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Lecture – 07
Statement of stress in three dimension

So, welcome to the lecture on state of stress in three dimension. So, we will also have some discussion over the state of stress in two dimensions on which we had discussed in the previous lecture. So, what we saw in the previous lecture that we discussed about the state of I mean stress in the two dimensional case, we found expression for sigma x prime, sigma y prime and tau x prime y prime.

And, we got the values of sigma x prime as when you have sigma x sigma y and theta is given in those cases and tau xy also is given in those cases sigma x prime will be sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 cos 2 theta plus tau xy sine 2 theta similarly, it will be minus in those cases. So, that will be your for sigma y prime that is sigma x plus sigma y by 2 minus sigma x minus sigma y by 2 cos 2 theta minus tau xy sine 2 theta. So, that is for sigma y prime.

And, similarly, we get tau x prime y prime and that is basically sigma y minus sigma x by 2 sine 2 theta plus tau xy cos 2 theta. So, that is what we got in the earlier lecture. We also came to know about certain properties like the invariants of the quantity, like we if you add sigma y prime and sigma x prime it will be same as sigma x plus sigma y. So, so that is how we saw that does not in basically depend on that theta. Then we also saw the we had defined something like the case of principle planes. So, basically when you are dealing with the two dimensional cases you have you know principal stresses you will have two values of principal stresses.

And, you will have the planes on which there is no shear stress these are known as the principal planes and the stresses which are active normal to these principle planes they are known as the principal stresses. So, basically they so, if you have a two dimensional case you have two principal stresses sigma 1 and sigma 2 and they occur at about 90 degree apart. So, that is how you get the value of sigma 1 and sigma 2 in such cases. Similarly, if you go for the three dimensional case, you will have the three values of the principal stresses sigma 1, sigma 2 and sigma 3 and one of them will be the algebraically the greatest one and another will be the largest one. So, will discuss that how that is required to find the value of the shear stress maximum shear stress such situations.

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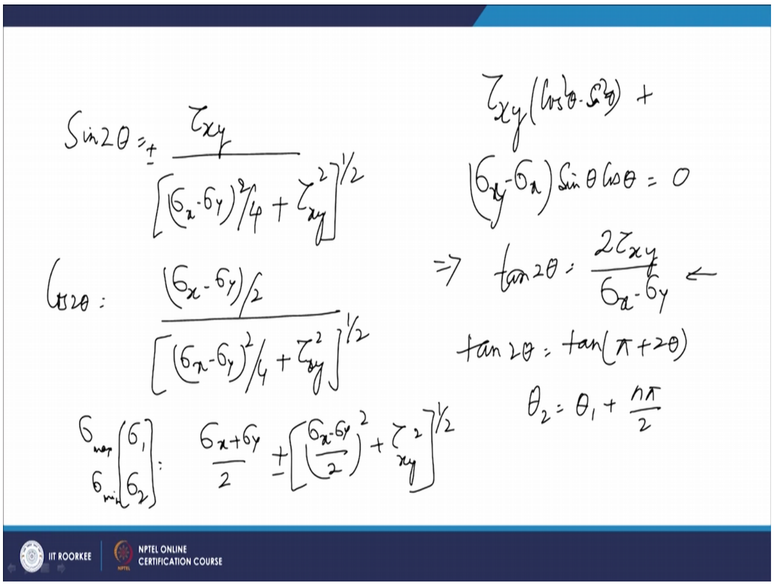

Now, if you talk about the case of these principle planes. So, as we know that the principle planes are one where the shear stress component is 0, so, if you look to that value the principle planes. So, if the shear stress component is 0, in that case you will have the tau xy and then cos square theta minus sine square theta and similarly you have tau sigma x minus sigma y minus sigma x and that is your sine theta cos theta so, that is to be equated to 0. That is what we got this expression for tau x prime y prime that will be equated to 0.

And, from there this way from here you can get directly tan 2 theta so, that you can get as 2 tau x y upon sigma x minus sigma y. So, that is what you get these directions for the you now principle planes. Now, in this case what we see from here you will have these values that talks about the inclination of I mean that those angles at which there will not be any shear stress component. Now, if you try to solve this, what we see that normally basically tan 2 theta will be again you can write has tan pi plus 2 theta.

So, basically you get the two values of theta, theta 1 as well as theta 2 and theta 2 will be theta 1 plus n pi by 2. So, something like you can get theta 2 will be theta 1 plus n pi by 2. So, this way you get the two values of these theta and they are the mutual mutually perpendicular stresses at these angles and they are free from the shear. Now, from this expression that is tan 2 theta equal to 2 tau xy upon sigma x minus sigma y from there you can get you can use the Pythagorean principal and you can get the value of sine 2 theta as well as cos 2 theta.

So, if you use that formula. So, sine 2 theta will be basically tau xy upon and again you will have sigma x minus sigma y square. So, you can see from here. So, this will be used by 4 and plus you will have tau xy square and whole raised to the power 1 by 2. So, this way you can get the value of sine 2 theta. Similarly, and its value will be plus minus. Similarly, you will have the value of cos 2 theta and cos 2 theta will have only the you know in the numerator you will have changed.

So, it will be sigma x minus sigma y by 2 and then you will have the in the denominator you will have same thing sigma x minus sigma y square upon 4 plus tau xy and then square and raised to the power half. So, this way you get the value of the sine 2 theta and cos 2 theta and this can be you know substituted in the expression for tau x prime and I mean sigma x prime and sigma y prime. So, in the sigma x prime you know that it is sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 cos 2 theta. So, in place that we will have this cos 2 theta and plus again tau xy sine 2 theta. So, in place of sine 2 theta we will put there.

So, if you put that you can get the expression for the sigma maximum and sigma 2 principal is stresses and that will be sigma 1 and sigma 2 and these value you can get as sigma x plus sigma y by 2. So, that is common in both the cases and then you will have plus minus and then you will have the expression if you substitute. So, you will get it in the term of sigma x, sigma y and tau xy. So, that you will be getting as sigma x minus sigma y by 2 square and plus tau xy square and raised to the power half. So, this way you get these sigma max and sigma min you can get it.

So, this is how you get these value of these principle stresses in the case of two dimensions and you can get the value of theta also because you will have two values of theta. Now, if you try to see the you now you have to also see that you have to have the condition of the maximum shear stress.

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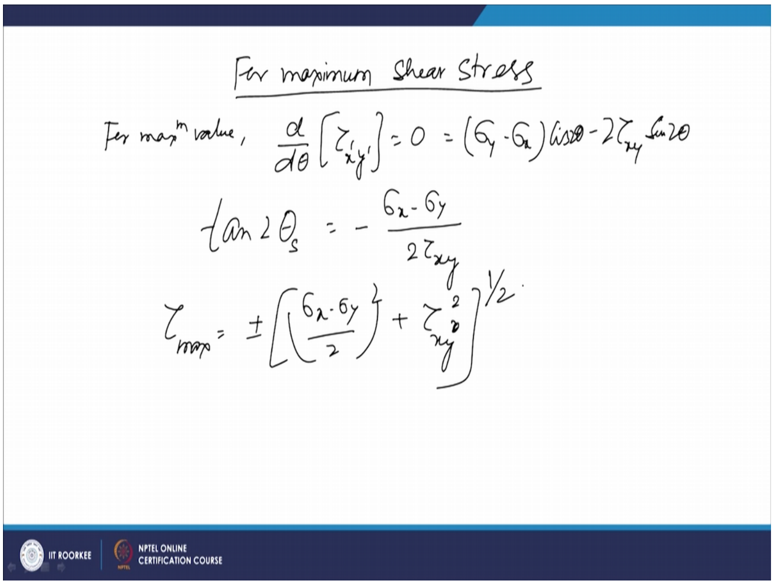

So, if you try to have the condition for the maximum shear stress. So, for maximum shear stress if you look at the expression for tau x prime y prime so, for maximum condition it is you know derivative with respect to theta has to be 0. So, d by d theta of tau x prime y prime must be 0. So, that is for maximum value your d by d theta of tau x prime y prime must be 0. So, if you equate that to be 0, you can further see that that will be equal to sigma y minus sigma x and then that will be cos 2 theta minus 2 tau xy sine 2 theta.

So, so basically that is to be equated 0 and from here you get this tan 2 theta and then since we are getting for shear condition. So, we write it as theta s in those cases and when we talk about the normal stresses in those cases principal stresses. So, in those cases we write it as theta n now in this case if you take this is equated to 0. So, sine 2 theta by cos 2 theta will be 0. So, it will be having the value that is minus of sigma x minus sigma y divided by 2 tau xy that is what we get.

So, this is talking about that plane in which that is maximum shear stress and if you compare this angle with the principal plane in which there is no shear then what you see is that this is nothing, but the it is talking about the slope. So, if you would talk about if you see it is this tan 2 theta as this one and the another one for the principal planes if you multiply them you see it to be minus 1.

So, what we see is that this is the negative reciprocal of that value and that indicates that basically they are perpendicular to each other. So, basically the 2 theta n and 2 theta s they are basically orthogonal. So, they work at you know they are at ninety degree to each other. In fact, that means, that if you divide by 2 so, theta n and theta s they will be separated by 45 degree. So, that is what it means and if you to put these values you can get directly the tau max value as plus minus of sigma x minus sigma y by 2 square and plus tau xy square. So, that is raised to the power half. So, this is what you get the maximum value of the shear stresses.

Now, if you so, this is I mean if you have a problem based on sigma x sigma y and tau xy given then you can have the value of these you know sigma 1 sigma 2 or tau max that can be found out. Now, coming to when we talk about the three dimensional case now, in the case of three dimensions basically what happens that you have three principal stresses. So, just as in the case of two dimension you have two principal stresses in the case of three dimension you have three principal stresses.

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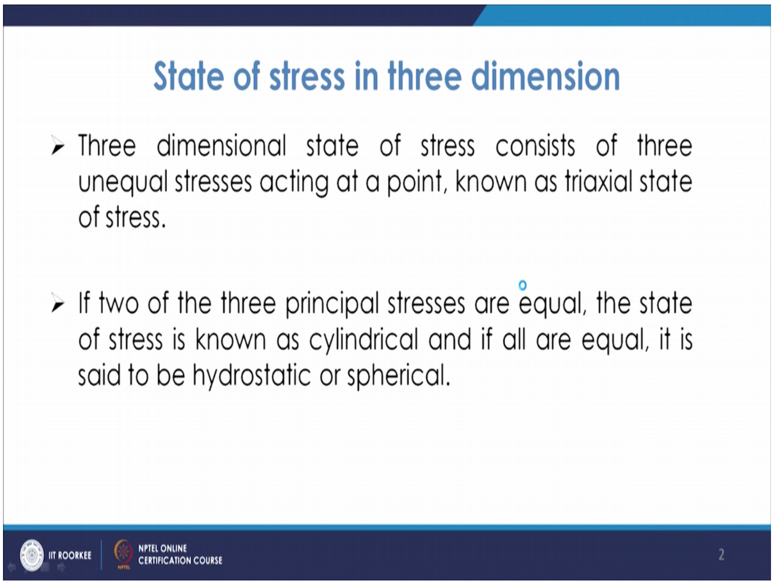

And, if you try to see the three dimensional case now what happens in those cases so, what we see that you have in the three dimensional state of stress condition you have three unequal stresses acting at a point and that is known as triaxial state of stress. Now, if the two of these three principal stresses are equal, then state of a stress in known as cylindrical and if all are equal, then it is said to be hydrostatic or spherical so, that is what the three dimensional case. Now, its simpler analysis is somewhat similar to that in the case of the two dimensional cases the example that the thing here is that you will have three components sigma x, sigma y and sigma z. So, you will get an equation which will have three roots and then that will be solved to get these three values of the principal stresses.

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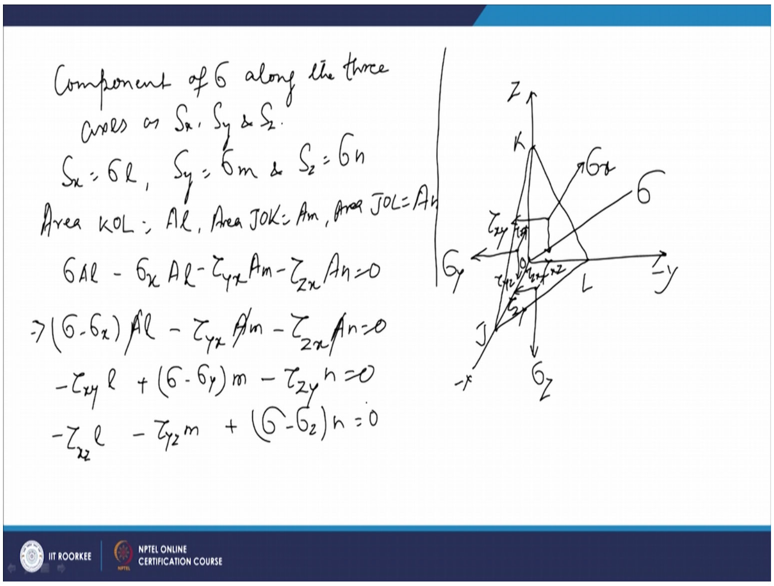

So, you can see we can have a three dimensional you know representation for that and suppose, if you take the for example if you have a three dimensional case. So, if you look at a plane. So, suppose you take a plane like this, where this is your negative y direction and this is your negative x direction and this is your z direction.

So, if suppose now in this planes you take this J, K, L plane so, now what we do is that we are taking a diagonal plane that is J, K, L of some area a we are defining that area to be a and here in this you have a principal stress acting that is sigma this is the principal stress acting normal to this plane J, K, L and now for this what we do is you define the direction cosines to be l, m and n. So, basically this is the cosine of the angle between sigma and the x, y and z axis. So, that way we define these l, m and n that is are the are the direction cosines for this of that sigma

Now, the thing is that again you will do the equilibrium analysis for the forces and you will have the component of sigma ah. So, if you see the we have discussed about the representation of the stresses on these different planes suppose if you look at so, if you suppose this is the this is the phase. So, in this phase now in this the sigma x will go like this because this is the in the in that direction sigma x that is the normal value and if you look.

So, you have you have two you know shear components now this is working in the y direction so, you will have tau xy and similarly this is working in the z direction, so, this will be tau xz. So, similarly on all these phases you can represent the values. Suppose, for example, on this phase you will have this coming as sigma y and similarly, you will have two shear stress components. So, that will be tau yx and similarly, you will have tau and this is working in the z direction. So, this will be tau yz.

So, in the similar way if you have this phase on this phase you will have sigma z working in this direction and you have two component of shear. So, this is working in the y direction again. So, that will be tau zy and similarly, you will have tau zx. So, that will be working like this will be tau zx. So, like this we have already discussed about it is presentation of the stresses on the plane.

Now, what we do is basically we are talking about the component of sigma. So, if you take the component of sigma. So, that is along x along the three axis. So, we are taking these values as S x, S y and S z. So, S x, S y and S z. Now, what happens that so, if they are they are components? So, we know that l, m and n are the direction cosines it is nothing, but this cosine of these angle between these axis so, respective axis with sigma. So, you will have S x as sigma l and S y as sigma m and S z as sigma n. So, this way you get similarly you will have the different area this is our O. So, area KOL will be something area KOL so, this will be you know if the area is this is A l similarly, you will have other areas like area JOK. So, that will be am and similarly, area JOL this will be A n.

So, this way you get these values and then what you do is you are taking the summation of the forces in the you know x direction, y direction and z direction and you are getting the equation of for these forces. So, if you do that if you do in the x l direction what you see is it will be coming as sigma A l so, that will be it is sigma component in the x direction, then further you will have minus sigma x that is A l and then you have this minus tau yx that is A m and then tau zx A n.

So, that becomes equal to 0. So, if you do that analysis in such cases now this comes out to be sigma minus sigma x A l minus tau yx A m minus tau zx A n that will be equal to 0. If you are doing the summation analysis in the x direction you are getting this equation. So, you can get similar equation if you do in the y x direction and z direction you will get the similar equations. So, so what you get is minus of tau xy and l then you have plus sigma minus sigma y that will be you know further m and because here you can take the a out. So, so this a will go out from all the terminologies and then again you will have minus of tau zy n will be 0.

And then the third equation which you get for the z direction will be minus of tau xz l minus tau yz then m and then you will have the sigma minus sigma z of n. So, that will be equal to 0. Now, you will have so, this is one you will have three homogeneous linear equations and they are in terms of l, m and n and l, m, n and cannot not be 0. So, you will have the only nontrivial solution which will be obtained by setting the determinant to be equal to 0.

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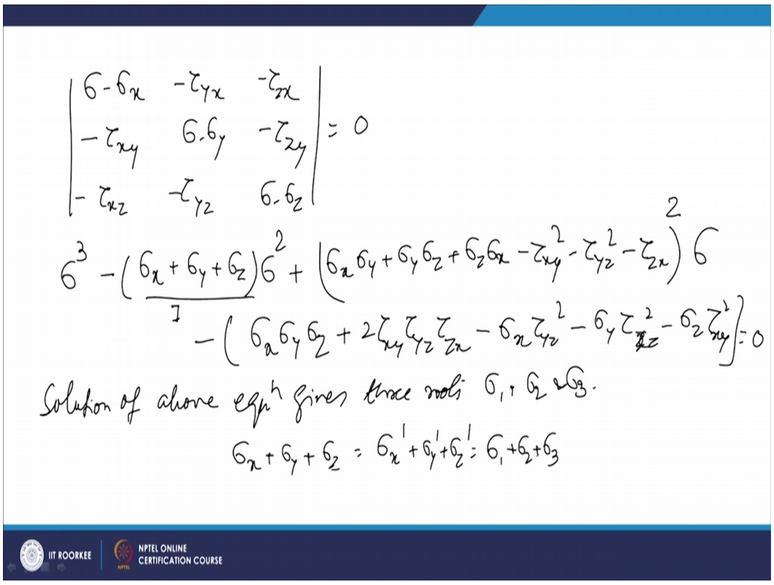

So, if you set the determinant as 0 what you see is that sigma minus sigma x then you will have minus tau yx then you have minus tau zx. Similarly, minus tau xy sigma minus sigma y and minus of tau zy and you will have again minus of tau xz minus of tau yz and sigma minus sigma z. So, this determent has to be equal to 0 and that is the only nontrivial solution which we can get by equating this determinant to be 0 and from here if you expand solution if you get the try to get the solution of this determinant what you get is that sigma cube minus then you get sigma x plus sigma y plus sigma z and into sigma square, then you get sigma x sigma y plus sigma y sigma z plus sigma z sigma x minus tau xy square minus tau yz square minus tau zx square.

And this multiplied by sigma and again you have the term sigma x sigma y sigma z plus 2 tau xy tau yz tau zx minus sigma x tau yz square minus sigma y tau z xz square and minus sigma z tau xy square. So, this way you get this is the equation which will be solved and being the cubic equation you will have three roots sigma 1 and sigma 2 and sigma 3 and these 3 values will be the principal values they are the 3 principal stresses value.

So, solution of the above of the equation solution of above equation gives three roots sigma 1, sigma 2 and sigma 3 these are the three principal stress values which you get and if you we can we can further try to get the determine the direction with respect to the origins original x, y and z axis in which the principal stresses acts that is what we have done for earlier for the two dimensional cases and that can be done again with one condition that is l square plus m square plus n square equal to 1. So, keeping that in you know combination you can solve and you can try to get the values of these the directions of all these principal stresses.

Now, if you try to analyze this equation which is solved to get the principal values what we see is that we have three you know coefficients this is of this is first coefficient and this is the second one and this is the third one. Now, these are known as the invariant coefficients you know. So, basically what you see is that this is comprising of all these three value of the sigma x, sigma y and sigma z. So, these are known as the stress invariants and they are the three stress invariants what we see in such cases.

So, when we talk about the you know two dimensional state of stress we had seen that sigma x plus sigma y is an invariant quantity. Similarly, here you see that sigma x plus sigma y plus sigma z is seen and that also is one invariant quantity what we get see that some of the normal stresses this sigma x, sigma y and sigma z. Now, this for any orientation in the coordinate system of axis so, that will be equal to the some of the normal axis that is what the first invariant of the stress talks about.

So, what you can write from there that sigma x plus sigma y plus sigma z that is the first invariant of the stress. So, it will be same as sigma x prime plus sigma y prime plus sigma z prime or it will be same as you take the three values of the principal stress is stresses sigma 1 plus sigma 2 plus sigma 3.

So, basically you can get these this these values as constant and they are known as the stress invariants. Now, if you try to find further you try to further do the analysis for finding the shear stress values what you see is that we can also get the value of the principal shear stress is stresses and for the principal shear stresses. Now, you will have the total shear will be total stress will be component will be the combination of these normal stress as well as the shear stress. So, the shear stress can be taken as the total stress minus the so, resultant of the these total normal stress and shear stress will be the total stress.

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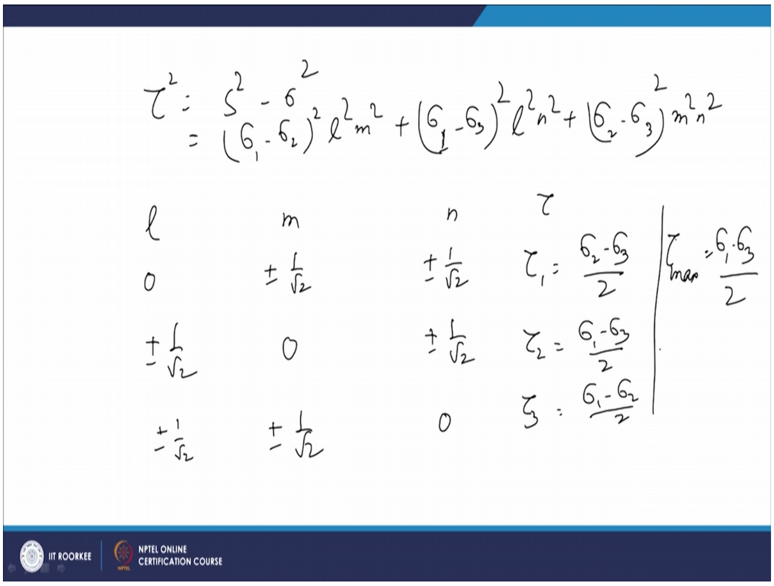

So, the total shear stress now, tau square will be you know as you can defined as s square minus sigma S square. So, by this so that can be you can have you can further study and this you get as sigma 1 minus sigma 2 into square l square m square plus sigma 1 minus sigma 3 square l square n square plus sigma 2 minus sigma 3 square and then m square n square. So, this way you try to get the tau square and this l, m, n, we know that they are the direction cosines.

And, principal stresses basically occur for certain combinations and those combinations are like you have l, m and n values. So, if you may have l value of 0 and m value as plus minus 1 by root 2 and similarly, this value also as plus minus 1 by root. So, if you try to get the value of the principal you know the there was shear stress values now this value tau 1 you can get if you put in this equation so that you can get as sigma 2 minus sigma 3 divided by 2. Similarly, if you can keep this 0 also and this 0 also and in those case this is plus minus 1 by root 2 and plus minus 1 by root 2 and similarly, you have plus minus 1 by root 2 and plus minus 1 by root here. So, in those case says you get tau 2 and tau 3 and get tau 2 has sigma 1 minus sigma 3 by 2 and tau 3 as sigma 1 minus sigma 2 by 2.

Now, the thing is that they are talking about the you know maximum shear stress is and if the sigma 1 is the algebraically the maximum of the principal stress and sigma 3 is the minimum of the shear stress in that case maximum shear stress you can say as sigma 1 minus sigma 3 by 2 if sigma 1 is the maximum and sigma 3 is algebraically the minimum of these values and in those case says you can also get the value of tau max you know once you get the value of theta you have these maximum value of the tau max as the sigma 1 minus sigma 3 upon 2. So, that is what we normally we normally do for the analysis in the case of the three dimensional state of stresses.

So, this basically this maximum shear stress normally will be used when we do the plastic forming analysis because you know that deformation normally is done by the shearing mechanism so, that is used in that concept. So, we will talk more about it in our coming lectures.

Thank you very much.