Principles of Metal Forming Technology

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Lecture – 08
Description of Strain

Welcome to the lecture on description of strain. So, in the last lecture we discussed about the state of stresses in the case of 2 dimension as well as in the 3 dimension, now we will discuss that about the strain values. So, when we talk about the you know the continuum body a concept of continuum is there in the case of analysis now in that basically you have displacement of points are there and that result from basically either from the rigid body translation or rotation or deformation.

So, you know deformation maybe again made up of either the change in volume or change in shape. So, the change in volume is you know known as dilatation or you have the change in shape. So, you will have many you know situations and when you have the situation of translation and rotation, then they are treated in the branch of mechanics that is known as dynamics and in the when we talk about the deformation to a very small you know quantity, then we that is treated in basically in the theory of elasticity and when this is you know of the larger scale larger deformations are there they are treated in the case of I mean in the theory of plasticity.

So, that basically you they are applying to all types of this continuous media now what we try to understand in the case of strain is that I mean what happens in the case of you know if you have a solid body in the fixed coordinates. So, suppose we can understand the strain like this. So, suppose you have a solid body. So, suppose you have a body.

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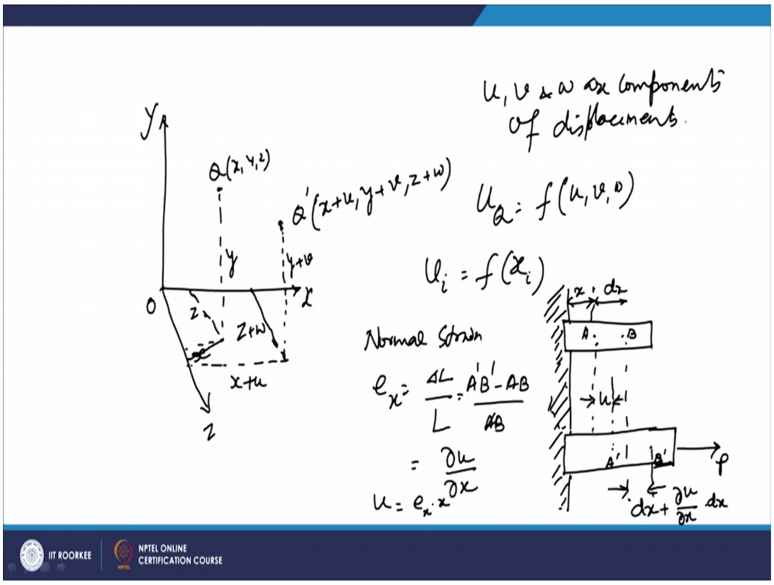

So, what happens that, you have this as the axis now you have a point that is Q and it has the coordinate that is x y z.

So, if suppose you have a solid body in fixed coordinate that is x y z. So, this is x y z and you have these are the axis. So, this is your y axis, this is your x axis and this is your z axis. So, now, if this point Q it has the position vector that is x y z. So, you will have this as the origin, this as has the coordinate axis x y and z.

Now, suppose you have a combination of deformation and movement and that basically is displacing this point Q to the another point Q prime. So, suppose this is displaced to this point Q prime and this Q prime has the coordinate x plus u, y plus v and z plus w. So, basically u v and w they are basically the displacements. So, you can say that if you have if you take this point. So, it will have one point in this plane and from this plane you get further, you can have it is components in the you know y and z direction that way.

So, you will have this as z and similarly you will have this as. So, this is basically horizontal this is horizontal note, this one this is your x component this is horizontal one this is your in the z direction and this is your y component. Now this is changed to this Q prime and Q prime becomes x plus u y plus v and z plus w and so, again it is if you take it is you know it is projection in this plane. So, this plane you will have this coordinate as x plus u. Similarly you will have it is component if you take it is this. So, this will be you know z plus w and this will be your y plus v.

Now, what you see in this that you have in a body you have this point Q and under the action of deformation or stress is in the combination of movements, this point is changed to you know this point Q displaced to Q prime and you have this these u v and w are the components of displacements.

So, what we see that you know displacement of this Q it is a vector u Q and this u Q will be the a function of u v and w. So, this is u Q is basically the displacement of Q. So, that is the function of u v and w. Now if it is constant for all the particles of the body it means there is no strain in the body, but in general this u i i is different for from particle to particle and that is why this u i i will be a function of x.

So, normally u i i is a function of x i because it does not remain the same for all the points. So, that is why u i i is taken as the function of x i. Now in the case of the elastic deformations, now this u i i is where you have very small displacement, this u i i is a linear function of x and you have homogeneous displacements and that is why this is displacement equations are linear, but otherwise for other materials that may not be even linear.

So, we will try to have a case with the simple one dimensional analysis and if you took that analysis suppose you are we are dealing with a body that is you have this is this is the 2 point A and B. So, they have the in between they have. So, this is at a distance of you know x from the origin. So, you this is your so, similarly you will have the another body that is.

So, same body is there and it is you know under the action of this, when it is default. So, this A goes to A prime. So, and then this B goes to B prime. So, this becomes A prime and this becomes B prime now in this case again.

So, this is your now, what we see is that this in the case of undeformed state you have these 2 points A and B and this distance of A from this position is x and distance between A and B is dx. Now this is changed and when is the force is applied in the x direction basically. So, if this is after applying the force P in the x direction. So, there will be movement in that, and this A prime moves to A; A moves to A prime and B moves to B prime and since the displacement is that is u which is one dimensional case. So, it will be a function of x in such cases.

So, if you look at. So, B will be you know B will be displaced slightly larger than A and in that case if you talk about if you try to define the normal strain. So, if try to define this normal strain and in the in the x direction. So, that is known as ex it is nothing, but it is change in length by original length. So, it will be delta L upon L. Now the thing is that here your A B is changed to A prime A prime, it means the change will be A prime B prime minus AB and originally it was AB.

Now, the thing is this is x and this is dx and what happens for this now what is u? U is the displacement of this one. So, this part A is moving to A prime. So, this part is u. So, basically B is further moving and that is why this will be u plus du by dx into dx. So, this will be your between the B and the B prime this distance this will be basically dx plus du by dx into dx. So, into dx. So, this is how you find the displacement of these 2 points.

Now, if you find the normal strain in such cases. So, ex will be A prime B prime minus AB upon AB and then if you take this A prime B prime as you know dx plus du by dx into dx plus all these. So, once you do that you will get the value as du by dx. So, what you see you see that you know in such cases these strain which you get now if you look at this one dimensional case you will find the displacement u to be e x into x if you integrate it. So, u will be e x into x.

So, basically if you generalized generalize it if you generalize this to A. So, what you can write is you will have you can write in that case u as ex into x now if you generalize this to a 3 dimensional case you can write you know these component of the displacements u is basically the displacement.

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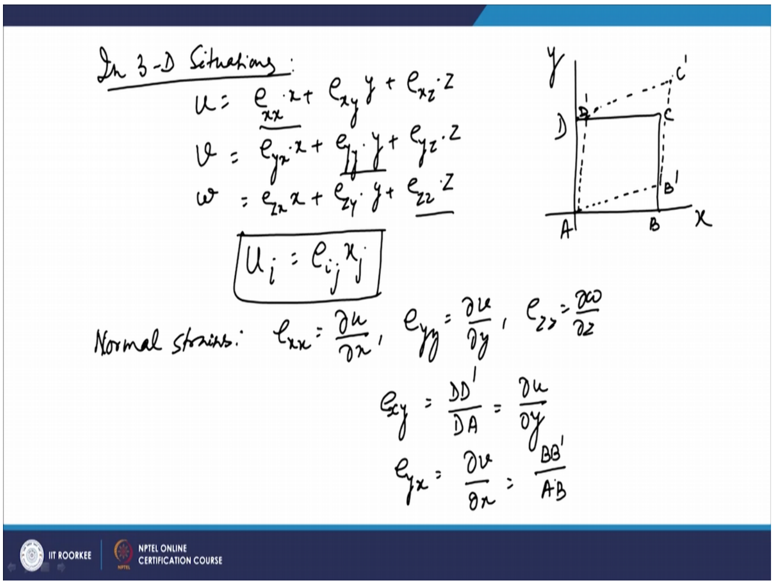

Now if you generalize in the 3 dimensional case in 3 d situations now, in the case of this 3 dimensional situations what you see that u will be e x x into x plus e x y into y plus e x z into z. So, so we saw that u is basically e x into x. So, if you are going to the 3 dimensional case you can represent this u as this similarly you can represent v also. So, v will be e y x into x plus e y y into y plus e y z into z and similarly you can write w as e z x into x plus e z y into y plus e z z into z. So, you can write that is why you can denote it as u i as i j e i j and x j.

So, this way you can denote these displacement you know vector that is made of component that way to find that strain now this coefficients which are relating these displacement with the you know coordinates of the body point in the body. So, they are basically component of the relative displacement tensor. So, basically they talking about the general notations if you take all these components it will be again a 2 dimensional 2 rank tensor or so, as in the stress you will have the 9 components here also you will have the 9 components 3 raised to about 2 that is 9 you have the components to define the strain at a point.

So, what we see is that you have in this what we you see that these components which have xx or yy or zz now here you see that they are basically the normal strains. So, normal strains are you have 2 types of the strains. So, you have normal strains and normal strains you have e x x. So, that will be du by dx similarly e y y. So, that will be dv by dy and e z z that will be du w by dz. So, that is what you get in the case of normal components.

Now, you have other 6 coefficients. So, they are required to be understood what are these components, now if you talk about now if we try to find what are these 6 components you can say that if suppose you have an element in the xy plane. So, suppose you have in xy plane and this is x and this is y, now this element in this plane now this is your element. So, if suppose this is your A B C D now this you know this is distorted by shearing suppose. So, if have given in the shearing and this has gone to this type of you know shape. So, if it has taken such shape. So, this C has come to C prime now this B has come to B prime and now, in this case it is not it meets here not here.

So, this is your d prime. So, basically this also moves like this the situation now what happens that because of the application of this shear stresses it has gone into one angular distortion type of you know situation and if you take if you see these displacement now here the displacement is parallel to x direction here similarly displacement here is parallel to the y direction or so, now in such cases you can find these e x y now e x y if you look at. So, e x y where you see that in this case e x y you have y component du by dy.

So, du by dy an in the u component what is that change in that u side. So, that is why and dy. So, dy will be. So, you have divide by this. So, basically what you get is d e x y will be DD prime. So, that is your change in the u displacement component in the x direction.

And similarly you have to divide it divide it by DA. So, that is what you get this component divided by this component and that becomes as du by dy. So, this you get as e x y similarly you can get e y x and if you see that it will be you know dv by dx. So, here you have change in the in y component. So, that is. So, that becomes basically equal to BB prime. So, you have the displacement in the y direction. So, this your displaced quantity and then it is for this AB, so, BB prime divided by AB, so this way you define this e x y or e y x.

So, similarly you can define the e y z or e z y or e x z or e z x and then also there is certain sign to these shear components and this shear displacements are basically positive when they are rotating from positive to one positive axis towards another positive axis then we take that shear displacements as positive and otherwise they are negative. So, if you come to the displacement tensor if you find the displacement tensor now what we have to understood we have try to understood these how we get these e x y or e y x.

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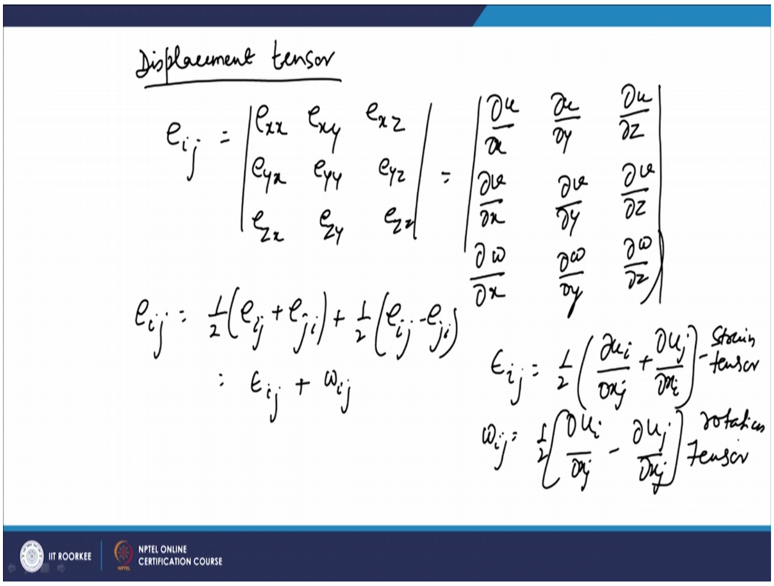

So, what we get is in the displacement tensor as. So, you have e x x then you have e x y e x z similarly e y x e y z yy and e y z similarly e z x e z y and e z z. So, this can be written as e x x we know that this du by dx this is du by dy and this is du by dz similarly dv by dx dv by dy and dv by dz similarly dw by dx dw by dy and dw by dz, now what we see is this is basically known as the displacement tensor in such a cases.

Now, basically these displacement components like xy e x y or e y x they are producing both shear strain as well as the rigid body rotation. So, what we do is basically we try to divide it into 2 components. So, one will be your strain tensor and. So, that is creating the shear strain and another part will be basically talking about the rotation part. So, that will be rotation tensor.

So, one will be talking about the strain tensor another will be for the rotation tensor and for that as we know that this matrix can be any matrix can be you know represented in terms of sum of 2 matrices one will be symmetric another will be skew symmetric. So, what we do is we are. So, this is the second rank tensor this we are basically decomposing this i j e i j into 2 parts one is the symmetric matrix another is the skew symmetric matrix. So, one will be half of i j e i j plus e j i and then another will be half of i j e i j minus e j i. So, this way we get one is this is your strain tensor and then the second part will be your rotation tensor that is we write it as omega i j.

So, we have basically to define this strain which we have getting by this this shearing stresses since we are getting that into 2 ways we have we have to represent them in 2 ways. So, we are representing them as epsilon i j plus omega i j then epsilon i j will be basically epsilon i j if you try to find if you you add these 2 and then divide by half. So, it will be basically you can (Refer Time: 21:05) rotate as du i by dx j plus du j by dx i. So, this is known as this known as the strain tensor.

And similarly if you take the skew symmetric part so, here omega i j and this will be coming as du i by dx j minus du j by dx i. So, basically this comes as the rotation tensor now the thing is that if you now represent try to represent this epsilon i j if if you look at this whole this strain tensor in that case. So, the strain tensor becomes we you can represent as that is epsilon i j.

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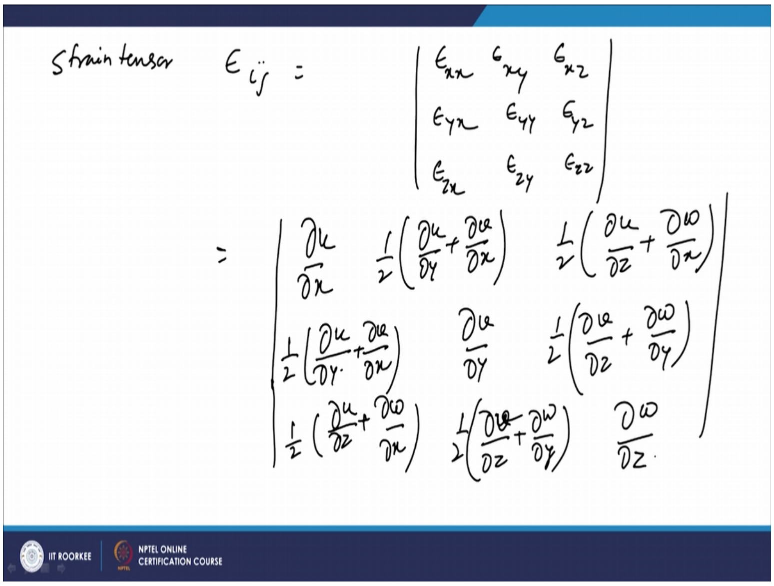

Now, this will be nothing, but you know epsilon xx epsilon xy epsilon xz similarly epsilon yx epsilon yy epsilon yz similarly epsilon zx epsilon zy and epsilon zz and we have seen that this is nothing, but half of du y by dx j du j by dx i. So, if you do that what you see is here it will have du by dx and here it will be half of du by dy plus dv by dx and similarly half of du by dz plus dv by dw by dx.

Similarly, the other part will be here half of du by dy plus dv by dx and here it will be dv by dy and this will be half of dv by dz plus dw by dy and then here it will be half of du by dz plus dw by dx similarly half of du by dz dv by dz basically plus dw by dy and this will be dw by dz.

So, what you see is that this strain tensor is a symmetric type of matrix you can see that this and this is same similarly this and this is same this and this is same. So, this is the strain tensor if you go to find the you know rotation tensor.

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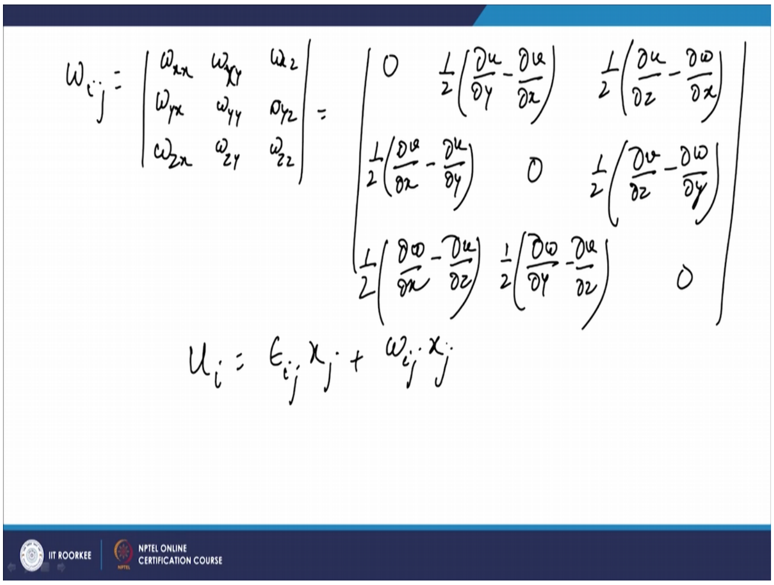

So, rotation tensor will be omega i j and you will have that can further we represented as omega xx omega yy omega xy omega xz

Similarly, you have omega yx omega yy omega yz omega zx omega zy and omega zz. So, that will be basically by subtracting in that case the diagonal elements become 0. So, in this case you get the diagonal element as 0 and here you get half of du by dy minus dv by dx similarly here will be will be half of du by dz minus dw by dx.

So, then it will be you know of this one. So, it will be half of, so, this will be negative of this. So, half of dv by dx minus du by dy and this will be again 0 the diagonal term and this will be half of dou v by dz minus dw by dy similarly it will be half of negative of this. So, half of dw by dx minus du by dz similarly half of negative of this same as this one, but negative. So, half of dw by dy minus dv by dz and diagonal component will be again 0.

So, this part this is known as the rotation tensor and the summation of this 2 tensors they are the displacement tensor and what we see in this case this is omega at this omega i j this part is negative of this part is negative of this part is negative of this. So, this way you get these value, now if you try to generalize what we see is that you get u i as basically epsilon i j x j plus omega i j x j because your strain tensor is epsilon i j plus omega i j.

Now, what we further try to we will try to further define this you know shear strain. So, if you try to see the different types of you know you know angular changes from the right angle you can see that you have many cases which are basically originated and the shear strain basically that is defined as if you talk about the shear strain.

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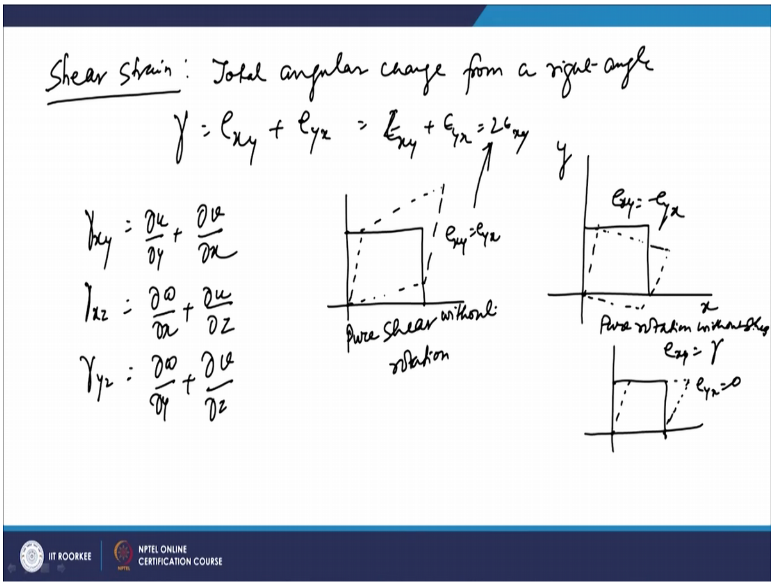

So, normally this shear strain is defined as the total angular change total angular change from a right angle. So, if you look at now the suppose gamma it will be e x y plus e y x.

Now, in the earlier figure what we have seen you have e x y and you have e y x in both the cases. So, total angular change will be your shear strain in those cases. So, basically it becomes you know 2 times. So, it will have epsilon xy plus epsilon yx and that becomes 2 epsilon xy in those in that situation if you look at the situation different situation for a now in this case if you look at here where you have this is as the element and here if you look at this element goes like this, so, in such situations what you have seen previously.

In this situation e xy is same as e yx that is why this leads to you know 2 times epsilon xy there is no rotation in this in such cases. So, that is why this your 2 times epsilon xy now the thing is that. So, gamma xy you can you can write as du by dy plus dv by dx similarly you will have gamma xz you can write as dw by dx plus du by dz and gamma y z you can write as dw by dy plus dv by dz.

So, that you can refer by some other cases like you can take the example of such deformation where you have this is as an element and this has gone to such situation where it has gone like this and this has moved to such case. So, in this case what we see is if this is the y and this is your x. So, in that case you can write. So, in this case you can write that e xy is minus of e yx.

And if you take the example of such deformation where your this is how the deformation occurs in this case your e xy will be as gamma and e yx is basically 0 because there is no such displacement in that y component. So, this way you have different types of you know situation this is a situation where you have this is the situation of pure shear without rotation.

So, and then there is no rotation here now in this case you have pure rotation without shear. So, that will be your pure rotation without shear and if you take the simple this is the case of simple shear that. So, here you have a shape change because of the displacement. So, this is how you try to have the expression for the shear strain as the you know in terms of u v w and x and y and z.

Further you can have the transformation of I mean transformation principle applied which we apply in the case of tensor and there also we try to have the you know expressions for the strain you have the invariance for the when the strain tensor that is also you can you know we get it like we have 3 invariance of strain tensor and similarly you have the sigma x plus sigma y plus sigma z is the first strain tensor second you have.

So, this way we will have the 3 invariance of strain tensor you will have the principle shearing strains also that also can be found out and based on that you may have other strain components like you know volumetric strain can be found and you have you know for the deformation and as well as for the you know volumetric component based on that the strain deviator is also you know defined. So, basically strain is normally tensor will be basically in the form of 2 components one will be hydrostatic or mean tensor and another will be the deviatoric type of tensor which will be for these shear components it will be talking about.

So, that way you can you can you know represent this strain tensor in terms of the hydrostatic component or as well as the, you know deviatoric type of components. So, that way you can study to have a better understanding about the strains.

Thank you very much.