**Radiative Heat Transfer**
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**Lecture - 11**
**Network Analogy**

Welcome friends in last 2 lectures we discussed radiative heat transfer between black and gray surfaces. For a problem involving 1 or 2 surfaces the solution method is relatively straightforward and one can solve analytically, but it is very difficult to keep track of the equations when the number of surfaces involved in radiative heat transfer are large.

In this chapter, in this lecture we will discuss a method based on electric networks to solve the radiative heat transfer problem in a gray enclosure where the enclosure is divided into a number of surfaces.

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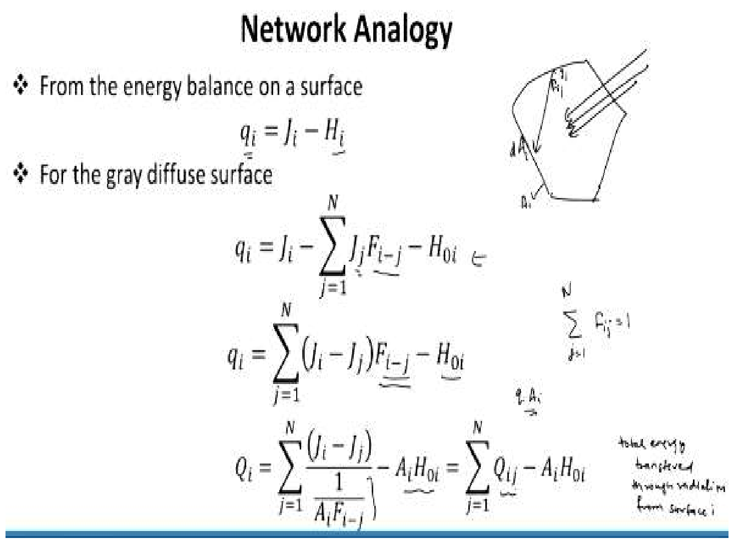


So the discussion of this method starts from the same equations that we discuss in the previous lecture on gray surfaces. So we write an energy balance on a surface, we have an enclosure and on surface element we can write the energy balance and the flux qi is basically the difference between the radiosity which includes the emission from emittance and reflection – total irradiation. (okay)

So this is the heat flux or energy balance on surface i, qi = Ji – Hi, now for the great diffuse surface we have to substitute for irradiation Hi. So irradiation can be written as summation over radiosities from all the surfaces. So summation j = 1 to N, Jj and the view factor between the 2 surfaces Fij, so Fij and Jj okay, - H0i which is the radiation coming from outside okay, this is outside radiation, which may be coming from some outside source okay.

So qi = Ji – summation Jj Fij – H0i okay. We can also write this expression as qi = summation j = 1 to N, Ji Jj reason being the summation rule, summation i j = 1 to N, Fij = 1. So using this summation rule and substituting it in this equation we can write down qi in terms of difference between the radiosities between 2 surfaces. So Ji – Jj is the difference in radiosities between the 2 surfaces and multiplied by the view factor between the 2 surfaces.

We can write down the heat flux on surface i and definitely the outside radiation H0i. Now what we do is the total energy transferred through radiation from surface i okay, so from surface i the total amount of energy transferred through radiation can be written as qi times Ai so we multiply by the area Ai and we write it as Qi. So Qi = Ji Jj and we have taken the view factor in the denominator, we write this as 1/Ai Fij – Ai $H_{0i}$ okay.

Or we can also write this as summation J = 1 to N, $q_{ij}$ now this is the total energy transfer between 2 surfaces. So Qij is the total energy transfer between the 2 surfaces i and j – Ai H0i. Now there is a specific reason why we have written these equation in this particular form okay. So it will be clear on the next slide.

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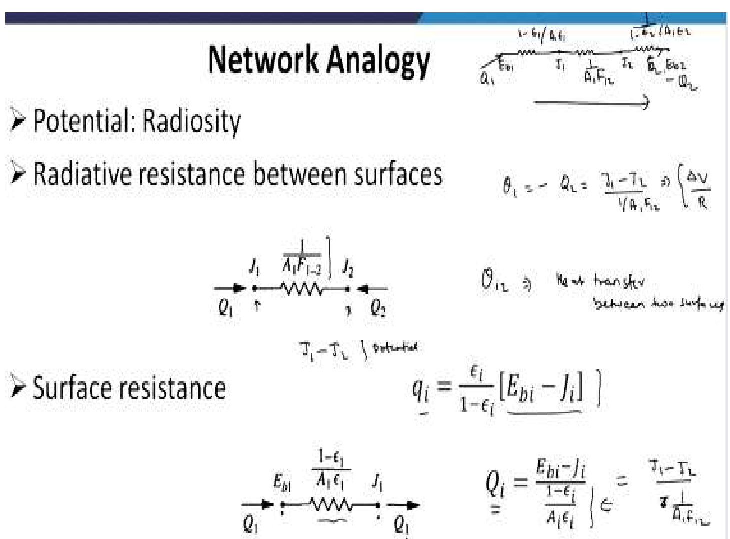


So $q_1$ or so this is the heat transfer between 2 surfaces okay. So definitely if one body is gaining energy, the other body maybe losing the energy and the direction of $q_1$ and $q_2$ are opposite to each other, if $q_1$ is losing energy, $q_2$ will be gaining energy okay. We have this factor $1/A_1 F_{12}$ okay, this factor A basically defined as radiative resistance between the 2 surfaces okay.

And $J_1$ is radiocity and $J_2$ is radiosity, we also define $J_1 - J_2$ as potential okay or radiative potential. So what we have basically done is, we have used the concept of electric networks to define this problem okay. So we have radiosities of the 2 surfaces as potential. The difference between radiosities is the potential difference and the view factor $1/A_1F_{12}$ as the radiative resistance okay.

If we know the radiosities using the method of electrical analogy, we can calculate the current or heat transfer $Q_1 = -Q_2 = J_1-J_2$ upon this resistance $1/A_1F_{12}$. So this is the potential we basically write in electrical analogy as delta V and this is the resistance we write in electric analogy. So V = IR, so here also we have the potential difference $J_1-J_2$ divided by the radiative resistance 1 upon $A_1F_{12}$ okay.

Now we have another equation that relates back body emissive power and radiosities Ji to the heat flux. So qi = $\epsilon_i$, where $\epsilon_i$ is the emittance of the surface, upon $(1 - \epsilon_i)$ and then the difference in radiative $(E_{bi} - J_i)$, the emissive power of the body and radiocity of the body okay. This particular equation can be simplified okay by writing the total energy transfer qi which is same as in the previous case okay, the energy transferred is same.

So Qi = $(E_{bi} - J_i)/(1 - \epsilon_i /A_i \epsilon_i)$, this particular term $(1 - \epsilon_i)/(A_i \epsilon_i)$, in the denominator is called surface resistance okay and definitely this is going to be same as $(J_1 - J_2)/ (1/A_1F_{12})$ okay. So the current is going to be same okay. So we write this as in electrical analogy, the electric network methodology, the two points are at potential $E_{b1}$ and $J_1$, $E_{b1}$ is the black body emissive power, $J_1$ is the radiosity.

The resistance is $1 - \epsilon_1/ A_1 \epsilon_1$, this is the called surface resistance and $Q_1$ is basically the current or the heat transfer okay. We can combine these 2 resistances into single network like this. So we have $E_{b1}$ followed by a resistance of $1 - \epsilon_1/ A_1 \epsilon_1$, this we have is $J_1$ and then we have radiative resistance of $1/A_1F_{12}$ and we get to $J_2$ and then again we may have $E_{b2}$.

Again we have, $(1 - \epsilon_2)/(A_2 \epsilon_2)$ okay and if there is no other surface, the current throughout is going to be the same okay. If $Q_1$ is current here, so this current will be uniform here okay. So it will be $-Q_2$, where $Q_2$ has direction in the reverse direction okay. So for the 2 surfaces, if there are only 2 surfaces the current is going to be uniform because it represents the same network connection okay.

There is no parallel connection into this network. So it is just a single serial connection or current is going to be the same or radiative transfer is going to be the same.

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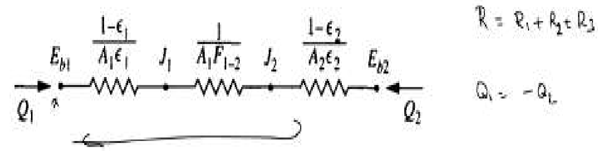
## Network Analogy

- For infinite parallel plates :

$$Q_1 = \frac{E_{b1} - E_{b2}}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{1-2}} + \frac{1-\epsilon_2}{A_2\epsilon_2}} = -Q_2$$




- Denominator represents total radiative resistance between surface $A_1$ and $A_2$

So for infinite parallel plates we can combine the resistance in series okay as i have drawn there, this network, we have 3 resistances, the 2 resistances are surface resistance, one resistance is radiative resistance as is also show here okay. So 3 resistances can be added together, so combined resistance are in series is $R_1 + R_2 + R_3$ in series.

So we have added the 3 resistances and then the total potential difference between the 2 ends $E_{b1}$ and $E_{b2}$ and the current or radiative heat transfer $Q_1$ at this end is simply the potential difference $(E_{b1} - E_{b2})$ divided by the total resistance or at the other end it will be the same because it is the same network, no parallel connection. So we have $Q_1 = -Q_2$ okay. So this is the how we can basically apply the method of electric network to solve simple problems okay.

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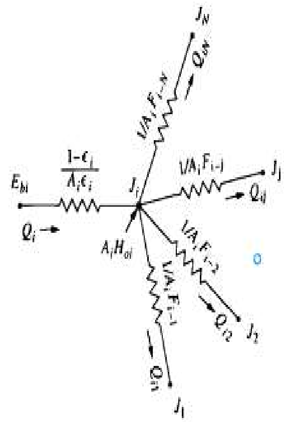
## Network Analogy

- Total heat flux at the surface $i$:

$$Q_i = \frac{E_{bi} - J_i}{\frac{1-\epsilon_i}{A_i\epsilon_i}}$$

$$= \sum_{j=1}^{N} \frac{J_i - J_j}{\frac{1}{A_i F_{i-j}}} - A_i H_{oi}$$

$$= \sum_{j=1}^{N} Q_{i-j} - A_i H_{oi}$$




We can have multiple surfaces in an enclosure, we can represent the enclosure with the finite area surfaces, there may be n surfaces in all. So for these n surfaces we can define junctions okay. So we have in this particular network, the current that is reaching at this node we can write this current as the difference between the potential across this resistor. So $E_{bi}$ – Ji is the potential difference and then the surface resistance $(1 - \epsilon_i)$/(Ai $\epsilon_i$).

Now this current or heat transfer between these junction has to satisfy the summation law of the current that means the radiative heat transfer at this point should be sum of the radiative heat transfer in all the networks okay. So we write using the summation law J = 1 to N. The potential difference in each arm Ji – Jj divided by the radiative resistance of each arm or each network okay.

Now the surface may be exposed to external radiation from sun or some other source outside the enclosure, that will be added separately to the radiosity. So we have added the component of external radiation separately at the radiosity of surface i okay. So total heat transfer from surface i can be written as Qij, where Qij is the radiative heat transfer between surface i and j. All the surfaces we have taken into account and then extra radiation from the outside source.

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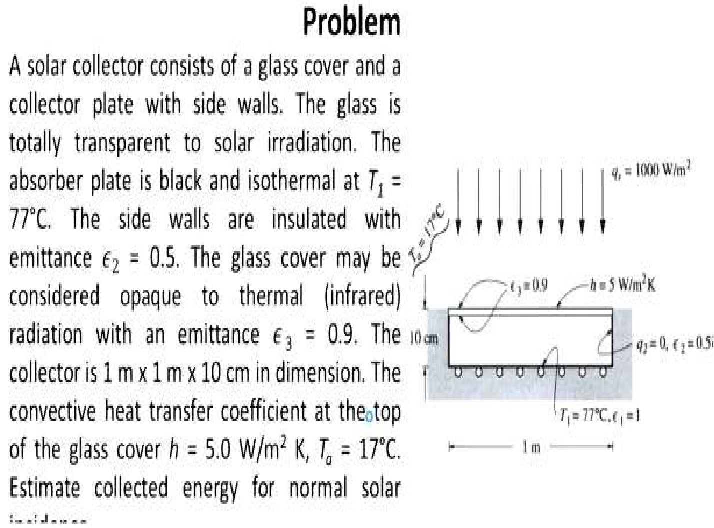


So this is a very simple method that basically allows us to solve complicated problems having multiple surfaces, multiple gray surfaces and we found to solve for heat flux or unknown blackbody emissive power of a particular surface. This method can be applied to find out potential that is the temperature or current that is the heat flux in any arm or any surface we can apply this method.

So we will apply this method to this problem, in this problem what we have is again a solar collector. The solar collector basically consists of collector plate okay, the plate has a temperature of 77 $^0$C, this is the collector plate. It has emittance. It is black, the emittance is 1. Now as we have already discussed in previous lecture we want solar collectors to absorb maximum radiation, but we also want that they should not emit significant amount of radiation okay.

So what we basically has been done, in this case the solar collector has been covered with the glass which is totally transparent to solar radiation okay. So we have this is the glass plate okay. The solar radiation is falling vertically with an intensity of 1000 W/m$^2$, that atmospheric temperature is given as 70 $^0$C. There are 2 side plates also, these are the 2 side plates on the collectors which are insulated and have an emittance value of 0.5 okay.

The reflectance of the glass plate is given as 0.9, the dimension of the collector is 1 m x 1 m and 10 cm is the height of the side plates. In addition to radiative heat transfer, we have also convective heat transfer in this problem, although this is the point of discussion of a different lecture, we will combine radiation with convection, but here we have a convection on the top surface. So top surface have convection with the transfer coefficient of 5 $W/m^2$ K.

Any effect of free convection inside the enclosure, so free convection inside the enclosure is neglected. What we have to find out is total amount of energy collected by the collector with this glass cover and without glass cover. So let us solve this problem using the electric analogy. So first we will draw the network and then we will basically solve this problem. So let us draw the network first.

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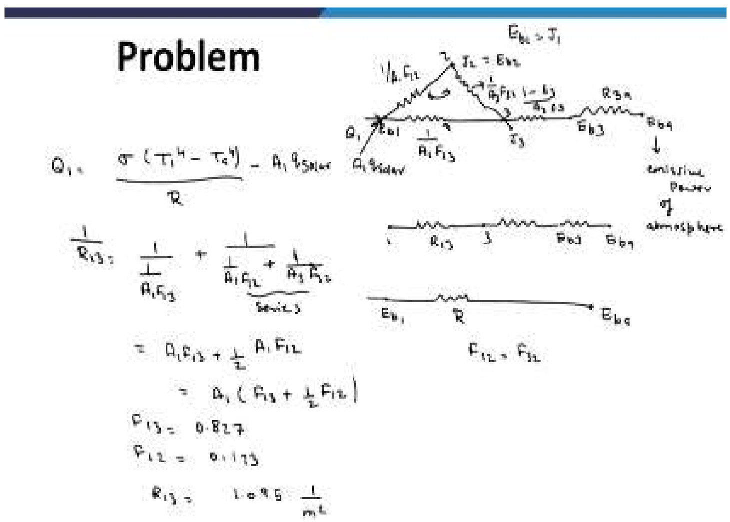


So we have surface 1 which is at the bottom and let us call this $E_{b1}$, it is black, so there is no difference in radiocity and the emissive power. So $E_{b1} = J_1$ that is they are equal, because the surface is black okay. These are the side plates, we call it $J_2$ okay, here also we have $J_2 = E_{b2}$. There is no potential difference because the plates are insulated. So for 2 cases we have $E_{b1} = J_1$ and the surfaces are black and when the surface is insulated.

So the surface 3 or side plate surface 2 is insulated and that is why $J_2 = E_{b2}$, there is no flux or current in this arm. This is $J_3$ and this is connected by the radiative resistance $A_1$, $F_{13}$ okay and then this is $E_{b3}$ and this value is the surface resistance $(1 - \epsilon_3)/(A_3\, \epsilon_3)$ okay. Now for a radiative problem, this network is good enough okay. We have radiation from sun.

So for radiative heat transfer this problem is good enough, but we have convection also. So we have to draw another network here. So we will draw it here from the top surface where $E_{ba}$ is the emissive power atmosphere, which is maintained at 17 $^0$C okay and this resistance is $R_3$ let us say atmosphere okay. So this is the network for the problem. Now we will solve this problem for the heat flux $Q_1$.

Because we are interested in how much energy is collected, so $Q_1$ can be written as the total amount of energy transfer from surface 1 that is the collector = $\sigma(T^4 - T_a^4)$. So we are basically dealing with this point and this point okay. This is the network, we can just draw it again $E_{b1}$ and then this is the combined resistance and $E_{ba}$ okay. This we can call as R okay.

So we are trying to simplify the above network with the simplified network where we have multiple resistances in parallel and series. The end to end potential difference is ($E_{b1}$ – $E_{ba}$), that is = $\sigma(T^4 - T_a^4)$, that is end to end potential difference and then R okay, this is the amount of flux and then we have also the radiation from the sun okay. Outside radiation we have taken it separately.

Let me just show you this point once again, so we are taking the external irradiation separately okay, that is what we have done here also. So A1 is the area of the collector and Qs is the Q solar is the solar radiation which is 1000 $W/m^2$ okay. Now we first try to find out this value of R okay. So let us see, so first of all let us call this point 3, this is point 2. So is the resistance effective resistance between point 1 and 3.

So what we have is these 2 are in series and this is in parallel okay. So 1/R13 = (1/A1 F13) okay + (1/A1 F12) + (1/A3 F32) okay. So these are then the series okay, this is the resistance which is = (1/A1 F12) and this resistance = (1/A3 F32) okay. So these 2 resistances are in series that is why we have just added them and then taken inverse and these 2 are combined with A1 F13 which is in parallel.

So this is that effective resistance and we can just simplify this as A1 F13 + (1/2) A1 F12 where we have assumed or observed that F12 = F32 because of the symmetry in nature okay. So F12 = F32 okay, so we have (1/2) F12 and this becomes = A1 F13 + 1/2 F12 okay. Now we can find out the values again using the view factor tables. The value of F13 can be calculated and it comes out to be 0.827 and value of F12 = 0.173 okay.

And with this the resistance R13 = 1.095 1/meter square. So note down the dimension of the radiative resistance. It has units of 1 upon area, so 1 upon meter square. So value of R13 is

1.095 1/meter square okay. So with this out network looks like this, this is 0.1, this is 0.3, this is R13 okay and then we have another resistance to Eb3 and then another resistance to Eba. Now this is a serial network, so we will solve this. So the resistance R3a is little complicated.

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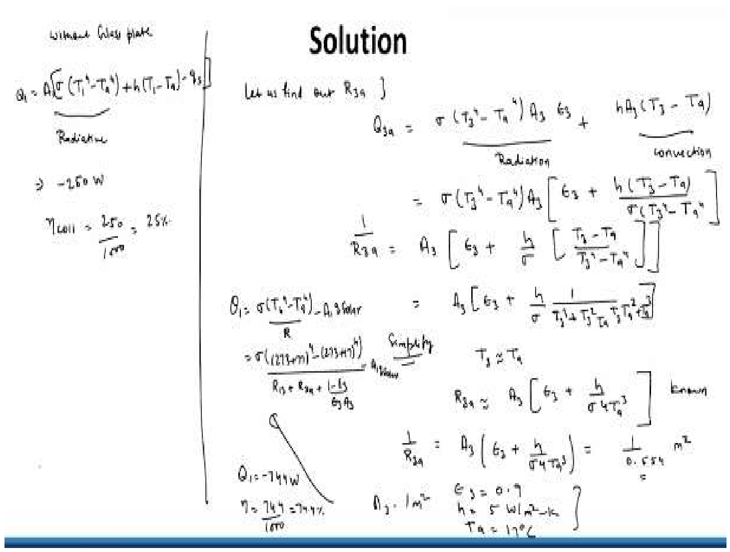


So first of all let us find out R3a, now this point we will understand in a different lecture, basically here we have combined radiative and convective heat transfer and the resistance R3a includes the fact of radiation as well as convection. So it is important that we write an energy balance and try to find out this resistance. So the heat transfer through convection and radiation from the top surface = $\sigma(T_3^4 - T_a^4)$, okay A3.

This is due to radiation, so we are writing the energy balance on the top surface, top plate, top surface, that is exposed to the sky, not that surface of the top plate which is exposed to the cavity okay. So this is the top surface. So this is the radiation part. The heat transferred from the top surface through radiation and definitely we have to multiply by epsilon 3, the emittance plus the convection part which will be = $hA_3$ ($T_3$- $T_a$) okay.

This is the convection part okay. So we write this as in terms of radiative potential we have to write. So write it this as $\sigma(T_3^4 - T_a^4)$, okay [A3$\epsilon_3$ + h ($T_3$ – Ta)/ $(T_3^4 - T_a^4)$ okay. So this is what we have basically defined okay. Now we define the resistance( R3a) = A3 [$\epsilon_3$ + (h/ $\sigma$) (T3 – Ta)/ $(T_3^4 - T_a^4)$] okay.

So this is the complicated expression for the resistance combined radiative and convective resistance at the top surface okay. Now we have to simplify this expression A3 [$\epsilon_3$ + (h/ $\sigma$) (T3

– Ta)/ $(T_3^4 \; - \; T_a^4)$]. So we write this as = A3[$\epsilon_3$+ h/sigma. Now T3 – Ta we taking the denominator and this becomes = 1/($T_3^3$+ $T_3^2$Ta +T3 $T_a^2$ + $T_a^3$) cube okay. So this is the thing. As compared to the other resistances like radiative resistance and surface resistance which is a surface property or property of orientation of the 2 surfaces.

This depends on temperature okay. Through the convective heat transfer coefficient. So this resistance is the function of temperature. So we have to kind of simplify this. So how do we simplify this. So we simplify here by assuming that the temperature difference between atmosphere and this glass plate is not going to be significant and it is not going to affect the resistance R3a in a big way.

So we assume this value and we just approximate R3a as A3 [$\epsilon_3$ + h/$\sigma^4 T_a^3$] okay. Where atmospheric temperature is known. So all the quantities now are known okay so this is now known okay, otherwise the problem cannot be solved by hand, you will have to incorporate computer to solve this problem because the resistance will then become the function of temperature $T_3$ which is not a known variable okay.

So from this we can calculate 1/R3a as = so this is actually I did a mistake here, so this is 1/ R3a, so 1/R3a = A3 [$\epsilon_3$ + h/$\sigma^4 T_a^3$]. $\epsilon_3$ value is given as 0.9 okay. The value of h is given as 5 watt per meter square kelvin and Ta is given as 17-degree C. So we substitute these values okay in to this A3 is given as 1-meter square. So all these value we substitute and we get the value of 0.554-meter square.

So this is the resistance between the top surface combining radiation and convection okay. Now we have all the variables, we have this resistance, we have R13 okay. So we can combine this into single and calculate the value of heat flux using this expression. So the value $Q_1$, I will just solve it here. $Q_1$ = from this expression, $Q_1$= [$\sigma(T_1^4 \; - \; T_a^4)$/ the combined resistance R] – A1$q_{\text{solar}}$ okay.

So $\sigma T_1^4$, T1, the temperature of the plate we know okay, that is 77 degree C. So $\sigma$ (273 + 77) power 4 – the 17 degree (273 + 17) power 4 okay, combined resistance, now they are in series okay. So we have to just add the 3 resistances [R13 + this resistance (1 – epsilon3/A3 epsilon

3) and R3a] and when we have to multiply by A1 and $q_{\text{solar}}$ okay. Now all these quantities we know R13 we have already calculated.

R13 value is 1.095 over meter square, R31 we have already calculated 0.554, 1 upon 0.554 and epsilon 3 is also known 0.9. So we have all the quantities Q solar is 1000 watt per meter square, we substitute the values and we get Q1. So this will be -744 watt and the efficiency of the collector we can define eta as 744/1000 because the total amount of solar radiation coming is 1000.

So the efficiency is 74.4%, which is reasonably very good okay. Now if we do not use this glass plate over the collector how much energy will be basically absorbed okay. So I will just do, so without glass plate the energy balance Q1 will be = the radiation part A1 σ( $(T_1^4 - T_a^4)$, this is the radiative energy balance and then the convection part h (T1 – Ta).

Now the collector plate will be directly exposed to the environment with the same heat transfer coefficient that we have assumed – $q_{\text{solar}}$ okay. So the area will be common okay. So we substitute the values here and we get the value as -250 watt. So collector efficiency is simply = 250/1000 which is = 25% okay. So compared to 75% with the glass plate the efficiency has reduced significantly to 25% okay.

So that is why the glass, the solar collectors are always constructed in such a way that they have 2 layers of materials one, which absorbs radiation and another surface which basically has little bit of transparency into it. So thank you for your kind attention. I end this lecture on the network methods. In the next lecture we will study some more methods, some more accurate methods to solve radiative heat transfer between black and gray surfaces.