Principles of Metal Forming Technology

Dr. Pradeep K. Jha
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 06
Description of Stress

Welcome, to the lecture on Description of Stress. So, in the last lecture we had discussion about the concept of stresses and also strains. So, now, we will discuss about the stresses how they are described specially how what are the notations for the stresses and we will also discuss about the state of stress in two dimensional and we will also see that how we are finding the value of a stress, when you have the case of two dimensions so, that we will discuss later.

So, what we discussed in the last lecture that stress had a point it can be resolved into normal component as well as the shear component. So, that is what we discussed about it.

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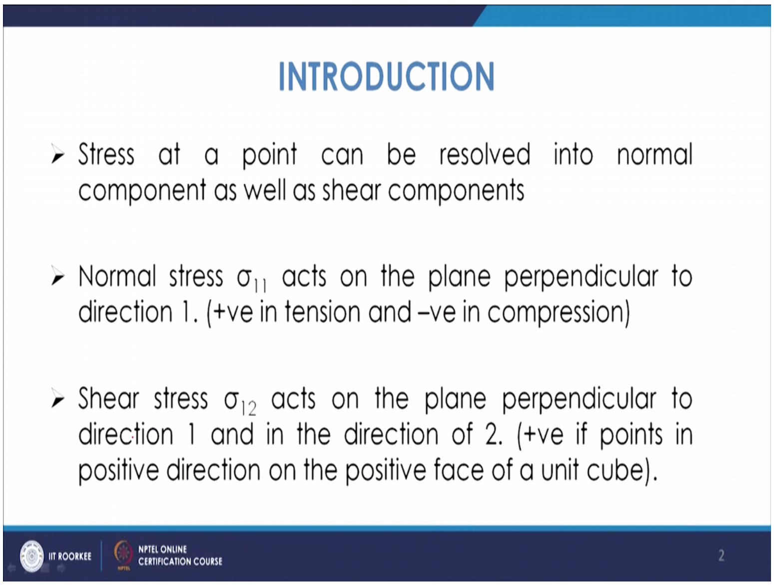

Now, normal stress that is sigma 1 1 or sigma x x, that will be acting on the plane perpendicular to the direction 1. So, if suppose you have a cube in that case you have the x directions. So, that will be if suppose you have y, z plane and then. So, x direction will be normal to it and the stress which will be acting in the perpendicular direction to that plane that will be the normal stress and similarly, you have the sigma y and sigma z then you have its nomenclature, its sign so, we discussed that if it is tension we normally take it as positive and if it is compression we take it as negative.

Then shear stress; now, shear stress normally will be denoted by sigma 1 2 or tau xy or so and it will be acting on the plane perpendicular to direction 1 and in the direction 2. So, sigma 1 2 or tau 1 2 or tau xy normally we denote as tau. So, it will be you have one is your plane perpendicular to the direction and then another is subscript will be talking about the direction in which it is acting.

So, again it will be positive if it points in the positive direction on the positive face and otherwise it will have the, you know negative type of you know notation.

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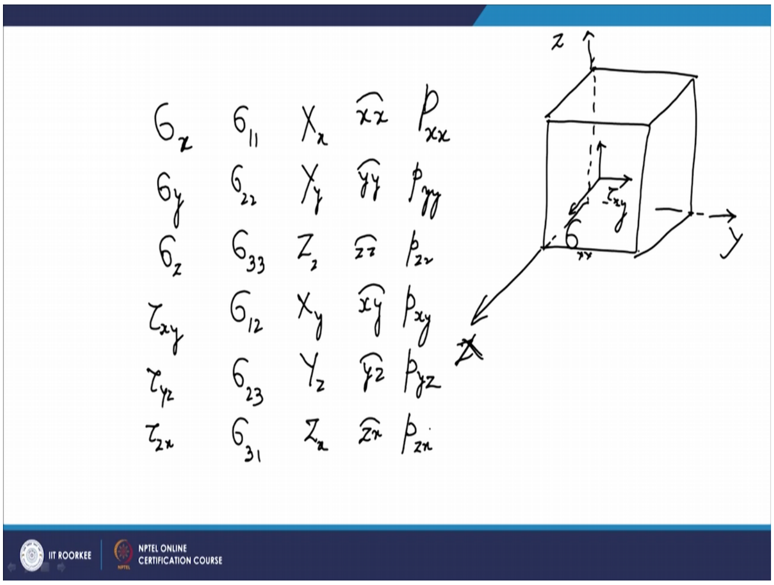

Now, the thing is that if you to talk about how this stresses are represented. So, what we see that suppose you have when we try to denote the, you know stress. So, in that case what we see is that suppose you have a cube and so, you will have so, this is how you have this is one is your suppose this is z axis and this is so, so this is x axis basically this is x axis, this is y axis and this is your z axis.

Now, in that case if you look at you will have the notations for the stresses in in all the directions and if you look at so, if this is the planes this is plane is the yz plane and in that basically you will have one stress which is in this direction. So, this will be your sigma xx. So, because this normal stress is parallel to I mean perpendicular to this x plane. So, I mean this is perpendicular in this plane which is and this x direction is perpendicular to this plane. So, you have sigma xx or which is also known as sigma 1 or you can call it as sigma x.

Now, you have two type of stresses. So, you will have to basically shear stresses. Now, if you look at that so, you call it this. So, because this is perpendicular to this x direction this plane so, the first subscript will be x and since it is working in the y direction so, you will have here y. So, this way you will have the value of the notation of the normal stress as well as the shear stresses and the same way it will be sigma x it is will be tau x and then it is in the that direction of tau xz. So, this way on any plane you will have the you know value of normal stress as well as the shear stress.

As far as we denote notation we denote it as suppose sigma x or we also denote as sigma 1 1 we some time also denote as X x then we also many a places you see the notation as like this or we also denote as xx. So, this way you have different types of notations for these normal stresses. Similarly, you have you have sigma y or this will be again then sigma 2 2, you will have you have you have.

So, this will be your Y y and similarly this will be yy and then this will be p yy. So, this way you will have the normal stress in the z direction that will be sigma z and you can also call it as 3 3 then you will have this is Z z so, this will be z z like this and this will be p zz. So, these are the normal stress components and in the different situations you denote in a different manner.

Coming to the shear stresses, they are denoted as tau xy. So, that is what we discussed that we denote it as tau xy we can also denote them as sigma 1 2, then we also denote them as X y. So, this will be y will be the direction we also denote as like this and then we also denote as p xy. So, these are the different way you denote these stresses and similarly, you will have tau yz and then tau zx. So, this is how. So, tau yz will be again it will be sigma 2 3 and this will be Y z this will be y z and this is p yz. So, this way it will be 3 1 and this will be Z x will be zx and then this is p zx.

So, this is the notation of the stresses in such cases now this is about the notation of stress.

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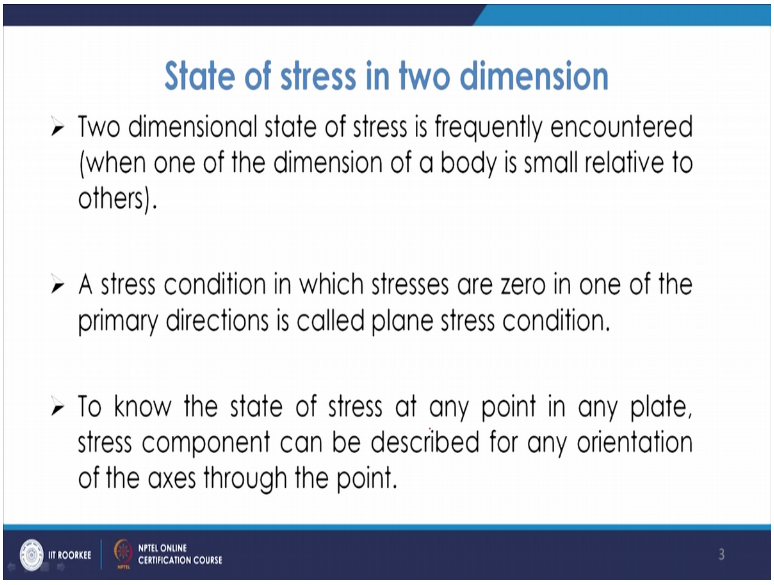

Now, we will discuss about the state of stress. Now, state of a stress in two dimension. So, when we talk about the stresses this two dimension two dimensional state of stress is a frequently encountered when one of the dimension of a body is small relative to other. Many a times we see that you are you are facing those situations where one of the dimension is very small as compare to the other two, like a very thin sheet if you look at. So, in that case the stresses are in that in that plane itself there will not be stress in the in the direction perpendicular that plane of the sheet. So, so, this is a case this is a case known as the plane stress condition where you have you know stresses are zero in one of the primary directions when the stress value is 0, then they are known as the plane stress condition.

Now, to know the stress that state of stress at any point in any plate the stress component has to be described for any orientation of the axis through the points. So, as we know that to if you have to find the state of stress at any point in the you know in the plate. So, what we do is that you find the stress component you will because that may be oriented at any angle with one of the axis and in that case you try to find what will be the, you know component of that in those inclination angles. So, that can be found out by coming to such example.

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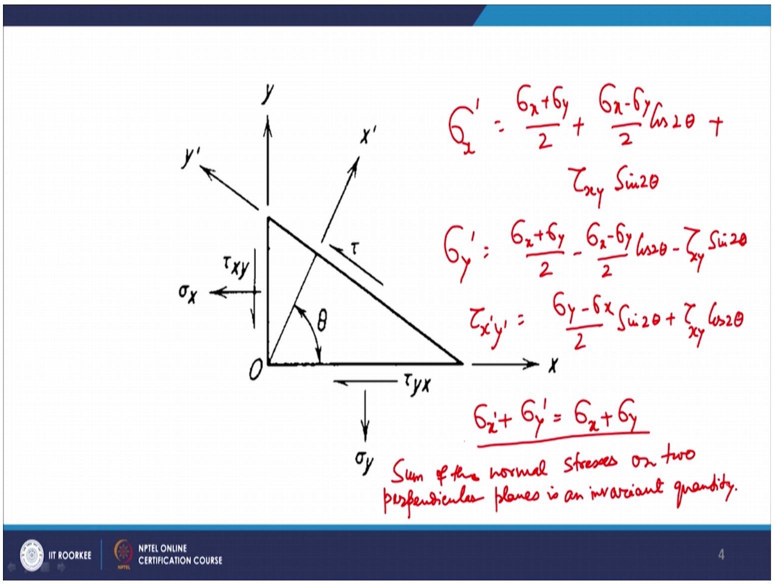

Now, what happens that if you look at such situation now, this is basically it is showing you know thin plate. So, now in this thin plate you have its thickness is in the normal to the paper. So, its thickness is like in this direction. So, its thickness is very very small. Now, in this case as we discussed that we need to find the state of stress at point O. So, for that we are we should able to describe the stress components at O you know for any orientation of the axis.

Now, what we do here is we are defining basically we are considering one oblique plane this plane which is there and normal to this plane basically is making angle theta with this x axis. So, this plane is basically you know taken and this oblique plane is normal to the plane of the paper and it is at an angle theta between the x axis and the outward normal to this plane. So, this outward normal is this x prime and this x axis this direction is making angle theta with this you know normal.

So, for that now what the purpose is that you have two directions; one is x prime and another is you know in the in the direction of the plane that is y prime and we have to find basically you have to find this stress components in those directions x prime and you have y prime directions and that can be basically you know measured. So, now what we is do here for that if you know revise the concept of your strength of material or so, you must have discussed about it and what we do here is basically what we have the direction cosines defined and this direction cosine basically is represented by l, m and n and this l, m and n, l will be so, that is a function of the cosine of this angle theta.

Now, for that what we do is you will have the component of the area in that particular direction. So, the direction cosine between x prime and x and y axis are basically l and m. So, l will be cos theta and m will be sine theta. So, basically what we have to do is we have to take the summation of the forces and based on that you further find the expression for sigma x prime and sigma y prime and also the shear stress which is acting in that particular direction.

Now, if you so, what we discussed that you must have studied about that. So, if for the, this is the angle theta in that case this sigma x prime this is coming to be sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos 2 theta and then you have tau xy sine 2 theta. Similarly, sigma y prime is also calculated and that comes as sigma x plus sigma y by 2 and then it will be minus sigma x minus sigma y by 2 cos 2 theta and minus tau xy sine 2 theta.

So, this is the expression for sigma x prime and sigma y prime this x prime and y prime we know that this is x prime is the direction this is normal to that oblique plane which is making angle theta with the x axis and y prime is the you know the direction in that along that oblique plane and then the expression for the tau x prime y prime also can be found out and that is basically coming out to be sigma y minus sigma x by 2 sine 2 theta plus tau xy cos 2 theta. So, this is how you try to find the you know component of the stress is sigma x is this is a normal component in the x prime and y prime directions and similarly, the shear stress component in that x x prime y prime plane.

Now, if you see this value sigma x prime sigma y prime and if you add them if you look at the addition of these two sigma x prime plus sigma y prime if you add them. So, what you see is that these two terms are cancelling. So, what you see is that this is coming as sigma x plus sigma y. It means if you are adding these two normal stresses ultimately you are getting one quantity, which is invariant which is not a function of theta. So, this is known as an invariant function. So, that is why the, we also write that the sum of the normal stresses on two perpendicular planes is an invariant quantity. So, this is known as the invariant of the stress.

Now, they are certainly some facts which can be you know found out by looking at these values. Now, the thing is if you take certain value of sigma x, sigma y and also the value of tau xy if that is given to you and if you try to see that, how they are varying with the value of theta then graphically you can have a feel and you can plot the value of sigma x prime, sigma y prime and also tau x prime y prime and from there basically you will have certain findings.

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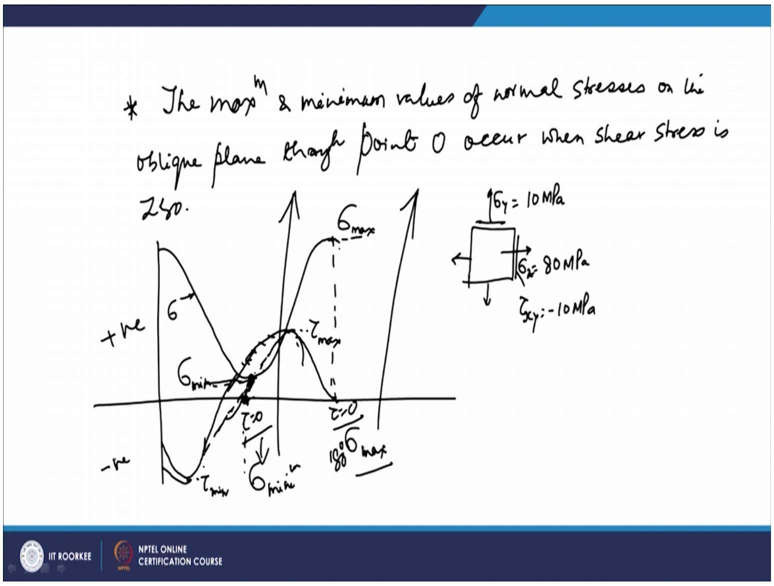

So, that finding you can have further though the findings are that the maximum and minimum values of normal stresses on the oblique plane through point O occur when shear stress is 0. Now, this can be found as we discussed that if you try to draw suppose you are taking one example suppose you have a case that you have a case where the sigma x, sigma y and tau xy is given suppose sigma x is given as 80 mega Pascal and sigma y also is given as you know 10 mega Pascal and then you have tau yx.

So, if the tau yx is or tau xy is to this will tau yx here it will have tau xy. So, tau xy is given as minus 10 mega Pascal. So, again here you will have sigma y, here you will have sigma x now for this case. So, as we see if you refer to the expression for sigma x prime and sigma y prime, then in that expression sigma x prime will be sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 cos theta 2 theta plus tau xy sine 2 theta. Similarly, sigma y prime will be sigma x plus sigma y by 2 minus of sigma x minus sigma y by 2 cos 2 theta minus of again tau xy sine 2 theta.

So, in that if you change the theta so, in that case what you see is you will get certain variation of the you know values and this you can try and you can draw it and if you draw you will get some curve like this and similarly, you will have the variation of the further the so, this is your sign of positive and this side you have negative, this is the value of sigma.

Now, you have another curve which is coming out to be. So, that will be here and then it goes like this and further it comes, comes and it comes down the maximum point. So, that will be you have to you just have to delete this points this comes the minimum here at this points where it is the minimum one.

Now, what we see that in such cases this is your minimum value this is r min and this is your sigma min this becomes as sigma minimum and this is your sigma maximum. Now, from the here you can have the you know and this line is for the tau. So, this your tau minimum and similarly you will have the tau maximum. Now, what you see is the first point which we have written that the maximum and minimum value of the normal stresses on the oblique plane through point O will occur when your this shear stress is 0. So, what you see is the shear stress is 0 when you get the maximum value, ok.

And, similarly the, this part where it is minimum basically it goes like not like this it goes like this. So, like this it goes not like this. So, basically at this point where you sigma is the minimum point, here also your, this line touches like this. So, here again at this point at this point the tau is 0 here also tau is 0. So, this point is your sigma minimum and this point is also sigma maximum. So, that is what this point tells that whenever you have for the maximum or minimum value of normal stress on any oblique plane through point O it has to be at that point when your shear stress is 0.

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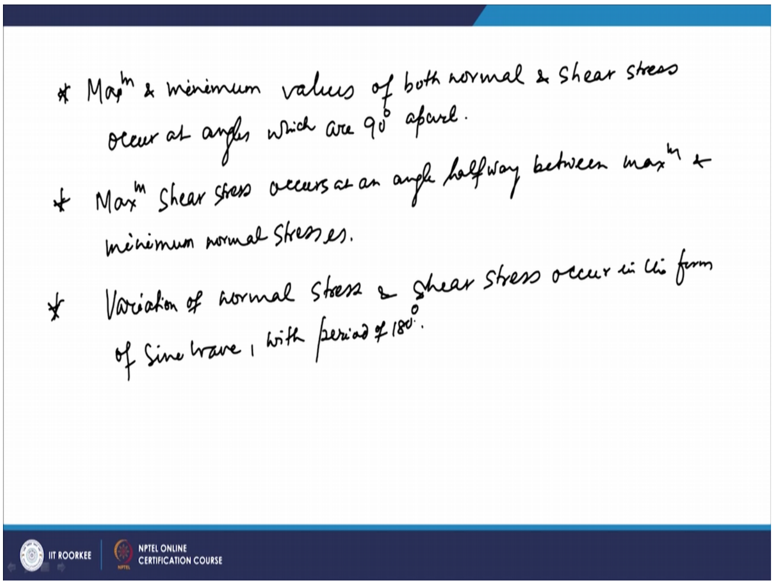

The second point which is also you know notable from this that the maximum and minimum values of both normal and shear stress occur at angles which are 90 degree apart. So, this is the second observation which is clear by looking at this you know graph that they are basically 90 degree apart. So, basically if you start from here this point is 90 degree upto this point this point is 90 degree, here where it is maximum and minimum is occurring that is 90 degree apart.

Third point is that the maximum shear stress occurs at an angle half way between maximum and minimum normal stresses. So, what we see is that the maximum shear stress value it will be occurring at an angle which is half way between the maximum and the minimum of the shear you know normal stress values. So, here if you see here or this one value or the maximum of the value it will minimum and maximum in between your this tau max occurs to be.

Then also the another observation which is clear from here that the variation of normal stress and shear stress occur in the form of sine wave. So, what we see is that this is going in terms of a sine wave with period of 180 degree. So, basically what is happening here is that, at this point it is coming as 180 degree. So, here is a maximum. So, this is also maximum you are getting the period of 180 degree after that it will again repeat. So, that is why it goes in the period of 180 degree.

Now, there is a concept of the principle plane and the principle you know plane actually when we talk about these findings now the thing is that we talk about these principle stresses. So, now the another concept is about the principle planes. Now, in this plane actually these are those planes, where there is no shear stress is acting. So, in the same expression which we have found for the, you know for the value of the shear stress tau x prime y prime now that is to be equated to 0.

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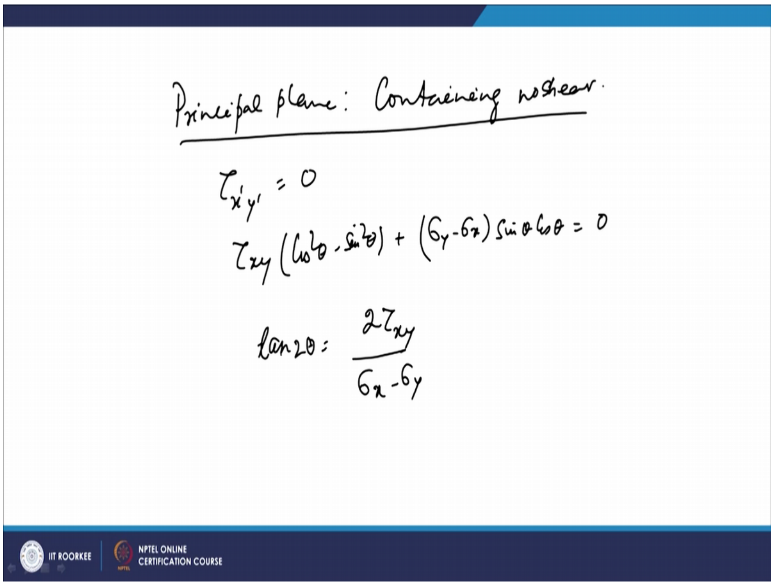

So, if you look at the principle plane that containing no shear, ok. So, basically it is about the you know principle stresses and their direction. So, as we know that in those direction there is no shear. So, now, the principle plane is defined by putting that 0 in the value of tau x prime y prime. So, in that expression we have seen that this tau x prime y prime is sigma y minus sigma x by 2 and sine 2 theta plus tau xy cos 2 theta and that is to be equated to 0.

So, that will be tau xy and cos 2 theta is cosec square theta minus sine square theta and plus sigma y minus sigma x and this is you know sine 2 theta so, that will be sine theta cos theta. So, earlier it was 2. So, that 2 come goes out. So, this way you have to equate it to 0 and in that case you get the value for the theta and from you here you get basically if you do the derivation you will get tan 2 theta will be 2 tau xy upon sigma x minus sigma y.

So, this is basically giving you the direction of this principle plane where there is no shear stress. So, that will have also certain significance when we talk about these things as you see that it is maximum value is if look at its value, the theta has to be 45 degree so, then it becomes tan 90. So, that is how you have the concept of you know deformation and all that. So, this is talking about that direction of this principle planes. So, we will discuss more about these 3D stresses in our next lecture.

Thank you very much.