Radiative Heat Transfer
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Lecture – 14
Non Gray Surfaces

Hello friends we are discussing radiative heat transfers between surfaces so far in this course we have focussed on black and gray surfaces. In this lecture we will focus on radiative heat transfer between non gray surfaces

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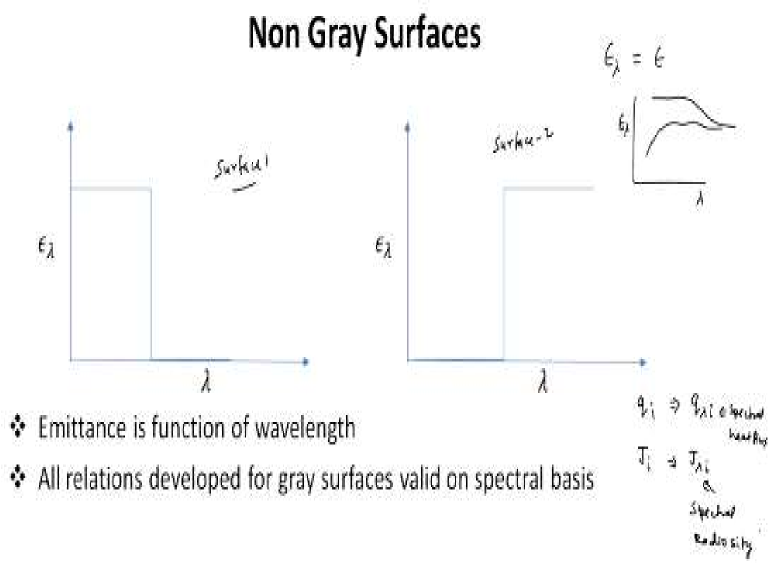

So, the gray surfaces are basically defined as where the emittance does not depend on wave length so $\epsilon_\lambda = \epsilon$, so the emittance is constant over a surface over a wave length. Such a surface is defined as gray, so far we have only discussed gay surfaces and black surfaces with diffused and specular reflection. Now we will look at the surface is when these surfaces have different magnitude of emittance over different spectral range.

What you see in this slide is basically an approximation of the emittance variation with wave length as we have seen in our lecture on properties of materials. The emittance may actually very continuously over the wave length. So, for some materials it will be constant at long wavelength region while wearing sharply in the short wave length region. While for some other materials it may be constant at short wave length region.

And then decrease at long wavelength region but in this example or in this course we will mainly focus on materials that have variation of emittance with wave length in the form of bands. So, as it is seen in this slide in these figures the emittance for surface one let us say this is surface one has large value at sharp wave lengths and almost negligible at long wavelength and for surface too.

The emittance is negligible like short wavelength while it has a large value at high wave length region so although emittance is a function of wave length but we have taken a simplified case. Where wave length is basically constant over a given wave length region so the analysis for this type of problems will be very similar to what we have done for the gray surfaces. The expressions that we have derived for heat flux (q) and Radiosity J_i are equally valid for this case.

Also the only thing is now we will have to solve for heat flux for each wave length for each surface. And we have to solve for radiosity for each wavelength for each surface so that the expressions that are developed are valid here. The only thing is we have to basically do a spectral calculation so this is called spectral flux spectral heat flux and this is called spectral Radiosity and we use the term spectral. To basically defined that the quantity is valid only for a given wavelength.

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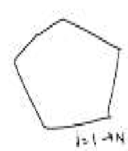
Radiative Heat Exchange Between Diffuse Non Gray Surfaces

$$\frac{q_{\lambda i}}{\epsilon_{\lambda i}} = E_{b\lambda i} - \sum_{j=1}^{N} J_{\lambda j} F_{i-j} - H_{0\lambda i}$$

$$q_{\lambda i} = \frac{\epsilon_i}{1-\epsilon_i}\left[E_{b\lambda i} - J_{\lambda i}\right]$$



$$\frac{q_{\lambda i}}{\epsilon_{\lambda i}} - \sum_{j=1}^{N}\left(\frac{1}{\epsilon_{\lambda j}} - 1\right) F_{i-j} q_{\lambda j} + H_{0\lambda i} = E_{b\lambda i} - \sum_{j=1}^{N} F_{i-j} E_{b\lambda j}$$

$$q_i = \int_0^\infty q_{\lambda i}\, d\lambda \qquad E_{bi} = \int_0^\infty E_{b\lambda i}\, d\lambda$$

Solve for $q_{\lambda i}$ for all wavelengths $\lambda_1, \lambda_2, \cdots \lambda_M$

Line by Line method

So, the relationship that we have already developed I am just reading them again so we develop this relation for a gray surface where there was no dependence of any quantity on wave length. Now when we have a non gray surface all the quantities be it flux, emissive power, Radiosity, irradiation or even the emittance has to be taken into account for each wave length. So, the first expression the first equation gives you a spectral heat flux.

That means the heat flux at a given wavelength divided by emittance spectral emittance again this is at a given wavelength is equal to a spectral emission power – summation of all N surfaces in the enclosure. So, we are basically talking about and closer divided into N surfaces, so we have j=1 to N, total N surfaces. And then we summed over all the surfaces Radiosity ($J_{\lambda j}$), radiosity, spectral Radiosity of each surface and the view factor.

The view factor is purely geometric so the wave length does not appear in this so we have a view factor or geometric view factor and then irradiation. Irradiation again may depend on wave length. So, this is the energy balance for a surface at a given wavelength we have expression in terms of Radiosity and emission power. We can basically eliminate the irradiance here so we can write the heat flux as $\frac{\epsilon_{\lambda i}}{1-\epsilon_{\lambda i}}[E_{b\lambda i} - J_{\lambda i}]$.

So, this is the emission power and this is the Radiosity now we can eliminate the Radiosity as we have done for gray and black gray surfaces. And we can get this expression for heat flux so these expressions we will solve for a non gray surfaces where we are trying to find out heat flux at a given wavelength. We have eliminated the Radiosity from this relation using the above two equations using these two equations we eliminated the Radiosity.

And we have calculated the heat flux the spectral heat flux once we know the spectral heat flux the total heat flux can be calculated by integrating the spectral heat flux or all the wave lengths flux qi at a given surface i is simply equal to integration from 0 to infinity. That is all the wave lengths and the integrant is being spectral heat flux. Similarly, the total black body emission power at a surface E_{bi} is integration over all the wave lengths of the spectral emission power.

Spectral emission power is basically nothing but the plank function these type of problems if you try to solve this for each wave length. So, we have to solve for q_λ for all wavelengths. So, wave length may vary from 0 to infinity. We have to take some finite number of wave lengths each wavelength maybe the interval from 0 to infinity maybe divided into λ_1, λ_2 and so on.

Okay times λ_{M}, there may be thousand wave length that you want to evaluate this integration through numerical scheme or there may be millions of points. So, this method where we have to find spectral heat flux at large number of wave lengths is called line by line method where we have to find out the heat flux at each wave length and then we find the total heat flux by integrating over the spectral heat flux.

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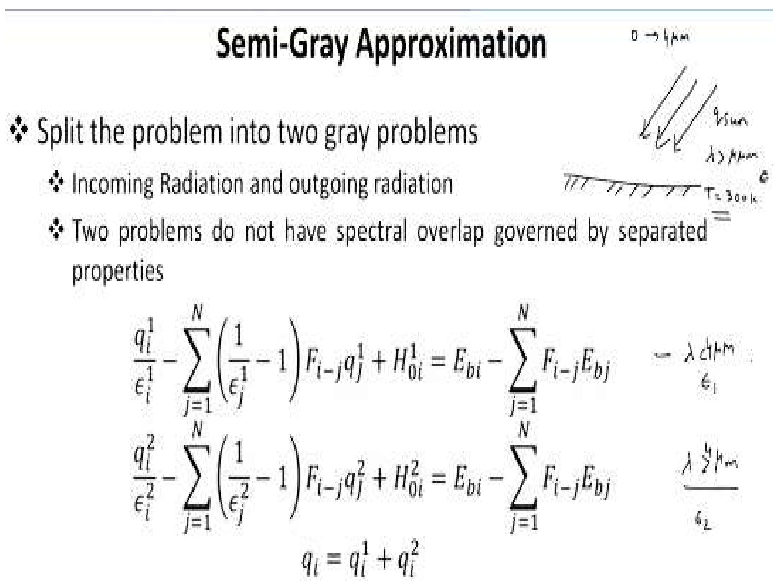

So, there are two ways of simplifying this problem we know that in this case the emittance is not continuously varying over wave length rather it is having a constant value in one spectral range and another constant value in another spectrum range. So, the first approximate method to deal with this type of problem is called semi gray approximation. For example, we have let us say surface maintained at same temperature of let us say 300 Kelvin.

And irradiated by solar radiation so we know that the solar radiation contains most of it is energy in the wave length region from 0 to let us say 4-micron. So, most of the solar energy comes into this a spectral range. While this plate which is maintained at 300 Kelvin will emit most of its radiation in the wave length region > 4 micron. So, what do we do is we split the problem into two parts?

One part contains heat transfer radiative heat transfer because of the solar energy which is in the 0 to 4-micron region. And another part which is > 4-micron region does not contain solar energy because we do not have solar energy in that spectral range. So, we have spirited the problem into 2 parts we apply the energy balance equation for heat flux on the two parts of the spectrum so this is the first part for lambda (λ).

Let us say 4 micron and this question is for lambda > 4 micron once we have solved these 2 equations for the heat flux. Please note there is no wave length here because the emittance is assumed to be constant so we have a value of emittance at epsilon 1. And here we have the value emittance epsilon 2 and which is constant over the wave length region < 4 micron or > 4 micron so the lambda (λ) does not appear here.

Because we have assumed that the spectrum can be divided into 2 regions each region can considered to be gray. Once we have calculated the heat flu the total heat flux q_i is simply the heat flux q_{1i} + q_{2i} ,that is the radiative heat flux from the two semi gray approximated fluxes. So, the 2 regions lambda (λ)< 4 micron gives you a heat flux q_{1i} and the other region lambda (λ) >4 micron gives you flux q_{2i} and we add the 2 components and we get the total radiant flux so this method is called semi gray approximation.

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Problem

Problem: A very long solar collector plate is maintained at temperature T_1 = 350 K. To improve performance at off-normal solar incidence, a highly reflecting plate is installed as shown. For a solar incidence angle of 30^0, calculate the energy received by the collector. Both the surfaces are diffuse

i) Without reflector = very poor

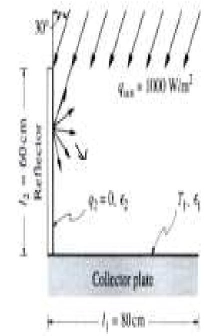
ii) Gray diffuse reflector

iii) mirror ⇒ specular reflector

iv) Non-gray diffuse

$$\epsilon_1 = \begin{cases} 0.8 & \lambda < \lambda_c = 4\mu m \\ 0 & \lambda > \lambda_c \end{cases}$$

$$\epsilon_2 = \begin{cases} 0 & \lambda < \lambda_c \\ 0.8 & \lambda > \lambda_c \end{cases}$$



We will solve one problem to demonstrate how a non gray surface basically can perform or increase the efficiency of a collector. Now this problem of solar collector we have been doing we have solved it for a number of cases. We have seen the solar collector problem on without reflector. So, this problem we have seen that without reflector it performs poorly well very poorly without the reflector the efficiency is very poor.

We have also saw this problem with the gray and diffuse reflector so we have done that gray diffuse reflector and we have seen that in previous lectures that once we put a great diffuse reflector some part of the energy is actually reflected towards the plate while some part of the energy is reflected back into the atmosphere. So, this plate great diffuse plate significantly improved the efficiency of the collector plate.

But the efficiency can further be improved by putting a mirror the advantage of putting a mirror is that it reflects only in a certain direction. That means it reflects most of its energy towards the collector plate and not towards the atmosphere. So, that is the advantage of the mirror which is basically a specular reflector. No the 4th type of modification to this problem that we are going to solve today includes non gray reflector.

So, no gray reflector it is diffuse we will assume diffuse but it is non gray so that means some part of the energy will indeed be reflected back to the atmosphere. So, first of all let me give you the governing equations we just learn.

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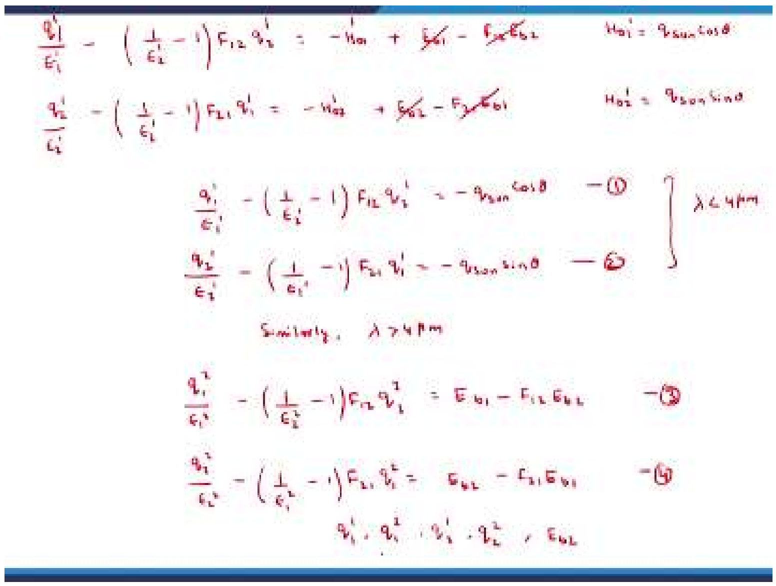

So the governing equation for this problem is q_1 or basically $\frac{q_1^1}{\epsilon_1^1} - \left(\frac{1}{\epsilon_2^1} - 1\right) F_{12} q_2^1 = -H_{01}^1 + E_{b1} - F_{12} E_{b2}$.this is the 1st equation and for surface 2, $\frac{q_2^1}{\epsilon_2^1} - \left(\frac{1}{\epsilon_1^1} - 1\right) F_{21} q_1^1 = -H_{02}^1 + E_{b2} - F_{21} E_{b1}$. so these 2 equations have been written for the 2 surfaces for the spectral range lambda (λ) > 4 micron and we know that within the spectral range both the surfaces will not remit any radiation.

That emission from the surfaces will be in the wave length range > 4 micron. So, these quantities can be neglected and H_{01}^1 is simply = $q_{sun} \cos\theta$ and H_{02}^1 = $q_{sun} \sin\theta$. so our equation now simplifies to basically $\frac{q_1^1}{\epsilon_1^1} - \left(\frac{1}{\epsilon_2^1} - 1\right) F_{12} q_2^1 = -q_{sun} \cos\theta$ that is called as equation 1 and $\frac{q_2^1}{\epsilon_2^1} - \left(\frac{1}{\epsilon_1^1} - 1\right) F_{21} q_1^1 = -q_{sun} \sin\theta$,this is equation 2 and these equations are for lambda (λ) < 4 micron spectral range.

Now similar equations similarly for lambda (λ) > 4 micron we can write it as q_1^2 (remember the super script here represents spectral range) $\frac{q_1^2}{\epsilon_1^2} - \left(\frac{1}{\epsilon_2^2} - 1\right) F_{12} q_2^2$ (the view factor does not change) = $E_{b1} - F_{12} E_{b2}$, so here E_{b1} and E_{b2} are non 0 because 2 surfaces will emit and absorb within this spectral range while the solar radiation coming from outside if taken 0 very less contribution with their in this spectral range and similarly for surface 2: $\frac{q_2^2}{\epsilon_2^2} - \left(\frac{1}{\epsilon_2^2} - 1\right) F_{21} q_1^2 =$ E_{b2}- F_{21} E_{b1}.

Let us call this equation 3 and this equation 4, so we have 4 equations and we have unknowns here 4 unknowns $q_1^1, q_1^2, q_2^1, q_2^2$ and there is another unknown here E_{b2} so we need a one more equation. So let us see how we will solve this so solve we will take a special case.

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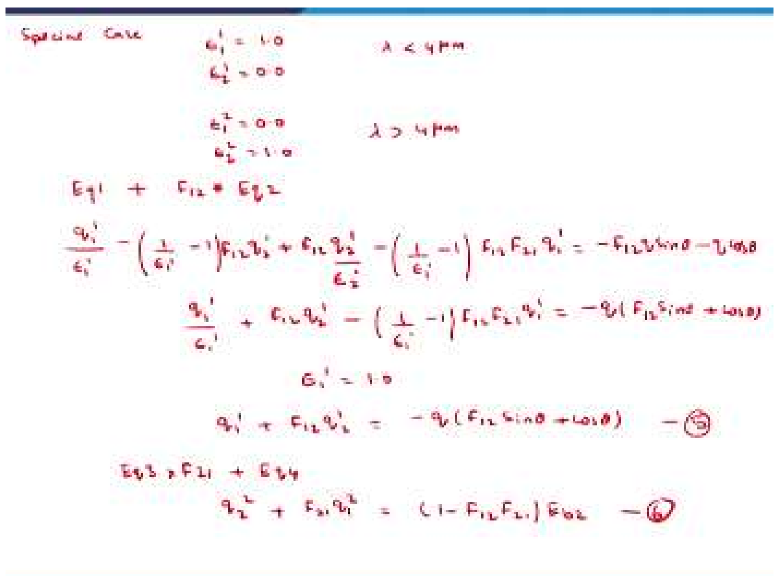

ϵ_1^1 =1.0 and let me just ϵ_2^1= 0.0 for lambda (λ) <4 micron and ϵ_1^2= 0.0 and ϵ_2^2= 1.0 for lambda (λ) > 4 micron that means surface act as a perfect black body perfect absorber or they act as a perfect reflector surface 1 act as a perfect absorber for lambda (λ) <4 micron and it act as perfect reflector for lambda (λ)> 4 micron so we will put this values in the governing equation.

So, let us add equation 1 + F_{12} times * equation 2 and we get when we do that $\frac{q_1^1}{\epsilon_1^1} - \left(\frac{1}{\epsilon_2^1} - 1\right) F_{12} q_2^1 + F_{12} \frac{q_2^1}{\epsilon_2^1} - \left(\frac{1}{\epsilon_1^1} - 1\right) F_{12} F_{21} q_1^1 = -F_{12}\, q \sin\theta - q \cos\theta$, where q is basically = $q_{sun.}$ simplification gives you $\frac{q_1^1}{\epsilon_1^1}$ + $F_{12} q_2^1 - \left(\frac{1}{\epsilon_1^1} - 1\right) F_{12} F_{21} q_1^1 = -$ q(F_{12} sin$\theta + \cos\theta$).okay, so this is what we have got we can put now ϵ_1^1= 1.0. So, we get q_1^1 + F_{12} q_2^1 and (then 2nd the quantity becomes 0) = $-$ q(F_{12} sin$\theta + \cos\theta$).

So, let us call this equation as equation number 5 now similarly we will write equation 3 * F_{21}+ equation 4, adding these 2 equations we get $q_2^2 + F_{21}\, q_1^2$= 1- F_{12} F_{21} E_{b2} let us call this equation as number 6 so now you should note that.

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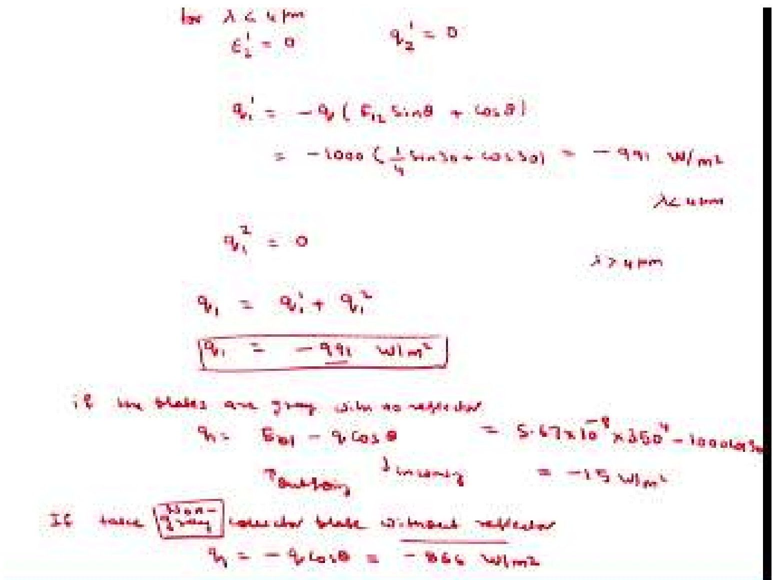

For lambda (λ) < 4 micron the reflector plate ϵ_2^1 = 0 that means it is a pure reflector whatever energy whatever solar energy falls on this reflector plate all of this energy is reflected back and we get q_2^1 = 0 so in this expression q_2^1 will be = 0 and we get q_1^1 = -q(F_{12} sinθ + cosθ), and this value given q = - 1000 [(F_{12} is) 1/4 sin 30+ cos 30] and this value comes out to be -991 W/m^{2}in the range lambda (λ) has 4 micron.

The energy absorb or the energy flux on collector plate is -991 W/m^{2}and how about q_1^2. Now q_1^2 is energy exchange with the collector plate in the spectral range lambda > 4 micron. In the 4-micron region there is no solar radiation coming and that is why q_1^2 will be simply = 0. q_1^2 is simply = 0 and that is why we get q_1 = $q_1^1 + q_1^2$ = -991 W/m^2, so this is very high factor efficiency for the semi gray approximation.

If the plates are not semi gray if the plates are gray that means they absorb and they emit radiation or all over the spectral range. Then what we will get is q_1 = E_{b1} (now the energy will be emitted by the plate) – q cosθ. so outgoing $-$incoming and this will be equal to 5.67 * 10^{-8} (Stefan

Boltzmann constant and the temperature of the surface is) 350^4 – 1000 cos30 and this value comes out to be -15 W/m^2, which is negligible.

So, just by making the plates semi gray we have increased the collector efficiency significantly. On the other hand, if we take gray plate gray collector plate without the reflector then our flexibility q_1= (simply) – q cosθ and that will be = -866 W/m^2so this is non gray collector plate without reflector and this grays are with no reflectors. So, both the cases if you do not assume reflector does not create that much difference.

The biggest difference in the character efficiencies coming by making the collector paid as non gray. Just by making the plate non gray we have increased the efficiency from - 15 watts per meter squared to – 866 W/m^2without reflector and -991 W/$\text{m}^2$with the reflector. So, this was an example on semi gray approximation method for non gray plates there is another approximate method and the method is called band approximation. So, here what we do is we in the previous example we have a typical case.

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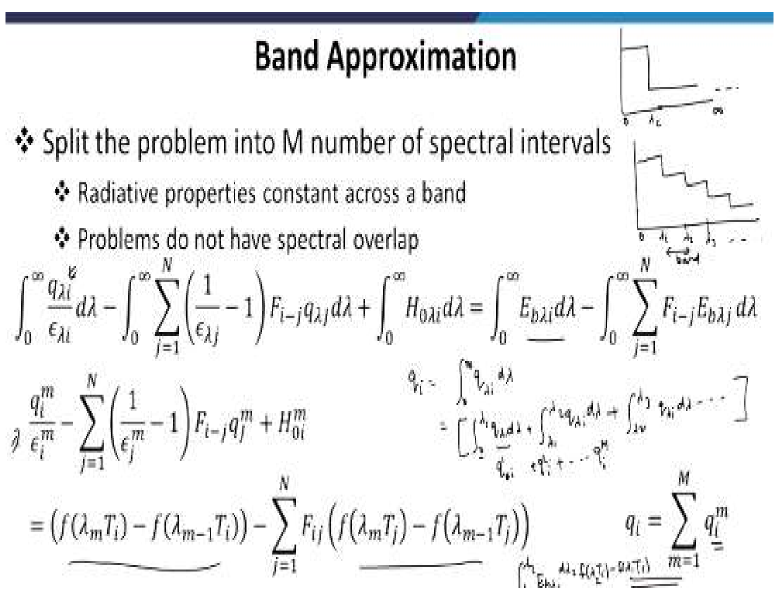

Where emittance was constant over only 2 spectral ranges so we have what we call 0 to lambda (λ) critical or let us say λ_c and then λ_c to infinity. We have only 2 ranges so we could apply the semi-gray approximation. Now it may happen many times that the wave length region may not be so benign of emittance value may actually vary like this or approximated like this. So let us call this 0 to $\lambda_1, \lambda_2, \lambda_3$and so on.

So, what we have in this case is basically what we call band approximation so in different bands so λ_1 to $\lambda_2$is basically a band spectral band. So, in different bands we have a gray value or a constant value of emittance. But there may be more than a 1 or 2 such bands. So, there may be multiple spectral bands so let us say we have m number of such spectral bands spectral intervals.

So, what we do is we take the equation of radiative heat transfer for non gray plates that is the spectral heat flux for the equation of spectral heat flux. And what we do is we integrate this equation our wavelength region 0 to infinity. Now what we do is basically we have to find out the total heat flux which is basically equal to the spectral heat flux integrated over all the wave length region.

Now what we have done is we have divided this integral [$\int_0^\infty q_{\lambda_i}\, \mathrm{d}\lambda$]into number of integrations 0 to let us say λ_1, [$\int_0^{\lambda_1} q_{\lambda_i}\, \mathrm{d}\lambda + \int_{\lambda_1}^{\lambda_2} q_{\lambda_i}\, \mathrm{d}\lambda + \int_{\lambda_2}^{3} q_{\lambda_i}\, \mathrm{d}\lambda$] and so on. There may be such bands so what we have done is we have split the interval or in their integration the spectral integration into number of small intervals.

Now each of these intervals do not overlap that is the time conditions 0 to λ_1, λ_1 to λ_2 these spectral intervals do not have any overlap. So, once we do that we can assume that the q_{λ_i} is independent of lambda for each spectral interval. That means within each spectral interval the flux q does not depend on the wave length. So, this can be just taken out of the integral.

And we can write down this as q_i^1 or lets us use a superscript q_i^1 + q_i^2 + and so on and then q_i^{M} so our heat flux basically summed over all the spectral intervals. So q_i total heat flux is basically sum M= 1 to N, M spectral intervals or bands and the average heat flux over each band so that is what we have basically done. So, the heat flux q_{λ_i} can be written as $q_i^{\mathrm{m}}/ \epsilon_i^{\mathrm{m}}$. Now the emittance also is constant over each spectral interval.

So, it can be taken out of the integral and we have on the right hand side $E_{b\lambda i}$ now once we have to calculate the q_{λ_i} over the spectral interval or band from λ_1 to λ_2, this non-dimensional or parameterized black body function appears. So, $E_{b\lambda i}$ integrated from λ_1 to λ_2 can be written in terms of difference between the parameterized black body emission power emission powers.

So, this is λ_1 to λ_2, $E_{b\lambda i}$ is simply equal to f function at (λTi) – f function ($\lambda 1$Ti). that is the one thing we have incorporated in this equation. So, our final equation for this band approximation becomes [f (λ_mTi) - f ($\lambda_{m-1}T_i$)] - j = 1 to N, similar thing for all the emitting surfaces. f (λ_mTj) where Tj is now the temperature of the surface j - f ($\lambda_{m-1}T_j$.

So, this represents the emission power within that spectral interval so this is how we can apply the band approximation method to solve non gray problems. Semi gray method was relatively easy band approximation method because of large number of bands. Maybe become a little more complicated than the semi gray approximation. So, thank you for your patients in the next lecture we will discuss about combined mode heat transfers.

We had conduction and convection is combined with radiation and we will see how in different applications this combined more heat transfer maybe useful and how to evaluate the governing equations for these types of problems. Thank you.