Radiative Heat Transfer
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Lecture – 10
Radiative Heat Exchange between Diffuse Gray Surfaces

Hello friends, in the previous lecture, we studied the radiative heat transfer between black surfaces, now in today's lecture we will study radiative heat transfer between gray diffuse surfaces just by converting the surfaces from black to gray, the problem of radiative transfer becomes much more complicated as you will see in the derivation of mathematical expressions, the problem involves not just algebraic equations but rather integral equations.

So, this class of problem is significantly difficult to solve as compared to the radiative heat transfer between black surfaces.

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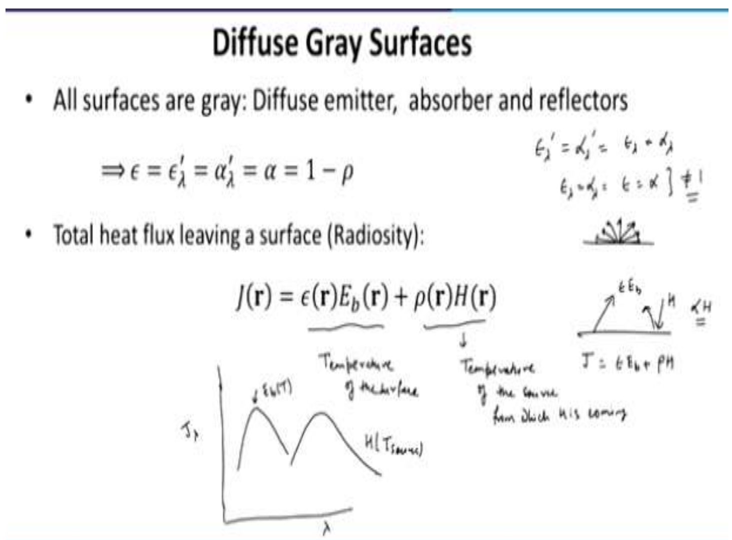

So, diffused gray surfaces, we have already studied the classification of the surfaces diffuse as we have already studied means, it emits and absorbs radiation equally in all directions that means,

$$\epsilon = \epsilon'_\lambda = \alpha'_\lambda = \alpha = 1 - \rho$$

so this is a diffuse emitter and absorber, the emittance and absorptance does not depend on direction, it is uniform in all the directions.

And the surfaces are gray that means, the emittance and absorptance do not depend on wavelength also so, we have emittance = epsilon(ϵ) and absorptance = alpha (α), they are equal as per

Kirchhoff's law but they do not depend on direction or wavelength, so this is we called diffused gray surfaces and this

ϵ and != 1

if it is = 1, then it will become black, they these surfaces are rather called gray.

They have emittance and absorptance value != 1 okay, so when we have emittance ! =1 or < 1, the amount of energy emitted by a surface is simply epsilon times the black body emissive power E_b, so this is the amount of energy emitted by a surface, okay this is going to be less than what a black body would emit and the amount of energy absorbed if let us say H is the amount of energy coming in falling onto a surface.

Then the amount of energy absorbed = $\alpha\ H$

A black body would have absorbed all the radiation, while the gray surface absorbs only a fraction of the incoming radiation and this fraction is called absorptance alpha so, we define so, part of this radiation H is basically absorbed, αH is absorbed and the rest of it 1 - αH is basically reflected back.

So, we define a quantity called radiosity J, as the sum of energy emitted by the surface and also energy reflected by the surface, so the total amount of energy leaving surface is basically defined as the radiosity, so it contains contribution from emission and it contains contribution from reflection okay, so the first quantity definitely depends on temperature of the surface, so this will depend on the temperature of the surface.

And this quantity depends on temperature of the source from which H is coming okay, so these two are different in the sense that the temperature of one surface may be significantly different from the other, so one value of the energy emitted will be in one spectral range and the other value may be in different spectral range, so amount of energy if somebody is interested in looking at the structure; the spectral structure of the radiation then this may look like this, okay.

If you look at the amount of energy emitted versus wavelength okay, you will have this kind of structure where this could be the black body emissive power at temperature T and this could be the spectrum for H at the source temperature okay, so they may be spectrally different regions okay but for this calculation diffuse gray, the ϵ and Rho (ρ) or α are independent of wavelength, while the black body emissive power E_b and irradiation H will depend on the temperature of the respective surfaces.

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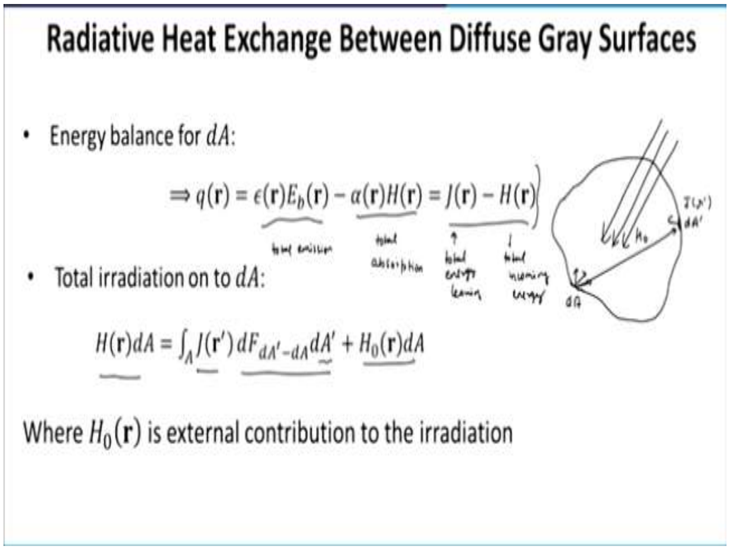

And definitely they will depend on wavelength okay, so we write energy balance for a surface dA, we take an enclosure just like we did for the black surface okay, it may have some opening from which external radiation may come in okay, let us say we have a small element here, okay let us call this dA, okay so, the total energy balance for this surface dA includes the

Energy emitted from this element = ϵE_b - energy absorbed.

Now,

$$\text{Energy absorbed} = \alpha\, H$$

now, this total irradiation H may include radiation coming from the entire enclosure as well as the radiation coming from outside H_0, okay so, it can include both the components so now, we will further simplify this equation, the same heat flux on this element dA can also be written in terms of radiosity, so radiosity is defined as total energy leaving the surface through emission and reflection.

And it is J and – H; H is total irradiation, so this is total energy leaving and this is total incoming energy of course, radiative energy, okay and this is total emission and this is total absorption okay, so we can write heat flux in terms of either black body emissive power and irradiation or radiosity and irradiation now,

Total irradiation on to dA:

$$H(\mathbf{r})dA = \int_A J(\mathbf{r}')\, dF_{dA'-dA} dA' + H_0(\mathbf{r})dA$$

as contribution from the entire enclosure just like we did in the case of blackbody.

So, we take view factor from *dA'* , let us say *dA'* is a small area on the enclosure and we have to integrate over the entire enclosure, so we multiplied by *dA'*, so we take the view factor between these 2 surfaces and $J(\mathbf{r}')$ is the radiosity from this element *dA'* okay, so this is the total amount of irradiation plus irradiation from the component coming from outside the enclosure, okay.

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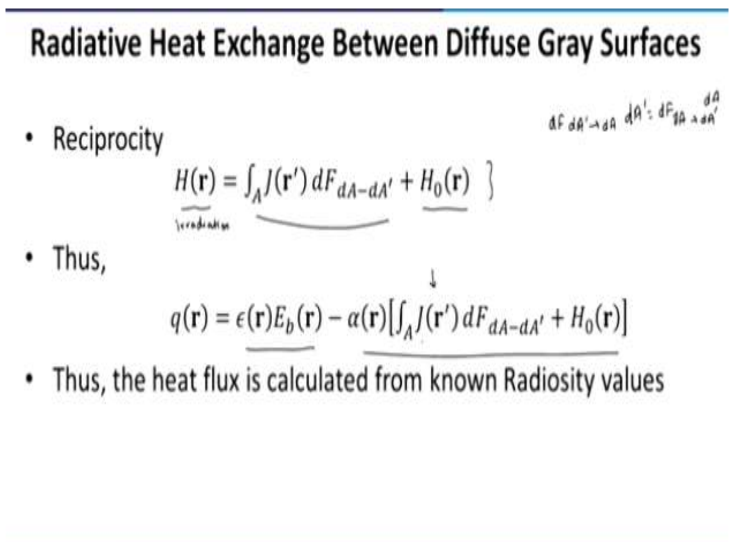

So, we substitute this value of irradiation, okay before we do that we simplify as we did in the case of blackbody enclosure, we write

$$dF_{dA-dA'}dA' = dF_{dA'-dA}dA$$

we substitute in this equation and we cancel out *dA* from both the sides, so we get the expression for irradiation, this is called irradiation at location of the small vector *dA*, radiosity integrated over the entire surface multiplied by the small view factor + outside irradiation, okay.

So, we substitute this expression in the expression of heat flux and we get this expression, so

qr = the energy emitted by the surface - energy absorbed by the surface

so this is how we can calculate the heat flux for a diffuse gray enclosure, now the difficulty in solving this expression is that it includes an unknown quantity radiosity in this integral equation, this type of equation is called integral equation.

And we have an unknown quantity J coming into this integral, so first of all we have to solve for J and then we can find out heat flux.

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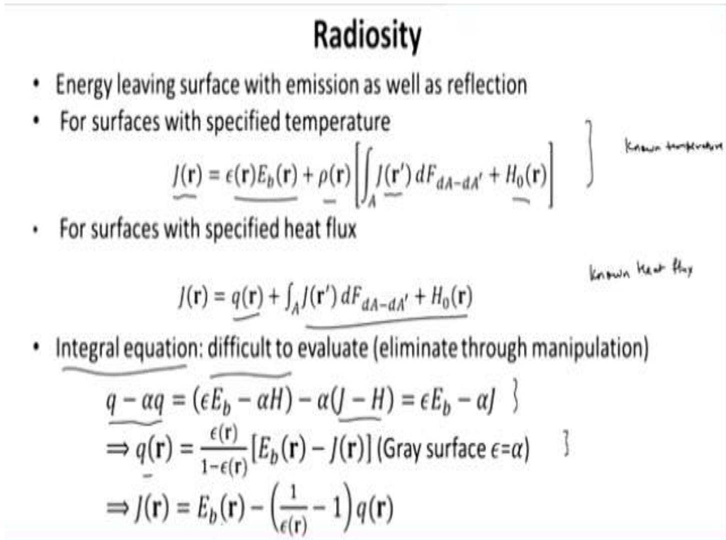

So, what we do is; we use the same expression okay, so what we do is; we use this expression okay, this equation we use okay, from this equation we can solve for the radiosity J, okay so radiosity can be written in terms of

$$q(\boldsymbol{r}) = \epsilon(\mathbf{r})E_b(\mathbf{r}) - \alpha(\mathbf{r})H(\mathbf{r})$$

and taking the Hr on this left hand side, we can solve for radiosity. So, using that relation we can solve for the radiosity

the radiosity at any location R = ϵ times the black body emissive power plus the reflected part of the radiation and outside radiation.

$$J(\mathbf{r}) = \epsilon(\mathbf{r})E_b(\mathbf{r}) + \rho(\mathbf{r})\left[\int_A J(\mathbf{r}')\, dF_{dA-dA'} + H_0(\mathbf{r})\right]$$

So, what we have just done is basically, substituted the value of H in this relation, okay we have substituted the value of H in this relation and solve for $J(\mathbf{r})$ okay, so once we solve for $J(\mathbf{r})$, we get the results for radiosity now, if on any surface the heat flux is specified rather than temperature, let us say there is a surface out in the enclosure or some element in the enclosure for which the; for which the temperature is not known rather the heat flux is known.

Then we can write down the radiosity in terms of heat flux also from the same equation, okay so from the same equation, the radiosity is simply $q + H(\mathbf{r})$, okay, so

$$J(\mathbf{r}) = q(\mathbf{r}) + \int_A J(\mathbf{r}')\, dF_{dA-dA'} + H_0(\mathbf{r})$$

The irradiation, so we can solve for radiosity using these two expressions, this is for known temperature and the other expression is for known heat flux but the problem is still not solved, why; this is not a normal equation, you cannot solve for radiosity just like an algebraic equation.

Because we have radiosity appearing on the left hand side but we have radiosity appearing on the right hand side and that too under an integral, these type of equations are called integral equations and they are not easy to solve, okay so it is very difficult to evaluate these equations because these are integral equations, unless and until we simplify, do some simplification it is very difficult to solve, okay.

Now, how do we solve it so, we basically write down the expression from previous, we take the same expression q =; this expression we take

$$q(\mathbf{r}) = \epsilon(\mathbf{r}) E_b(\mathbf{r}) - \alpha(\mathbf{r}) H(\mathbf{r})$$

okay and subtract the heat flux again multiplied by αq, so we solve for q - αq, okay this gives us $q - \alpha q = (\epsilon E_b - \alpha H) - \alpha(J - H)$

so from the same equation we have just; we are just trying to manipulate, okay and this simplifies to.

$$q - \alpha q = \epsilon E_b - \alpha J$$

So, from this we can solve for heat flux q

$$q(\mathbf{r}) = \frac{\epsilon(\mathbf{r})}{1 - \epsilon(\mathbf{r})} [E_b(\mathbf{r}) - J(\mathbf{r})]$$

kay so this is the expression for the heat flux, okay or in terms of heat flux, we can solve for radiosity

$$J(\boldsymbol{r}) = E_b(\boldsymbol{r}) - \left(\frac{1}{\epsilon(\boldsymbol{r})} - 1\right) q(\boldsymbol{r})$$

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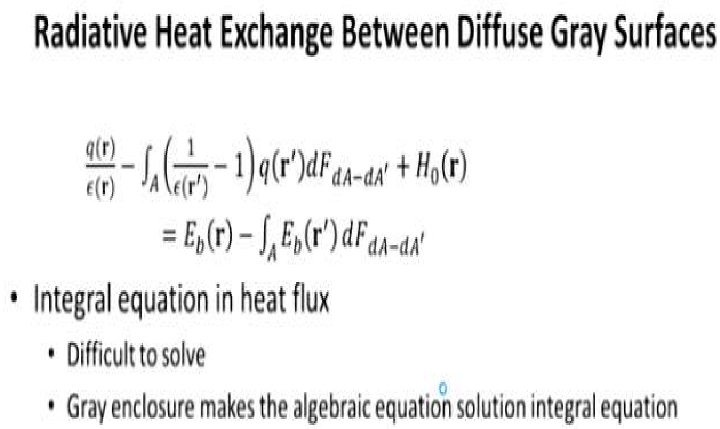

So, from this our problem has greatly simplified, what we have done is; we have found a solution for radiosity which was appearing in the integral equation okay, in terms of heat flux so, our objective is to eliminate radiosity because in calculations, radiosity is not our objective, our objective is the heat flux, so what we have done is; we have written radiosity in terms of heat flux, okay by manipulating the governing equation of this problem.

And once we have radiosity, we will substitute it back in the governing equation to calculate the heat flux, so when we put it back in this equation okay, so this is the equation for heat flux, now we have calculated J in for heat flux, we will just put it back okay, the nature of the equation is not going to change okay, the nature of the equation will remain integral equation, okay this is something you cannot simplify as compared to the blackbody radiation where we did an encounter this problem of integral equation.

The integral equation is inherent for gray problems okay so, this problem will remain, and our equation now in just q as a variable reduces to now,

$$\frac{q(\mathbf{r})}{\epsilon(\mathbf{r})} - \int_A \left(\frac{1}{\epsilon(\mathbf{r}')} - 1\right) q(\mathbf{r}') dF_{dA-dA'} + H_0(\mathbf{r}) = E_b(\boldsymbol{r}) - \int_A E_b(\boldsymbol{r}')\, dF_{dA-dA'}$$

this is not a unknown, I mean you can easily calculate this last term because E_b is known to you, if you know the temperature of the surface, you know the term and you can easily integrate it,

But this term is not easy to calculate because q is a variable and it is appearing inside an integral, so this is an integral equation okay, so let us see how we can solve problem of this type okay.

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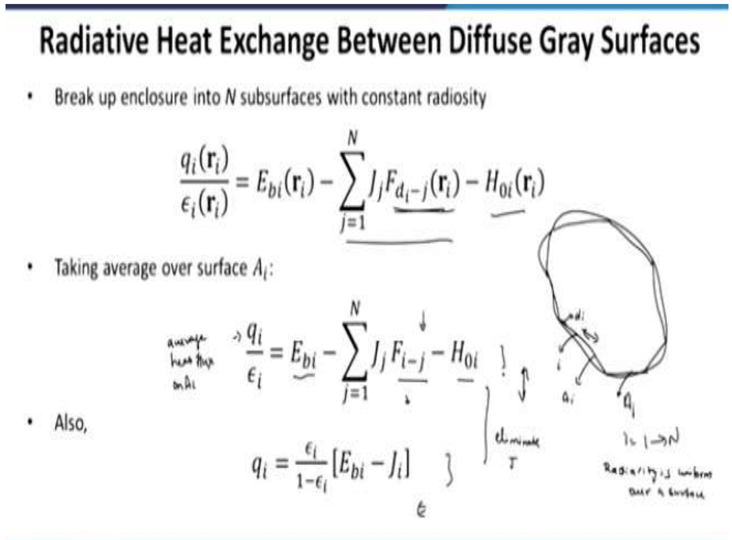

So, just like we did for the blackbody enclosure, what we will do is; we will break the enclosure into a number of flat surfaces okay, let us say *N* surfaces we will break, so this is going to be our first simplification, okay so, this is A_j where *j* varies from 1 to *N*, so we have divided the enclosure into *N* flat surfaces, okay. When we do that we also assume that the radiosity is uniform over a surface, okay.

This is just like an assumption we make in the blackbody case, where we assume that the temperature is uniform over the surface, so here we are assuming radiosity is uniform over the surface okay, so this equation, the value of E_b can be taken out okay, it is independent of the location, it is just a variable for a given surface, okay and then we have summation over all the surface, the view factor from a surface d_i, let us say this is surface *i* and this is the element d_i okay.

So, we are calculating the view factor F_{di-j} as per the argument that we made in the case of blackbody that the view factor is not uniform, you can have radiosity uniform, you can have temperature uniform over the surface but you cannot have the view factor uniform over the surface, you can have an average view factor but not a uniform view factor. So, first of all we will assume that the view factor varies over the surface.

So, we just left with F_{di} and *j* okay and minus the incoming radiation,

$$\frac{q_i(\mathbf{r}_i)}{\epsilon_i(\mathbf{r}_i)} = E_{bi}(\mathbf{r}_i) - \sum_{j=1}^{N} J_j F_{d_i-j}(\mathbf{r}_i) - H_{0i}(\mathbf{r}_i)$$

now we will take the average, so what we will do is; we just take an average over the surface A_i over the surface, this is the average heat flux on A_i, okay, average emissive power, then this is view 2 factor between finite element areas A_i and A_j, okay again, this is going to be an average of the small element view factors and then average value of the irradiation, okay.

$$\frac{q_i}{\epsilon_i} = E_{bi} - \sum_{j=1}^{N} J_j\, F_{i-j} - H_{0i}$$

So, this is the first expression basically what we have got okay, now we have another expression again coming from the same relation, from this relation we have this another expression for q, okay so

$$q_i = \frac{\epsilon_i}{1-\epsilon_i}\left[E_{bi} - J_i\right]$$

now this expression we have to basically find out, what we have to do is; eliminate J between these two equations, this is the same exercise that we are doing, we did for the integral form of the equation.

Now, we are just doing it for the summation, okay because we have divided the enclosure into many, many surfaces, so we have to eliminate J_i between these 2 equations and once we have what we get is basically an equation in just heat flux, okay so, we have eliminated using this equation, substitute the value of J_i into this expression, okay in terms of q_i, okay.

$$\frac{q_i}{\epsilon_i} - \sum_{j=1}^{N}\left(\frac{1}{\epsilon_j} - 1\right) F_{i-j}\, q_J + H_{0i} = E_{bi} - \sum_{j=1}^{N} F_{i-j} E_{bj}$$

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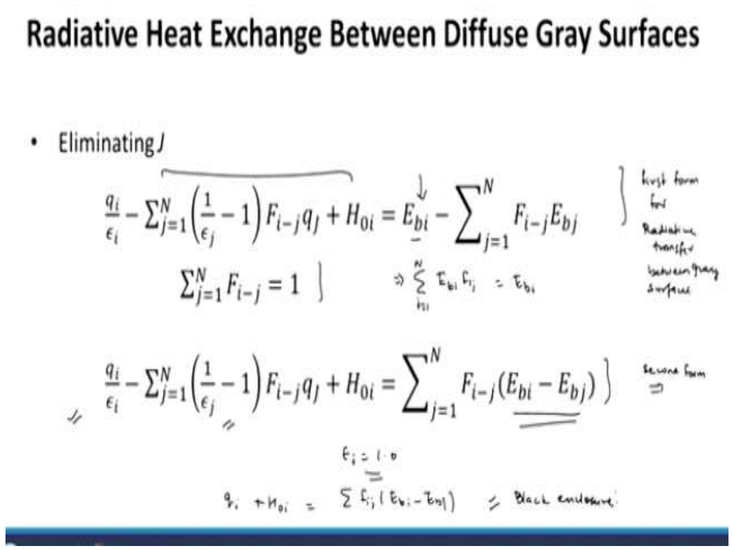

And what we get is basically this kind of equation, okay so q_i upon epsilon i from this term okay, E_{bi} and then we have substituted for the J_i in terms of q and we have taken it on the left hand side, so this is our final equation okay, so for gray surfaces this will be used as the equation to solve for, okay, this is the first form for radiative transfer between gray surfaces.

We have finite number of surfaces in an enclosure, all are gray and this is the first form now, second form as we did are derived in the black body case, we write F_{ij} as per the summation rule and we can just write it as

$$\sum_{j=1}^{N} E_{bi} F_{i-j} = E_{bi}$$

we substitute for E_{bi} using this expression and we get this relation in terms of the difference of emissive power of the two surfaces, this is the second form okay.

So, the two forms we derived earlier also now, also we are basically deriving the same, this is the second form okay. Now, let us see what happens when we substitute

$$\epsilon = 1$$

in this case that is the surfaces are black okay, I will just use this equation so, in this equation let us put

$$\epsilon_i = \epsilon_j = 1$$

we get q

$$q_i + H_{0i} = \sum F_{ij} (E_{bi} - E_{bj})$$

And this is same as for the black enclosure that we derived in the previous lecture okay, so just by substituting the $\epsilon = 1$, this relation has reduced to the same relation that we derived for the black enclosure, so these 2 equations we will use to solve for radiative heat transfer in a gray enclosure, the enclosure is divided into a finite number of surfaces, $j = 1$ to N, okay.

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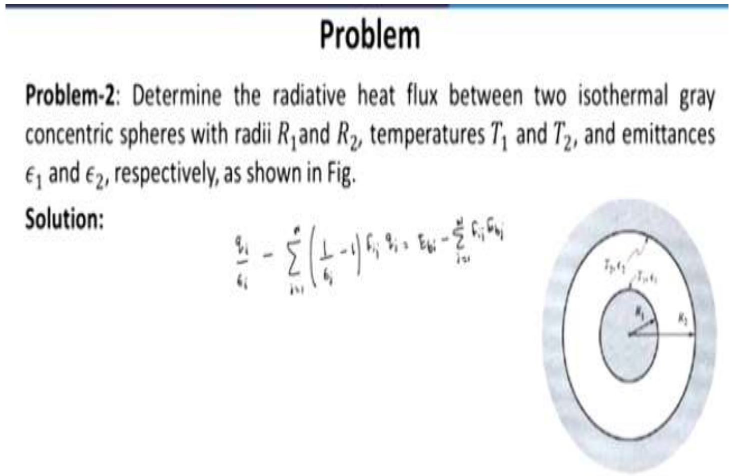

So, let us solve one problem, we have two isothermal gray concentric spheres okay, the inner sphere has radius of R_1 and outside sphere has radius R_2, the temperature of the inner sphere is T_1 and emittance is ϵ_1, outside sphere has temperature T_2 and emittance ϵ_2 and everything is uniform okay, so we have to find out radiative heat flux between two spheres okay.

So, let us solve this problem, so we will use this equation, let me just write down the equation first, so this equation we are going to use, so the equation we will use is

$$\frac{q_i}{\epsilon_i} - \sum_{j=1}^{N}\left(\frac{1}{\epsilon_j} - 1\right)F_{i-j}q_i = E_{bi} - \sum_{j=1}^{N}F_{i-j}E_b$$

there is no H_{0i}, so we will just ignore

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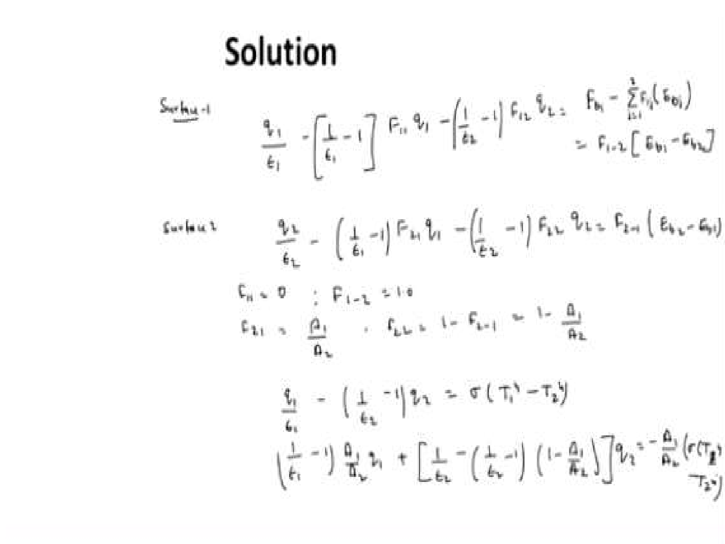

So, surface 1, that is the inner sphere, we write

$$\frac{q_1}{\epsilon_1} - \left[\frac{1}{\epsilon_1} - 1\right] F_{11} q_1 - \left[\frac{1}{\epsilon_2} - 1\right] F_{12} q_2 = E_{b1} - \sum_{j=1}^{2} F_{1j} E_{bj} = F_{12}[E_{b1} - E_{b2}]$$

Then similarly for surface 2,

$$\frac{q_2}{\epsilon_2} - \left[\frac{1}{\epsilon_1} - 1\right] F_{21} q_1 - \left[\frac{1}{\epsilon_2} - 1\right] F_{22} q_2 = F_{21}[E_{b2} - E_{b1}]$$

So, now let us look at the view factors, so

$$F_{11} = 0$$

$$F_{12} = 1$$

then, $F_{21} = A_1/A_2$

this is from the tables okay, so we can easily calculate the view factors,

and $F_{22} = 1 - F_{21} = 1 - A_1/A_2$

okay so, this is how we have calculated the view factors now, let us simplify the equations, so we have

$$\frac{q_1}{\epsilon_1} - \left[\frac{1}{\epsilon_2} - 1\right] q_2 = \sigma(T_1^4 - T_2^4)$$

$$\left[\frac{1}{\epsilon_1} - 1\right]\frac{A_1}{A_2} q_1 + \left[\frac{1}{\epsilon_2} - \left(\frac{1}{\epsilon_2} - 1\right)\left(1 - \frac{A_1}{A_2}\right)\right] q_2 = -\frac{A_1}{A_2}(\sigma(T_1^4 - T_2^4))$$

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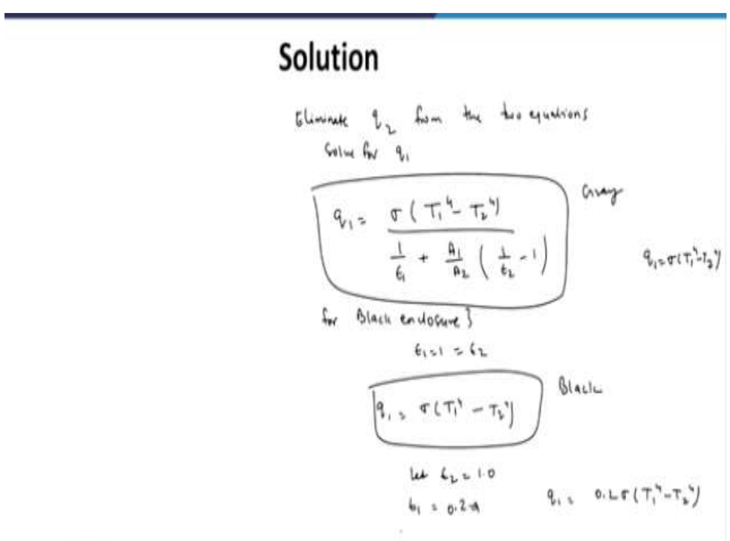

So, this is how we have basically written these 2 equations now, we have to solve for either q_1 or q_2, so we can eliminate either q_1 from this equation or we can eliminate q_2 equation q_2 from this equation.

So, let us eliminate q_2 from this equation okay, so and we will solve for q_2, so we get q_1 =; and this is going to depend on the areas of the sphere,

$$q_1 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

so this is basically the expression for the heat flux for black enclosure now, just note down that q_1 should reduce to the black body relation that is for the black enclosure.

$$\epsilon_1 = \epsilon_2 = 1$$

The second term in the denominator will be 0, okay so we just get

$$q_1 = \sigma(T_1^4 - T_2^4)$$

okay so, this is for the black and this is for the gray okay, so now one can find out the magnitude of the denominator and see what is the effect of this gray emittances on the heat transfer between the two spheres okay, so just to see the effect of the gray emittance is let ϵ_2 =1, so that the second term is 0, okay.

And let us say ϵ_1 =; let us say 0.2, okay so we get,

$$q_1 = 0.2\sigma(T_1^4 - T_2^4)$$

so this is reduced the heat flux from the surface 1, okay similarly, we can put ϵ_1 as 1 and take a value of ϵ_2 and find out what is going to be the effect on heat flux.

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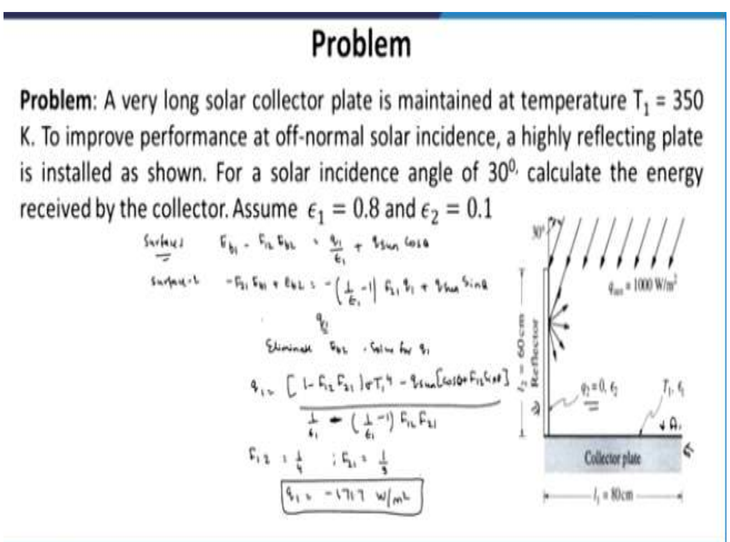

Let us do another problem okay, in this problem we have a collector okay, a solar radiation collector plate, the temperature of this plate is 350 Kelvin okay, now we want to improve the efficiency or performance of this plate by putting a vertical highly reflecting plate adjacent to this plate okay, so this is called reflector, okay and this is collector plate, okay so, collector plate has emittance which is ϵ_1= 0.8, okay.

That means, it is not a very good reflector, it is a good absorber, on the other hand we have put a vertical plate adjacent to it which has ϵ_2= 0.1, it is not a very good emitter or absorber of radiation but its reflectance is very high that is 0.91 - ϵ_2 = 0.9 and it is diffused; both surfaces are diffused and by putting this reflector, we assume or we are expect that some amount of radiation falling onto the surface will be reflected towards the collector plate.

And the amount of radiation received by the collector plate should increase, so let us see how to calculate this problem okay, so we will use the similar method that we basically used in the previous problem, so we will use the same relation surface1, we will write the heat flux okay, this is our surface 1;

$$E_{b1} - F_{12}E_{b2} = \frac{q_1}{\epsilon_1} + q_{sun}\cos\theta$$

where theta is the angle made by the solar radiation from the vertical direction, okay.

And surface 2,

$$-F_{21}E_{b1} + E_{b2} = -\left(\frac{1}{\epsilon_1} - 1\right)F_{21}q_1 + q_{sun}\sin\theta$$

it makes an angle of projection of cos theta on plane 1 of a surface 1 and sin θ on the vertical plate okay now, what we are basically interested in is the amount of flux on surface 1, q_1 we are interested in, so we will eliminate and q_2 is given 0, this is insulated.

So, the reflector plate is insulated, so q_2 is 0, so we eliminate E_{b2}, the temperature of the reflector plate is not given, so it is a variable but we do not need it so, we eliminate the variable E_{b2} from this equation and we solve for q_1, so we get

$$q_1 = \frac{[(1 - F_{12}F_{21})\sigma T_1^4 - q_{sun}(\cos\theta + F_{12}\sin\theta)]}{\frac{1}{\epsilon_1} - \left(\frac{1}{\epsilon_1} - 1\right)F_{21}F_{12}}$$

okay, now the view factors are easily available from the book, okay for two plates perpendicular to each other we can calculate the view factors.

The view factors values I am writing,

$$F_{12} = 1/4 \text{ and } F_{21} = 1/3$$

okay, it depends on the areas of the two plates okay, so F_{12} is 1/4 and F_{21} = 1/3, so substituting these values and the values of temperature and emittance, we get

$$q_1 = -171.7 \text{ watt/meter}^2$$

okay, this is the amount of radiation received by the surface, the collector plate, okay by direct absorption and by reflection from the reflector plate okay.

So, we have discussed the radiative heat transfer between gray surfaces, in the next lecture we will study the method of network and we will try to solve these problems, some more problems on this topic using the network method, thank you.