Radiative Heat Transfer
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Lecture – 15
Radiative Heat Transfer in the Presence of Conduction/Convection

Hello friends, in many problems more than one mode of heat transfer may be important there may be conduction together with radiation there may be convection together with radiation. And these two modes of radiative transfer and conduction or conductivity transfer maybe coupled to each other that means they affect each other and or they may be independent of each other in that case we may solve the problem of conduction and radiation or convection or radiation independently.

So in this lecture, we will study how the conduction and radiation problem can be solved together we will also study how the convection problem and radiation volume can be solved together.

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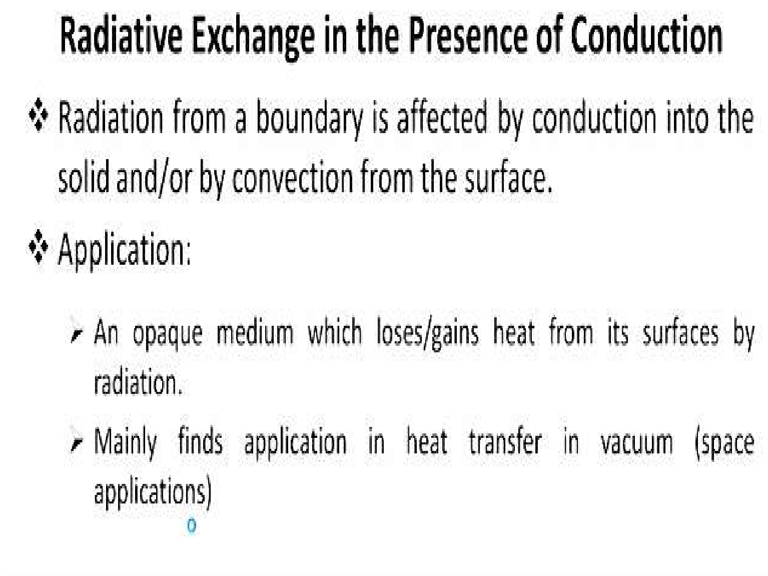

So there are many applications for example, in space the spacecraft has to dissipate heat generated within the space craft the heat transfer within this space craft is merely governed by conduction while the dissipation of heat from the surface of the space craft is mainly going by radiation. Such space crafts the dissipating surfaces of this is space crafts are normally pointed away from the sun.

So we can safely assume that they are not irradiated by any emission outside the spacecraft the solar radiation or I need addition coming from outside can be neglected. So these surfaces dissipate heat through radiation into the atmosphere or into space and energy transfer within the surface of the space craft is governed by conduction. Similarly, there may be some cases where for example in many applications related to combustion.

Where conductive heat transfer takes place inside some tube or a combustion chamber while the surface is radiate the energy into the environment and leads to heating or cooling of the wall.

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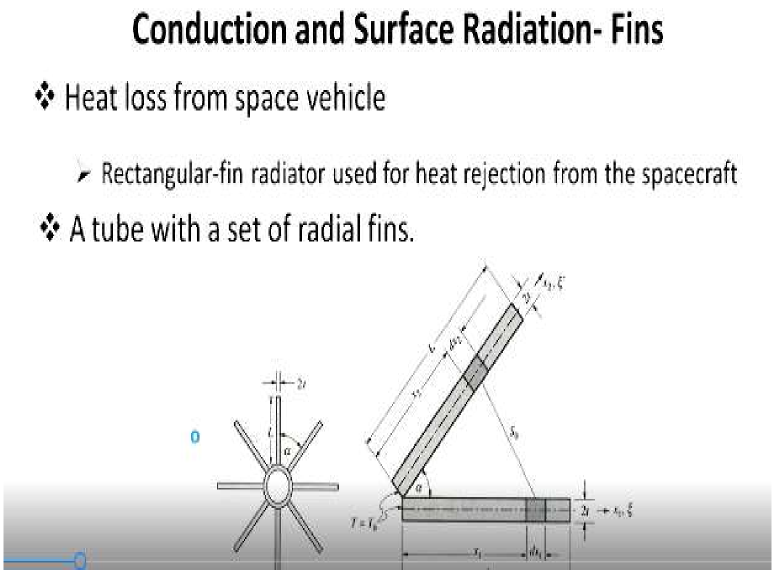

So first we will study the conduction and surface radiation we take an example of a fins which has which is a typically used to dissipate heat in space craft. In this case we have taken a special configuration we have actually symmetric a configuration radially located fins around the tube the fins have a what we call a rectangular cross section. The rectangular cross section of the fin have thickness (2t). the thickness of this rectangular cross section of the fin has (2t) and we will make certain assumptions as this customary in many conduction problems.

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Conduction and Surface Radiation- Fins

❖ Assumptions

1. Fin temperature is a function of radial distance only. T(0
2. Negligible end losses from fin tip by (convection + radiation). $\frac{\partial T_i}{\partial x_i}(L) = 0$
3. Thermal conductivity (k) is constant.
4. $T_1(0) = T_2(0) = T_b$, and $T_1(x_1) = T_2(x_2 = x_1)$.
5. No external irradiation falling into the fin cavities ($H_0 = 0, T_\infty = 0$).
6. Opaque, gray, diffusely emitting and reflecting surface with emittance ϵ.

We assume that the temperature of the fin varies in the radial direction only that means the temperature is not a function of theta (θ) that means it does not vary from fin to fin at a given radial distance and it also does not vary in the axial direction the axial direction is rather taken very long. We assume that the N loss is from the tip of the fin are negligible by convection as well as radiation.

The conductivity of the material of the fin is known and is constant the base temperature of the fin so we have this fin which is radiating energy from both the surfaces. Here we do not have energy so we have assumed that the N losses are negligible and the best temperature is maintained at constant temperature =T_b. There is also no external irradiation that means solar energy is not falling onto this fin.

And the space is at very low temperature approximately close to 0 so there is no radiation coming from the space also okay they also assume that the surface of the fin the radiative properties are Gray diffuse and the fin itself is opaque.

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Conduction and Surface Radiation- Fins

❖ Energy balance on small element of volume dV with unit axial length

$$2tk\frac{dT}{dx}\bigg|_{x} = 2tk\frac{dT}{dx}\bigg|_{x+dx} + 2q_R dx$$

Conduction in = conduction out + net radiative loss from top and bottom surfaces ($2dx$)

$$\boxed{\frac{d^2T}{dx^2} = \frac{1}{tk}q_R}$$

where $q_R\,(x)$ =net radiative heat flux leaving a surface element of the fin

$$q_R(x_1) = \frac{\epsilon}{1-\epsilon}[\sigma T_1^4(x_1) - J_1(x_1)]$$

So the energy balance for a cross section of fin so we take this cross section of fin and we write down the energy balance for this fin. So basically what we have is energy coming in through conductive heat flux and energy going out through conductive heat flux and then from the surface we have energy leaving the fin through radiation. So we write this energy balance equation.

So the equation is energy coming in through conduction is = energy going out to conduction and radiation combined. Now we intentionally chose the thickness of the fin as 2t so factor 2 basically cancels out so we get this differential equation[$\frac{d^2T}{dx^2} = \frac{1}{tk}q_R$](d square t/dx square) so temperature is varying only in the direction radial direction of the fin and the radial direction here is represented by a coordinate x.

So[$\frac{d^2T}{dx^2} = \frac{1}{tk}q_R$] where q_R is the radiative heat flux that we need to solve through Radiosity relation. Now in this problem we see that the problem is going to be a coupled problem why a couple because in conduction equation we have a radiative source term that is appearing so conduction equation this is conduction equation where we have a source term for a radiative heat flux okay.

Similarly, in the radiative equation the radiative equation is written by as we have already developed for Gray surfaces so q_R the flux at any location x we are talking about two fins so if you look at the configuration of this geometry they will be radiative heat transferred between two adjacent fins okay. So we call one fin aligned in the x direction as and another fin which is adjacent to it is x_2 okay.

And due to symmetry we do not have to worry about the fin on the other side. So there will be exchange of radiation between two adjacent fins. So we have introduced a Radiosity J_1 okay which depends on radiation on adjacent surface okay and we have emission from the surface okay now because of the unknown temperature of the fin T the problem is coupled to the conduction okay.

So we have an unknown temperature T_1 which is the temperature of the fin surface at any location x_1 which is linked to the conduction equation. So this is the you can call it radiation equation. Okay so definitely these type of problems are very difficult to solve analytically so we have to solve this specially because of the dependence of radiative flux on Radiosity which is continuously varying with x.

This will give us again a Fredholm type of integral equation which we know how to solve using numerical methods. So the combined couple problems is going to be much more difficult.

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Boundary conditions and Non-Dimensional Form

$$T(x=0)=T_b, \qquad \frac{dT}{dx}(x=L)=0$$

$$\theta(\xi)=\frac{T(x)}{T_b} \qquad \mathcal{J}(\xi)=\frac{J(x)}{\sigma T_b^4} \qquad N_c=\frac{kt}{\sigma T_b^3 L^2} \qquad \xi=\frac{x}{L}$$

- Where θ and $\mathcal{J}$ are non-dimensional temperature and radiosity
- N_c is conduction to radiation parameter (plank number).

$$\frac{d^2\theta}{d\xi^2}=\frac{1}{N_c}\frac{\epsilon}{1-\epsilon}\left[\theta^4(\xi)-\mathcal{J}(\xi)\right]$$

- $\theta(\xi=0)=1,$
- $\frac{d\theta}{d\xi}(\xi=1)=0$

$$\mathcal{J}(\xi)=\epsilon\theta^4(\xi)+(1-\epsilon)\int_{\xi'=0}^{1}\mathcal{J}(\xi')K(\xi,\xi')d\xi'$$

So the boundary condition we have is temperature of the fin at x=0 is that is at the basis constant T_b and the end losses are negligible so dT/dx at x=L=0. So this is the boundary condition for the conduction problem a two dimensional conduction problem requires two boundary conditions a second order boundary equation requires two boundary conditions. So these are the two boundary condition for the conduction problem.

So we will not attempt to solve this problem because its very difficult to solve this problem analytically we will just leave the development of the problem by giving a nondimensional form of these equations. We define a non-dimensional temperature as temperature of the fin[$\theta(\xi) = \frac{T(x)}{T_b}$]at any location x divided by its base temperature. So this is the non-dimensional temperature theta where theta is basically a non-dimensional length x/L.

Non-dimensional Radiosity [$\mathcal{J}(\xi)$=$\frac{J(x)}{\sigma T_b^4}$]is defined as actual Radiosity at location x divided by black body emissive power at temperature T_b .and then we define one parameter non-dimensional parameterN_c. In the numerator we have the conduction term where we have k conductivity times thickness 1/2 thickness t and in the denominator we have a radiation term $\sigma T_b^3 L^2$. Now this particular non-dimensional parameter is often called plank number.

Which is basically a ratio of conductive heat transferred to radiative heat transfer. Now with this non-dimensional parameters we can write our equations the conduction equation as this is modified non-dimensional conduction equation so non-dimensional temperature $\frac{d^2\theta}{d\xi^2} = \frac{1}{N_c}\frac{\epsilon}{1-\epsilon}[\theta^4(\xi) - \mathcal{J}(\xi)]$.

So of course this is a nonlinear equation and it has to be its all using numerical methods the non-dimensional Radiosity relation can be written we have already discussed how to evaluate this in case of Gray surfaces so we have what we call emission term theta is unknown here so it is coupled to the conduction problem and we have this function what we call Kernel function that depends on view factors and all so this is for the irradiation term or absorption

Okay so this becomes an integral equation so we have to solve these equations conduction equation which is non-linear and radiation equation for Radiosity which is basically integral equation. So both the equations are difficult in their own sense and have to be solved using some kind of numerical scheme. The boundary conditions the modified boundary conditions in non-dimensional parameters becomes θ =1 for $\xi = 0$ and $\frac{d\theta}{d\xi}$ =0 for ξ =1.

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Fin efficiency

$$\eta_f = \frac{\text{Heat loss from actual fin}}{\text{Heat loss from the ideal fin (a black fin, isothermal at } T_b)}$$

- Heat loss from an ideal fin ($\epsilon = 1,\ J = \sigma T^4$)

$$Q_{ideal} = 2Lq_{R,ideal} = 2L\sigma T_b^4(1 - F_{12}) = 2L\sin\frac{\alpha}{2}\,\sigma T_b^4$$

- Actual heat loss from the fin, by applying Fourier's law

$$Q_{actual} = -2tk\left.\frac{dT}{dx}\right|_{x=0} = 2\int_0^L q_R(x)dx$$

Now just like we define fin efficiency in case of conduction and convection combined mode heat transfer we also define fin efficiency for conduction and radiation. So the fin efficiency is defined as heat loss actual heat loss from the fin divided by an idealised situation where we have a black fin at an isothermal temperature T_b.

So if such an idealized fin exists then the heat loss from that fin we compare our actual heat loss from that fin and the ratio is basically defined as the fin efficiency. Why we have taken denominator as black fin an isothermal fin because black surfaces basically radiate the maximum amount of energy. So the fin will be more of most efficient when its temperature will be highest when its emission power will be highest that is black body at isothermal temperature T_b.

That is going to give us the maximum amount of energy emitted by the fin. So that is why we have taken it in the denominator and the denominator can be evaluated analytically it will be basically = $\sigma T_b^4(1 - F_{1-2})$ because some part of the radiation will actually be interchange with the adjacent fin. So the factor$(1 - F_{1-2})$ appears because of that geometric configuration.

So this is our heat loss from idealize fin or black fin at constant temperature Tb okay now heat loss from the actual heat loss can be calculated in two ways once we have solved the problem the conduction and radiation problem. We know radiative heat flux as a function of facts we know temperature as a function of facts so either so we can solve dT/dx once we know T as a function of x and calculate the actual amount of heat transferred by calculating the gradient dT/dx at x=0.

Or we can basically integrate the radiative heat flux from 0 to L both the integration well give you the same results okay and this will appear in the numerator and the ratio is defined as the fin efficiency.

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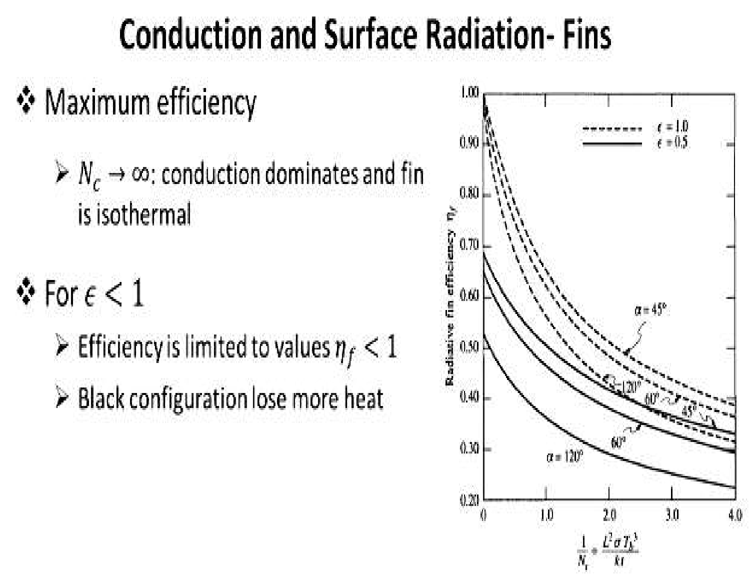

So as I said these problems can be solved numerically a very difficult to solve analytically I have I am showing you here when figure that basically explains how the efficiency of the fin varies on this chart we see that the efficiency is going to decrease when the fin goes from black to Gray okay if the emittance is 1 the fin is black the efficiency is highest and if the emittance is less let us say 0.5 the efficiency is going to go give go down.

And the reason is I already explained because the black surface emits most amount of radiation similarly the fin efficiency decreases with 1/radiation the plank number. So $1/N_c$ is the inverse of the plank number N_c is the plank number. So the radiative efficiency of the fin decreases with this number now the region is let us say if you have highly conducting fin or the value of k is large then $1/N_c$ is going to be very small.

The conduction basically dominates the fin will be isothermal and efficiency of the fin is going to be large okay. So with the radiation number with plank number 1/plank number the efficiency of the fin decreases okay. So numerically these type of problems can be solved we will not attempt to solve any example analytically in this lecture. Next, we will discuss his combined mode convective and radiative heat transfer.

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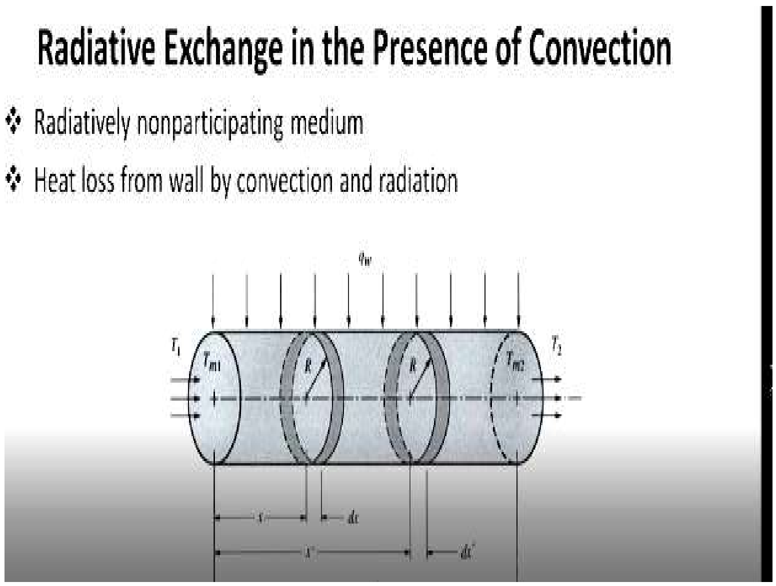

In combined mode convective and radiative heat transfer the surface radiation from a surface and this could be internal flow let us say flow through a pipe with the surface of the pipe is heated with constant heat flux as is shown here or it could be external flow over a flat plate with a surface of where the plates surface is heated with constant temperature or constant heat flux. Now due to radiation emitted from these surfaces either the plate surface.

Or the pipe inner surface that heat absorbed by the fluid is going to change okay because some amount of energy is lost due to radiation the temperature of the fluid or the temperature of the wall of the pipe or flat surface is going to change. Okay now in this particular configuration the fluid enters a pipe with a mean temperature of T_{m1} and it exists the pipe with a mean temperature of T_{m2}.

Now the surface on the left hand side the pipe inlet surface or inlet circle is exposed to an environment maintained at temperature T_1. It could be like this we have a basically let us say some kind of Furness and we have some environment and the pipe basically take some kind of fluid okay and this fluid basically takes fluid to the discharges the fluid to some environment we maintain that temperature T_1 okay.

So the two ends of the tube are going to be at different surfaces okay now what we have basically assumed here is that the fluid itself does not absorb radiation that is it is nonparticipating medium. If the fluid absorbs radiation, then definitely radiation will increase the temperature of the fluid. But here we are assuming that the fluid is nonparticipating radiatively and only role of radiation is basically heat loss from the ends of the pipe. So we write an energy balance equation for a small elemental volume let us call this elemental volume dV.

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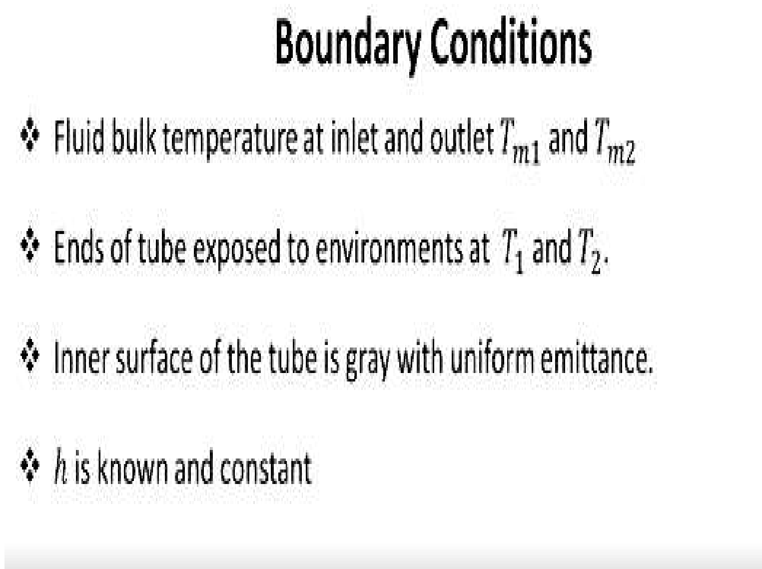

Okay, this is the boundary conditions that we will apply I have already explained this. This is constant heat flux problem, the tube surface is maintained at constant heat flux q_w while the temperature of the surface is changing with respect to x. So both T_m is a function of x and T_w that is wall temperature is a function of x okay and the heat transfer coefficient is assumed to be constant. So we assume that heat transfer coefficient is not varying with the pipe length.

Radiative Exchange in the Presence of Convection

- Energy balance on dV

Enthalpy flux in at x + convective flux in over dx = Enthalpy flux out at $x + dx$

$$\dot{m}c_p T_m(x) + h[T_w(x) - T_m(x)]\, 2\pi R\, dx = \dot{m}c_p T_m(\mathrm{x+dx})$$

$$\frac{dT_m}{dx} = \frac{2h}{\rho c_p u_m R}[T_w(x) - T_m(x)]$$

$$\dot{m} = \rho u_m \pi R^2$$

- Energy balance for the tube surface:

$$q_w = h[T_w(x) - T_m(x)] + \frac{\epsilon}{1-\epsilon}[\sigma T_w^4(x) - J(x)]$$

So writing the energy balance equation we take an elemental volume dV so in this we have enthalpy coming in which is equal to $\dot{m}c_p$ this is enthalpy coming in $\dot{m}c_p$ and the bulk fluid temperature T_m and enthalpy going out which is $\dot{m}c_p$ and the temperature is going to be changed here let say temperature is T_m here temperature is T_m at x+ Δx okay so the temperature is changing along the pipeline so then enthalpy is going to change.

And then we have heat added or removed from the wall through convection so the heat transfer coefficient is h and this is the temperature difference between wall and the fluid so T_w -T_m so this is the amount of heat removed per unit area from the wall surface and we multiply it by the contact area of this fluid element which is basically =$2\pi R$ and this thickness dx okay so this is the total amount of heat removed by this small element from the surface of the tube okay.

Now as I said the fluid is nonparticipating so no radiation term comes in this. So radiation basically comes into this equation through temperature only. So we can write down this equation simply $\frac{dT_m}{dx}$ so we expand this right hand side with Taylor series and we can write down the differential equation the derivative of mean bulk temperature of the fluid at any location x, $\frac{dT_m}{dx} = \frac{2h}{\rho c_p u_m R}$ where u_m is now average fluid velocity and ρ is the density of fluid.

Where R is the radius of the pipe okay so we can write down this differential equation now same type of energy we do for the surface element of the pipe that is this surface element this is the pipe surface the small element dx okay the area of this element will be of course $2\pi R$dx okay so the heat removed or the heat flux on the wall surface q_w assuming it to be Gray will have a radiation component.

Emission minus absorption term absorption of radiation coming from the two ends as well as absorption of radiation coming from other part of the pipe. So the Radiosity contains three components radiation coming from left hand radiation coming from right then and radiation coming from other parts of the pipe and then we have also heat flux due to convection. So heat removed from the pipe surface contains two components.

One is convection which is actually taken by the fluid and one is radiation which is not taken by the fluid because fluid is not participating it is basically lost to the environment.

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Radiosity and Non-Dimensional Equation

$$J(x) = \underbrace{\epsilon\sigma T_w^4(x)}_{\text{emission}} + \underbrace{(1-\epsilon)\left\{\sigma T_1^4 F_{dx-1}\right.}_{\text{reflection}} \left. + \sigma T_2^4 F_{dx-2} + \int_0^L J(x')\, dF_{dx-dx'}\right\}$$

$$\xi = \frac{x}{D} \quad \theta(\xi) = \left(\frac{\sigma T^4}{q_w}\right)^{1/4} \quad \mathcal{J}(\xi) = \frac{J}{q_w} \quad St = \frac{h}{\rho c_p u_m D} \quad H = \frac{h}{q_w}\left(\frac{q_w}{\sigma}\right)^{1/4}$$

$$\frac{d\theta_m}{d\xi} = 4\,St[\theta_w(\xi) - \theta_m(\xi)] \quad \theta_m(\xi = 0) = \theta_{m1}$$

$$1 = H[\theta_w(\xi) - \theta_m(\xi)] + \frac{\epsilon}{1-\epsilon}[\theta_1^4(\xi) - \mathcal{J}(\xi)]$$

$$\mathcal{J}(\xi) = \epsilon\theta_w^4(\xi) + (1-\epsilon)\left\{\theta_1^4 F_{d\xi-1} + \theta_2^4 F_{d\xi-2} + \int_0^{L/D} \mathcal{J}(\xi')\, dF_{d\xi-d\xi'}\right\}$$

So the expression for Radiosity J_x contains emission and the reflection part this is the reflected part we have the term in the bracket basically includes comprises of irradiation. So the fluid surface is basically eradicated by surface from inlet outlet and other parts of the surface of the pipe okay. So there are three components of irradiation, irradiation from outside radiation surface inlet surface 1 and inlet surface 2 and irradiation from the pipe itself okay.

So that will be basically an integral equation because Radiosity is varying along the pipe surface. So again this method this problem can be solved using numerical methods where we have to solve one integral equation the Fredholm equation and one equation we have to solve this for convection which is basically a linear equation. So these three equations we have to solve we non-dimensionalized the problem we define a non-dimensional length x/D.

We define non-dimensional temperature or yeah non-dimensional temperature $[\theta(\xi)]$as sigma $(\frac{\sigma T^4}{q_w})^{1/4}$.we defined non-dimensional Radiosity as $\frac{J}{q_w}$, where q_w is a known constant that is the heat flux at the wall. Then Stanton number is defined $\frac{h}{\rho c_p u_m D}$and one more non-dimensional we defined for the heat transfer coefficient $H = \frac{h}{q_w}(\frac{q_w}{\sigma})^{1/4}$

Okay these non-dimensional numbers we have defined.

And based on these non-dimensional numbers we write our governing equation. So the first equation for convection becomes $\frac{d\theta_m}{d\xi}$ where θ_m is now the non-dimensional temperature $=$ $4\, St[\theta_w(\xi) - \theta_m(\xi)]$ okay. So both θ_w andθ_m are unknown here. Then from radiative energy balance we can write down equation $1 = H[\theta_w(\xi) - \theta_m(\xi)] + \frac{\epsilon}{1-\epsilon}[\theta_1^4(\xi) - J(\xi)]$now theta 1 is known.

It is the temperature at the inlet - Radiosity non-dimensional Radiosity and then we have non-dimensional Radiosity which can be written as non-dimensional emission power $\epsilon\theta_w^4$ and emission term or irradiation term from the inlet and the irradiation from the pipe other part of the pipe. So this is the integral equation in non-dimensional form. So these equations we have to solve using some kind of numerical the problem is a couple problem.

Because the temperature the unknown temperatures T_m and T_w appear in both convection and radiation equation.

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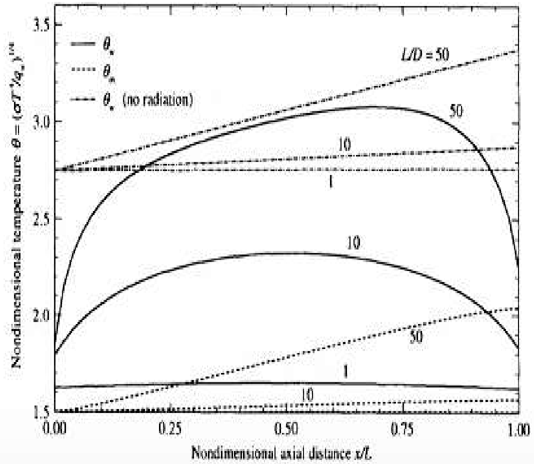
Radiative Exchange in the Presence of Convection



Effect of Radiation on Wall Temperature

We conclude this lecture by showing you the results its very difficult to solve this problem analytically but it can be solved through numerical procedures. So what I show you on this slide is basically the result for non-dimensional temperatures wall temperature and bulk fluid temperature with the actual non-dimensional actual length. So we expect that the fluid temperature will increase.

As its also observed the fluid temperature increases with the actual length because the fluid is going to absorb energy from the wall through conductive heat transfer. So fluid temperature will continuously increase what is surprising is the distribution of wall temperature how the wall basically varies if we do not include radiation into effect then we find that the temperature should also increase the wall temperature should also increase.

Because we have supplied constant wall heat flux and the bulk fluid temperature is increasing so the wall temperature should also increase and it is clear that the wall temperature is indeed increasing or rather linearly it is increasing the increase is significant for larger L/D ratio while the increase is not very significant if the pipe is short for long pipes this is going to have significant increase.

What is surprising and what is interesting to observe in this image is the variation of wall temperature when radiation is taken into account. So we see that variation basically leads to loss of energy from the pipe surface and we expect that the wall temperature should come down because of this loss of energy the wall will be cooler as compared to when radiation is not taken into account.

And this is pretty much clear where for all values of L/D 1, 10 and 50 the flux is significantly less than the flux value without radiation okay. The decrease is highest in pipes with L/D ratio of 1 and the reason is for shorter pipes so this is let us say L/D=1 this is L/D=10 and this is L/D of 50 okay so for L/D the fraction of radiation energy loss because now the surface is pretty much exposed to the ends.

The ends of the pipe are very close to the surface of the pipe so the fraction of energy lost by radiation is much more significant for smaller pipes okay. So here at radiation losses are significant and we can see that the temperature difference within without radiation is highest for the case L/D=1 here L/D=50 radiation is still there but it is mostly confined to the end regions and we see that the temperature drops significantly in the end regions okay.

There is a significant drop in temperature in the end regions but in between the difference between the temperature with and without radiation is not much. Okay because radiation effect is hardly observed in the middle of the pipe while at the end radiation is significant and it leads to a significant reduction of radiative a significant deduction of wall temperature at the end of the pipes.

So I conclude this lecture where we have discussed combined mode radiative transfer the problem are coupled and normally governed by Fredholm equation. Analysis of these types of problems have to be done through numerical analysis in the next lecture we will discuss problems where radiation passes through a medium where the medium is not vacuum rather the medium is emitting absorbing and scattering medium.

Such problems, we encounter in large number of applications like combustion furnaces atmosphere. So thank you for the time being, we will see you next time.