}$ is simply= $4\pi\kappa_\lambda I_{b\lambda} - \kappa_\lambda G_\lambda$.

So we substitute the value of G_λ here and we get σT and integrate over all the wavelengths $d\lambda$. So we also integrate over all the wavelengths $d\lambda$. So this will be= $\sigma T^4/\tau_R[2\tau_R + 1 - e^{-2\tau_R}]$ okay. So this is the expression for the radiative heat source term integrated over the entire spectrum at any location on the surface of the sphere. Now at the center of the sphere we have intensity I_λ from the of the intensity is= $I_{b\lambda}\,(1 - e^{-2\tau_R})$.

So this is the magnitude of intensity of radiation at the center of the sphere and this is going to be isotropic okay. It will not depend on theta so G_λ at the centre is simply= $4\pi I_{b\lambda}\,(1 - e^{-2\tau_R})$ and $\nabla.\boldsymbol{q}$ will be simply= $4\kappa\sigma T^4 e^{-2\tau_R}]$. So this is the $\nabla.\boldsymbol{q}$ at the center and this is $\nabla.\boldsymbol{q}$ at the surface.

So in this lecture we have seen how to apply the equation of radiative transfer to some special problems one dimensional problem where we have a sphere and gray problem where the absorption coefficient was not a function of wavelength and we did lot of simplification and solve for important parameters like heat flux at the surface and $\nabla.\boldsymbol{q}$. We will apply in subsequent lecture the same techniques to solve radiative heat transfer in cylindrical and plain parallel geometry.

So I thank you for your kind attention. We will continue in the next lecture for some special cases of radiative transfer one dimensional cases. Thank you.