Principles of Metal Forming Technology
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Lecture – 09
Hydrostatic and deviator components of stress and strain

Welcome to the lecture on Hydrostatic and deviator components of stress and strain. So, in the last lectures we discussed about the stress as well as a strain; we had some concept about the stresses the components of stresses. So, then further we also talked about the strain different types of strain.

Now when we talk about the plastic forming technology that we need to know the components of strain in the sense that some part of the strain or some part of the stress ah, you have the components of stress in you know categorized in different ways. And it can be also categorized based on what way it performs like one thing maybe causing the change in shape, whereas the another case may cause the change in you know deformation; it may cause the deformation.

So, based on that basically we try to define this stress or strain components into 2 parts; that is hydrostatic as well are the main component and also the deviator component. So coming to the strain because we have discussed about the strain in the last class; so, if you go to the strain part what we see in the case of strain.

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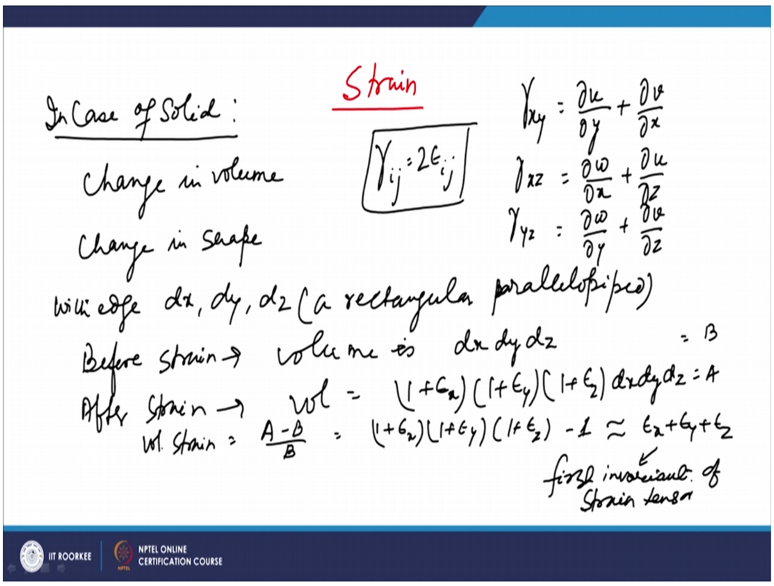

Now, in the case of strain what we see is that what we got the shear strain as gamma x y then gamma y z and gamma x z. So, since you have the 3 components of the shear strain and this is as we know that this is dou u by dou y plus dou v by dou x.

Similarly, you have this as dou w by dou x plus dou u by dou z and you have this as dou w by dou y plus dou v by dou z. So, this is what we had seen in the case of in the last lecture where we found the expression for shear strain and that came from this expression we got as gamma i j as 2 epsilon i j; so; so this way we found this is known as the shear strain. Now normally in the reason of elasticity, we discussed about this shear stress this is more common in those cases.

Further when we talk about this solid then normally you have 2 things, when we talk about the deformation is solid. So, in that you have one is your volume change and another is your change shape change of shape. So, in case of solid when we talk about the deformation in solid then you have one is change in volume and another is change in shape.

Now, the thing is that you know once you have the change in volume so, that way you will have the rising of the volumetric strain. So, you can find the expression for the volumetric strain. So, suppose how can you find this expression for volumetric strain? Suppose you have a rectangular parallelepiped and it has the edge of d x, d y and d z. So, if the with edge you know d x, d y, d z a rectangular parallelepiped is there.

So, suppose there is strain into on the sides and because of that; so, what will happen the when there will be strain there in the strain condition then their volume will be 1 plus epsilon x into. So, suppose. So, before strain its volume will be before strain the volume will be volume is d x, d y and d z.

Now suppose the there is strain component in x y and z direction; so, that is epsilon x. So, in that case the sites will be 1 plus epsilon x into d x 1 plus epsilon x y into d y and 1 plus epsilon z into d z. So, so the after straining, so after strain the volume will be 1 plus epsilon x, 1 plus epsilon y, 1 plus epsilon z into d x, d y, d z.

Now, so the volumetric strain what you get will be the, this minus this. So, if suppose this is B, and this is A; so, you can find the volumetric strain as A minus B upon B; so, this way you can find the volumetric strain. So, once you do this multiplication divided and subtract with this d x, d y, d z and if we be since epsilon x epsilon y and epsilon z will be small quantities.

So, any you know multiplication of epsilon x or epsilon y or epsilon y or epsilon z can be neglected in that case. So, you will have basically again d x d y d z plus that way you can have the components. So, you will have this is strain will come as 1 plus epsilon x into 1 plus epsilon y into z minus 1.

So, this way you can have; so, if you neglect the, you know product of the 2 strain components like epsilon x epsilon y or so or the 3 1. So, this can be written directly as epsilon x plus epsilon y plus epsilon z because this 1 and 1 will be cancelled and in that case epsilon all these. So, this way what you see is this is your volumetric strain; now what we see is that as we have seen that in the case of stress or strain, you have invariants. In the case of stress also be saw that we get the invariants the and first invariant what we saw is the sum of sigma x or plus sigma y plus sigma z or similarly in the case of strain also your first invariant of strain tensor will be epsilon x plus epsilon y plus epsilon z that is what it is.

So, this is basically the first invariant of strain tensor; so this is nothing, but first invariant of strain tensor. Now this component basically if it divided by 3; so, this is known as the mean strain. So, if you go to the further define that; so it is one third component will be the mean strain. So, mean strain you can defined as epsilon x plus epsilon y plus epsilon z divided by 3. So, we can further write.

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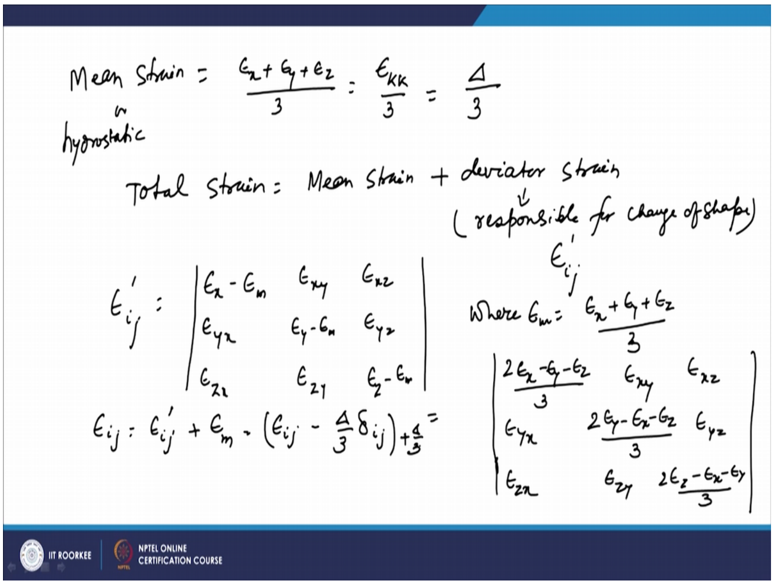

So, your mean strain or we also call it as hydrostatic strain because that is responsible for the change in the volume and that is basically epsilon x plus epsilon y plus epsilon z by 3.

So, this way you have the component that is known as mean component or hydrostatic component; so, or hydrostatic we also call it as hydrostatic or spherical strain. So, that way spherical component of the strain and this is divided as the epsilon x plus epsilon y plus epsilon z by 3. And it is also denoted you can denote it as epsilon KK upon 3 because it will involving this x x parts. So, KK by 3 and it is also divided denoted as delta by 3. So, you have the volumetric strain divided by 3; if you go now the thing is that now this is a part of the total strain.

So, it is subtracted from the total strain you will have another component of strain and that part is known as the deviator component of the strain. So, what we see is that you get this; so the deviator. So, total strain will be your mean strain and what is remaining is the deviator strain. So, that will be basically involved for the change in the shape rather than the volume change. So, one part of the strain is responsible for the volume change that is your mean strain or hydrostatic strain and then the remaining part is known as the deviator strain and that is responsible for the change in the shape.

So, this is responsible for change in shape; so, if you see the strain tensor component then in that case what we do is that this deviator strain component and this is denoted by epsilon i j prime. So, epsilon i j prime if you look at now this is nothing, but from the total strain part, you are going to remove the you know mean strain part. So, mean strain part if you remove; so, what will happen it will be like this it will be epsilon x minus mean and then it will be epsilon x y, this will be epsilon x z. Similarly epsilon y x and this will be again epsilon y minus epsilon m and this will be epsilon y z. So, similarly here it will be epsilon z x epsilon z y and then epsilon z minus epsilon m.

So, this is the tensor this is known as the deviator strain tensor. So, this is a tensor which is known as the deviator component of the strain tensor and as we know that epsilon m is where epsilon m is basically you know epsilon x plus epsilon y plus epsilon z by 2. So, it is basically you know along the diagonal that is working. So, epsilon x x epsilon y y epsilon z z; so, if you try to further you know see that. So, this is this strain tensor comes of the form you will get it as 2. So, your epsilon x minus epsilon x plus epsilon y plus epsilon z by 2; so, that will be. So, this 2 will be here; so you will get it will be 3 here. So, you will get 2 epsilon x minus epsilon y it will be minus epsilon z by 3.

Similarly, you have epsilon x y you have said x z then you will have epsilon y x; here it will be 2 epsilon y minus epsilon x minus epsilon z by 3 and this will be epsilon y z. Similarly epsilon z x epsilon z y and 2 of epsilon z minus epsilon x minus epsilon y divided by 3; so, this is known as the deviatory component of the strain this is a strain tensor, which is responsible for the change in shape. So, what we normally write? We normally write as epsilon i j as epsilon i j prime that is your deviator component of the strain plus epsilon mean. And we also write it as epsilon i j and minus delta by 3 and then we Kronecker delta we put it. So, we write this Kronecker delta that is you have a tensor where you have a matrix which has one as at the you know diagonal component and that is multiplied by delta by 3.

So, you will have a matrix which has all the you know you know elements are 0; except the diagonal elements and they are all the first diagonal this a this upper top left corner will be epsilon x by 3. Similarly you will have epsilon y by 3 and epsilon z by 3; so, that way. So, that is how it comes delta by 3; so that way it comes and then further you are going to add as delta by 3 delta i j. So, this is your, you know this part is basically what you get as the one which is the definition for the total strain part. And you will have the total strain as the deviator component as well as the, you know mean component.

Now similarly if you go to define the mean stress; so there also the stress also has 2 components. One is the mean stress component; another is the deviator component of the stress.

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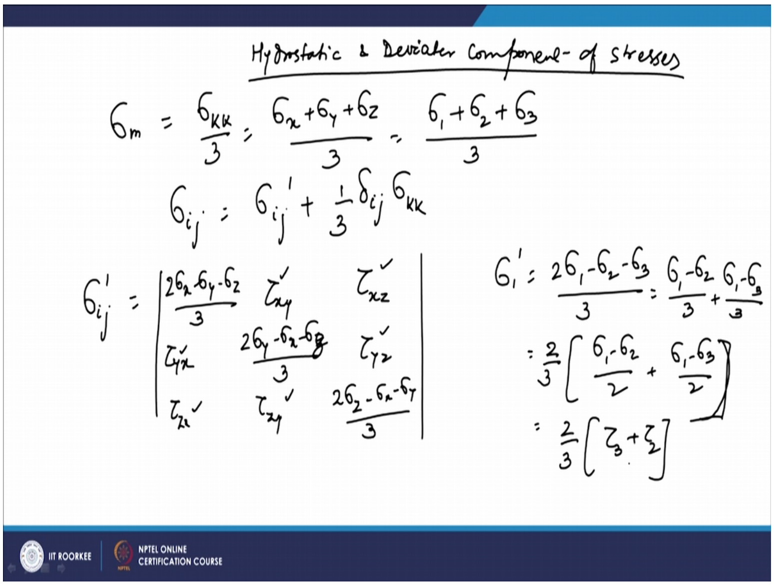

Now, so you have hydrostatic and deviator component of stresses. So, now again as we discussed that, in the case of strain, we found. Similarly in the case of stress also you have the 2 components; one stress will be required one stress, which is responsible for the change in the volume that is known as the you know hydrostatic stress or the mean stress and the stress component which is responsible for the change in the shape that is known as the deviator component of the stress.

So, again the hydrostatic component of stress that will be given as sigma 1 sigma KK upon 3 or we also write it as sigma x plus sigma y plus sigma z by 3. So, this way you write the expression for the deviator component of the, this mean component of the stress. So, further in the similar line if you try to decompose the total stress tensor into 2 parts; one will be your mean stress tensor another will be your deviator component. So, you can write it as sigma i j will be again you will have sigma i j prime plus 1 by 3 delta i j and sigma KK.

So, this way you find this is your stress tensors decomposition and now what we see that since you have the total stress and from there you are removing the mean stress. So, you are getting the stress deviator portion; so, if you try to define the stress deviator sigma i j prime. So, you can again write in the similar line 2 sigma x minus sigma y minus sigma z by 3. And similarly it will be tau x y this will be tau x z similarly you have tau y x and this will be 2 of sigma y minus sigma x minus sigma z by 3 and this will be tau y z and this will be tau z x, this will be tau z y and this will be 2 of sigma z minus sigma x minus sigma y upon 3. So, this way you get this expression for the deviator component of the stress.

So, what we have seen? Earlier we have seen that when we talk about the you know maximum value of you know shear stress that we had seen that sigma 1 minus sigma 2 by 2 or sigma 2 minus sigma 3 by 2 or sigma 1 minus sigma 3 by 3 2 and we had seen that the maximum value of the shear stress we. So, if you sigma 1 is the algebraically largest value and the sigma 3 is the smallest value; in that case the maximum value will be sigma 1 minus sigma 3 by 2. So, that way now what you see here if you look this component we know that all these components this one, this one, this one or this one or this one or this one; they are all very explicitly you can see that they are the shear components.

But you make think of that this involves the normal components; now why they are part of this is stress deviator and the stress deviator basically is responsible for the deformation. So, it must also be a shear component; now in that case you can see that if you look at this, this can be written as if you; if you look at this you can. So, the if you it is taken as sigma 1 prime. So, it will be sigma x minus sigma y plus sigma x minus sigma z by; so, by 3; so anyway if you can take the 3 part that side.

So, you can write sigma 1 minus sigma 2 by 3 plus sigma you know x minus sigma z by 3. So, so that way you see that sigma x minus sigma y that part again is a part which is something related to the shear stress part. Similarly sigma x 1 as sigma z also is a shear stress part; so, basically you see that they are also the sum of the shear stress you know type of forces and that is why they altogether, they are basically representing a stress component which is responsible for the change in the shape of the body.

So, if you look at the sigma 1 suppose prime you what you see is that 2 sigma 1 minus sigma 2 minus sigma 3 and by 3. So, you can write as sigma 1 minus sigma 2 by 3 plus sigma 1 minus sigma 3 by 3.

Now, so that you can write as 2 by 3 if you take it as common. So, you can write as sigma 1 minus sigma 2 by 2 plus sigma 1 minus sigma 3 by 2. Now, so what you see is you can write it as 2 by 3 of tau 3 plus tau 2. So, where tau 3 and tau 2 they are the principal shearing stresses; so that is what we had seen earlier.

So, what we see that normally your this part this sigma 1 prime part also has the components which has the shear components. Now this shear part which we have seen now it is also seen to be the strength deviator part, this is seen to be a second rank of tensor. So, now the principal value of these stress deviator; they are also the root of a cubic equation. So, basically if you try to find its roots. So, you have to further solve that and they will be the, you know. So, principal values of this you know stress deviator are roots of cubic equation.

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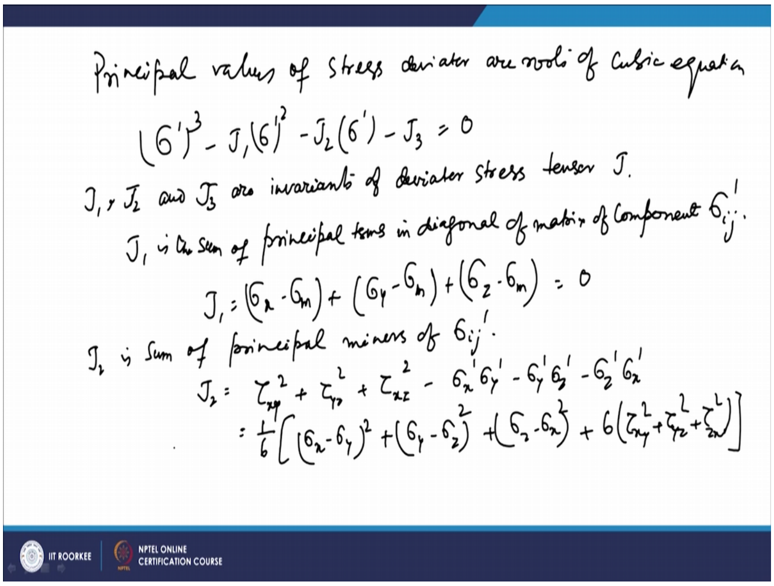

So, if you find that you will get sigma prime cube minus J 1 sigma prime square and then you will have minus J 2 prime sigma J 2 times sigma prime and then minus J 3 that is equal to 0. So, what we see is that here again the sigma J 1, J 2 and J 3; they will be working as the invariants of the deviator stress.

So, J 1, J 2 and J 3 are invariants of deviator stress tensor. Now the thing is that we have seen earlier the 3 you know components of 3 deviator this 3 invariants of this stress tensor total stress tensor there. And in that we got the different first second and third invariants of the stress and first invariant as we recall it was sigma x plus sigma y plus sigma z then second was again you had the expression for it.

So, similarly; so, that was the total now if you talk about this is stress deviator again it has these you know invariants and if you try to find the value of these invariants. So, what you see is that J 1 is the sum of; so, J 1 will be the sum of the principal terms in the diagonal of the matrix of the components. So, with there; you had seen that in the first case in the total case it was sigma x plus sigma y plus sigma z. So, here also J 1 will be basically the, this principal terms which means there along the diagonal of the term; so it will be the summation of that.

So, J 1 will be the sum of the principal terms in diagonal of matrix of component sigma i j prime. What we saw in the earlier case that this is your sigma i j prime and this component this component and this component. Now these 3 components its sum basically will be the J 1; so J 1 will be sigma x minus sigma m plus sigma y minus sigma m plus sigma z minus sigma m. So, if you take that that will be equal to 0; now you have the second component J 2 and J 2 will be again. So, there also we got that is some of the principal minors; so J 2 is sum of principal minus of sigma i j prime.

So there also we get these second invariant in the similar fashion and if you try to find the expression for J 2; so, J 2 you can get as tau x y square plus tau y z square plus tau x z square. Similarly then you get sigma x prime sigma y prime minus sigma y prime sigma z prime minus sigma z prime sigma x prime. So, that is what you get J 2 as the sum of principal minor of this sigma i j prime.

And that may come if you try to simplify this will be sigma x minus sigma y square plus sigma y minus sigma z square plus sigma z minus sigma x square and then 6 times tau x y square plus tau y z square plus tau z x square. So, that comes as the second invariant of the strain deviator; I mean stress deviator. Then third invariant will be basically the determinant of this equation; so the third invariant comes as the determinant of the equation.

So, this way what we see that you have 3 components. So, what we have discussed that in the case of these stress stresses; totally stresses if you look at. Now the thing is if you see when we talk we will talk later on about the you know different type of theories which relate to the failure of the material; in those cases because as we know that when we talk about the deformation it is it is by severe mechanism.

So, it is because of this component of the stress that is your deviated component of the stress. And this part the second invariant of the stress component part this part will be utilized when you go to find the theory for the failure of the material or for the; you know different type of theorems you have. So, you have Von Mises theorem and you have the Tresca. So, Von Mises theorem will be used in that case basically the theorem tells that this value; the second invariant of the stress tensor of deviatoric stress tensor that must reach certain critical value; now this is used in those components.

So, basically the mean component does not take part in to these you know change in the shape. Basically it talks about the change in the volume and based on that you have certain terminologies. So, that we will discuss in the later part and this this basically this deviator part, this will be mostly utilized for because they are basically responsible for; so that mean part is normally you know, they are mostly important when we talk about the smaller deformations per small change in volume or so, so that is in the elastic range.

And when we try to talk about the plastic range that time this part is mostly of use. So, that is about this mean part, as well as the deviator part or deviator component for the stress and strain which will be utilized its concept will be utilized in our subsequent discussions.

Thank you very much.