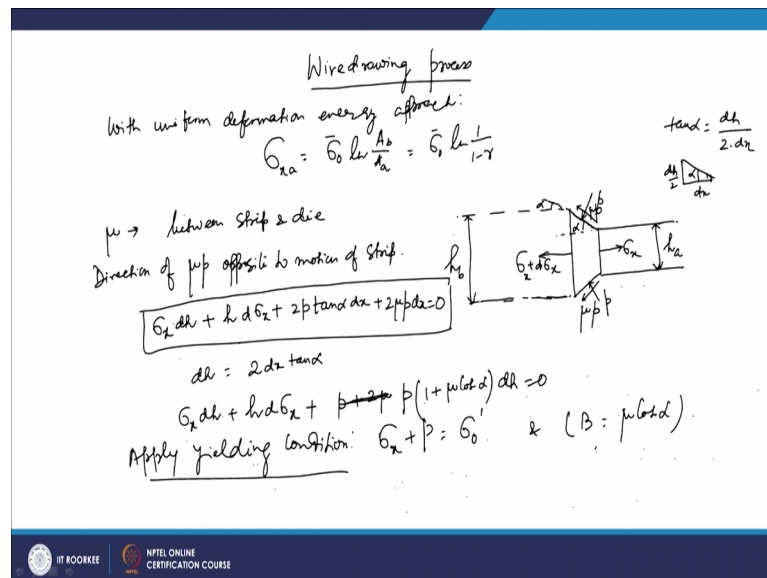


**Principles of Metal Forming Technology**  
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**Lecture – 36**  
**Analysis of wire drawing and tube drawing processes**

Welcome to the lecture of Analysis on wire drawing and tube drawing processes. So in this lecture we are going to discuss about the; you know finding that you know expressions, how to analyze this process when we draw what are the stresses which are caused and how can be find stresses values and so.

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So, if you talk about the wire drawing operations. So, we will start with wire drawing process, now what we see is that if you use this uniform deformation energy approach; then normally with uniform deformation energy approach, we can get the you know draw stress values  $\sigma_x$  and that will be basically average stress value and then  $\ln$  of  $A_b$  upon  $A_a$ .

So that is what we normally come across and this we also write it as so this is  $\sigma_x$  naught into  $\ln$  by  $1$  by  $1$  minus  $r$  so normally that is what we do so  $A_b$  by  $A_a$  is  $1$  by  $1$  minus  $r$ . So that is how we do using that uniform deformation energy approach and in this case we are not taking into account the friction. So we are basically neglecting the friction which is not very realistic type of consideration and also we are neglecting the

effect of transfer stresses which are you know generated and also the redundant you know deformation or shearing deformation. So, that is the neglected in this analysis.

Now if we take this friction into account then as per the line of the strip drawing what we have done earlier, while doing the analysis for the you know strip drawing process initially. We can do similar thing on the wire drawing process also and in that we will see that how we come; so suppose this also in that case also you have.

You have a conical die and so and then so what happens that this your so you have you know a rod which is of height or thickness  $h_b$  and then you are drawing with  $h_a$  so this is your  $h_a$ . So, that is how you know draw and in this case you have this angle as we take it as  $\alpha$ . So this is your angle that is  $\alpha$ , now what we do so this is your  $\alpha$  now the thing is that you apply this pressure  $p$  on on both this side.

And what happens that because of the you know movement of this in this zone, deformation zone [vocalized-noise] movement of the you know metal in the deformation zone you have the you know application of the frictional stress, frictional force will be acting and its direction will be opposite to the movement so the  $\mu p$  will be acting in this direction.

So, your  $\mu p$  is acting in this direction, so this  $\mu p$  and this is also  $\mu p$  so  $\mu p$  is the frictional you know stress which is acting in that case. Now apart from that we can take this as the at this point we can have the value of stress as  $\sigma_x$  and at this point we will have stress being of  $d_x$  you know this being  $d_x$ , so, this will be  $\sigma_x$  plus  $d_x$ .

So we can further do the you know you know force balance equations, you can put in and then again what we have done earlier we will use the equation of a equilibrium or the you know criteria of failure of material and from there you will get the expression. So you will have basically the [vocalized-noise]  $\mu$  is existing that the coulomb friction is there, so this is coulomb coefficient of friction it is between strip and die [noise], so this is your coulomb friction coefficient and we know that  $\mu p$  its direction will be opposite to the you know motion of the strip.

So direction of  $\mu p$  opposite to motion of strip, now the thing is that if you try to take into account the balance of the forces in the  $x$  direction what we see is that the  $\sigma_x$  into  $dh$  plus  $h$  into  $d\sigma_x$  plus  $2p \tan \alpha dx$  plus  $2\mu p dx$  equal to 0.

So, this is this equation you will be getting from this you know by if you assume if you take the balancing of the force because, you have this is your  $\mu p$  so  $\mu p dx$  so that will be your  $x$  component in that direction. You will have  $s d\sigma_x$  so because  $\sigma_x$  will be removed and then this way you will have this expression coming up.

Then now the if you take that  $dh$  if you take the  $dh$   $dh$  is basically  $dh$  will be  $2 dx \tan \alpha$  because  $\tan \alpha$  will be this by this, so this is your  $dh$  by 2 and this is your  $dx$ . So as you see that this  $dh$  by 2 and this is your  $dx$  and this is your  $\alpha$ , so  $\tan \alpha$  will be  $dh$  by 2 into  $dx$  so that is why you got  $dh$  equal to  $2 dx \tan \alpha$ .

So, if you put that what you get is so  $\sigma_x dh$  plus  $h d\sigma_x$  plus then it will be  $2 dx \tan \alpha$  will your  $dh$ , so it will be  $p dh$  so here it will be  $p$  and here it will be  $2\mu p$  so in that case this will be  $p$  into we can write further. So it will be here since  $dh$  is  $2 dx \tan \alpha$  so it will be our  $p$  here and if you take  $p$  into you know bracket, now this  $\tan \alpha$  will be there so you have the  $\cot \alpha$  here so that they you know cancel each other so it will be  $1 + \mu \cot \alpha$  into  $dh$  equal to 0.

So that way you can have so it will be  $p \mu \cot \alpha$  into  $dh$  so because this is  $dx$  so  $dx$  will be  $dh$  by 2 into  $\cot \tan \alpha$  will be in the denominators that will be  $\cot \alpha$ . So that way we got  $p$  into  $1 + \mu \cot \alpha$  into  $dh$  that will be equal to 0, so this is the term which you got. Now if you apply the you know the yielding criteria so if you apply the yielding condition.

Now, if you apply the yielding condition what you say we have already seen that you will have  $\sigma_x$  plus  $p$  will be  $\sigma_{naught prime}$  and also we take the  $\mu \cot \alpha$  as  $B$ . So we take  $B$  as  $\mu \cot \alpha$ , so it will be  $1 + B$ , so  $p$  will be  $\sigma_x$  minus  $\sigma_{naught prime}$ .

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$$\frac{d\sigma_x}{\sigma_x B - \sigma_0'(1+B)} = \frac{dh}{h}$$

$$\sigma_{xa} = \sigma_0' \frac{1+B}{B} \left[ 1 - \left( \frac{h_a}{h_b} \right)^B \right]$$

$$= \sigma_0' \frac{1+B}{B} \left[ 1 - (1-r)^B \right]$$

$$\sigma_{xa} = \sigma_0' \frac{1+B}{B} \left[ 1 - \left( \frac{D_a}{D_b} \right)^B \right]$$

So, if you rearrange this terms then what we get we get that  $d\sigma_x$  by  $\sigma_x B$  minus  $\sigma_0'$  into  $1+B$  it will be  $dh$  by  $h$ . So this is by rearranging this term so in this expression in place of  $p$  we will be putting  $\sigma_x$  minus, so  $\sigma_0'$  minus  $\sigma_x$  minus then into one plus  $\cot \alpha$  and  $\mu \cot \alpha$  will be  $B$  so  $1+B$  into  $dh$ .

So, that way you will be having  $dh$  term at one place and this  $\sigma_x$  at one place, so that way you can get this expression  $d\sigma_x$  by  $\sigma_x$  into  $B$  minus  $\sigma_0'$  into  $1+B$  equal to  $dh$  by  $h$ . Now we have to integrate it and if we are taking  $B$  and  $\sigma_0'$  as the constant values in that case we can directly integrate them to find the drawing stress and in that case what we get that expression as  $\sigma_{xa}$ , because again you have to put these boundary conditions in that  $x$  equals to  $B$  you will have  $\sigma_x$  values will be 0. So that can be put in and then accordingly once the integration constant will come then you can further get the expression, so this expression what you get is  $\sigma_{xa}$  will be  $\sigma_0'$  into  $1+B$  by  $B$ .

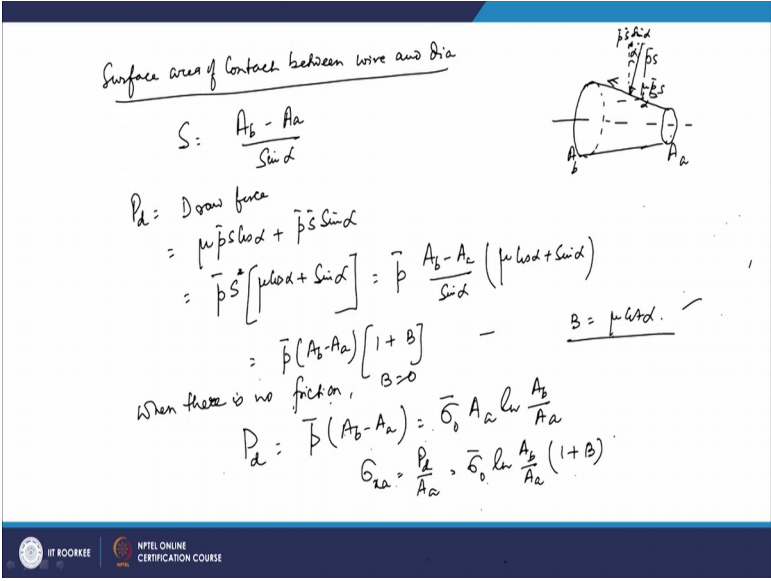
And then in the bracket you will have  $1$  minus  $h_a$  by  $h_b$  and that raised to the power  $B$ . So that is what we get the value of the you know; so this value we are getting in the case of the  $\sigma_x$ , and this can further be written as  $\sigma_0'$  and  $1+B$  by  $B$  and it can be written as  $1 - h_a$  by  $h_b$  will be  $1 - r$  so it will be  $1 - r$  raised to the power  $B$ .

So what we see here is that we can find the value of this  $\sigma \times a$  in that situation. We can also we can do that by using that another terminology, so if you take that circumference into picture in that case since you have the conical dice so taking the circumference of the die into you know picture you may have the expression like  $\sigma \times a$ ; so that will be  $\sigma \sin \alpha$  into  $1 + B$  by  $B$  and then you will have  $1 - \frac{D}{b}$  and then raised to the power  $2 B$ .

So that way you have the alternate expressions which are coming in that that case now coming to the different analysis by another researcher that is your Johnson and (Refer Time: 13:22) has given certain analysis and they have talked about the surface area of contact between wire and die.

So if based on this Johnson and (Refer Time: 13:34) analysis when the surface area between the die and the you know wire is taken into a count, if you look at that so suppose you take the you know 1 such case.

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Surface area of Contact between wire and die

$$S = \frac{A_b - A_a}{\sin \alpha}$$

$P_d$ : Draw force

$$= \mu \bar{p} S \cos \alpha + \bar{p} S \sin \alpha$$

$$= \bar{p} S [\mu \cos \alpha + \sin \alpha] = \bar{p} \frac{A_b - A_a}{\sin \alpha} (\mu \cos \alpha + \sin \alpha)$$

$$= \bar{p} (A_b - A_a) [1 + B]$$

When there is no friction,  $B = 0$

$$P_d = \bar{p} (A_b - A_a) = \bar{\sigma}_0 A_a \ln \frac{A_b}{A_a}$$

$$G_{ra} = \frac{P_d}{A_a} = \bar{\sigma}_0 \ln \frac{A_b}{A_a} (1 + B)$$

$B = \mu \cot \alpha$

So, now this is the you know wire which is drawn and you have, so here you have this is your here this is this area is  $A_b$  and this is area is  $A_a$ . So in that case and your  $p$  is being acted in this fashion, so what we see is that you have this as  $\alpha$ . So what you see is that this one this will be  $p \sin \alpha$ , so that is we will see how this you know  $s$  will be looking at.

Now this surface area of contact so if you talk about the surface area of contact between wire and die, so this surface area basically will be  $A_b - A_a$  divided by  $\sin \alpha$ , this is the you know surface area. Now the mean pressure so this will be now this you know which is acting here so this is basically this into as this force which is acting here the this multiplied by  $s$ .

Now, the draw force which is calculated if you try to find these draw force is  $P_d$  that is your draw force [noise] so it is basically to be balanced by the you know frictional force and the horizontal component of the you know pressure this one this normal pressure, so it will be basically  $\mu p s \cos \alpha + p s \sin \alpha$ .

So basically that is your you know draw force you have this frictional force as well as the normal pressure which is there on this it has its component and based on that that will be so you have frictional force acting that way this also is acting in that direction, so  $p$  has its  $\alpha$  part being normal part and its horizon its horizontal part it will be  $p s \sin \alpha$ .

Similarly, you will have  $\mu p$  will be acting in this direction  $\mu p s$  as will be acting in this you know direction and its horizontal component will be  $\mu p s$  into  $\cos \alpha$ . So  $\mu p s$  as is since it is acting in this direction this being  $\alpha$  and this is being  $\mu p s$  so it will be  $\mu p s \cos \alpha$  and  $p s$  into  $\sin \alpha$ . So what we can write is that we can write as  $p$  and  $s$  and then it will be  $\mu \cos \alpha$  here and you will have  $\sin \alpha$  here. So what we can write  $s$  we have already found  $A_b - A_a$  upon  $\sin \alpha$ , so it will be  $p$  into  $A_b - A_a$  upon  $\sin \alpha$  and then you will have  $\mu \cos \alpha + \sin \alpha$ .

So it can further be written as so  $\cos \alpha$  by  $\sin \alpha$  will  $\cot \alpha$  so we can further write  $p$  and  $A_b - A_a$  and then this  $\sin \alpha$  will be divided, so it will be  $\mu \cot \alpha + 1$ . So  $\mu \cot \alpha$  is we have defined as  $B$  so it will be  $1 + B$ ,  $B$  is  $\mu \cot \alpha$ . So this is how we can find these draw force ah.

Now, what we can come we can also find we can also draw the conclusion from this expression that if  $B$  is 0. Now  $B$  will be 0 when  $\alpha$  will be 0 or  $\mu$  will be 0, so when the frictional you know there is no friction in the case of absence of friction so when there is no friction in that case your  $B$  becomes 0.

So, your  $P_d$  draw force that becomes equal to  $p$  bar into  $A_b - A_a$ , so that is what it is coming and then you that you can write as  $\sigma_n A_a$  into  $\ln A_b$  upon  $A_a$ . So

this is the same one which is similar to the one which we had earlier done during the uniform deformation energy approach.

So, there we had got the so from in those cases if you ah, so this is for the without the case of friction so if you apply the friction you will have the this expression. Now in that case if you so we draw if you apply try to have the expression with draw stress with friction, so ultimately sigma x a it will be P d upon A a. So if you are trying to draw that I mean find the value of draw stress and in if taking friction into count so friction is here, so P d by a and it will be sigma naught prime l n of A b by A a and into 1 plus B.

So, that is expression when we try to if you try to see the approach by the Johnson and (Refer Time: 20:15) they have done the analysis we get the similar you know you know methods for similar expressions for finding the draw stress values. So you may be told to sometimes solve the problems based on such you know wire drawing operations,.

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Prob: Find Drawing stress to produce 20% red in a 10mm wire.  
 flow stress is given as:  $\sigma_0 = 1300 \epsilon^{0.30}$   
 $\mu = 0.09$ , Die angle  $= 12^\circ = 2\alpha$   
 B:  $\mu \tan \alpha = 0.09 \tan 6^\circ \rightarrow 0.009$   
 $\epsilon_1 = \ln \frac{1}{1-r} = \ln \frac{10}{9} = 0.105$   
 $\bar{\sigma} = \frac{K \epsilon_1^n}{n+1} = \frac{1300 (\epsilon_1)^{0.30}}{1.3}$   
 $A_0 = 10 \text{ mm}$ ,  $A_1 = 9$   
 $\sigma_{da} = \frac{\bar{\sigma}}{1+B} \left[ 1 - \left( \frac{A_1}{A_0} \right)^B \right]$   
 $\sigma_{da} = \bar{\sigma} \ln \frac{A_0}{A_1} [1+B]$   
 Result:  $240 \text{ MPa}$

So you may have certain problem suppose you may face with this problem suppose you have a problem where you have to find the drawing stress [noise] where you have to produce 20 percent reduction and in a 10 mm wire, so the flow stress may be given and and flow stress is given as; so the flow stress is given as suppose you may not able be 1300 epsilon e raised to the power 0.3. Suppose this is how the flow stress value given in terms of megapascal this is the flow stress given you have to find the drawing stress.

So, you can find the drawing stress with the help of the different you know approaches and so you have 1 by using that strip drawing approach another may be by the method which is given by the (Refer Time: 21:39). So suppose in that you know that you have being given this you know the so that angle is given as the 128 degree so the die angle will so  $\mu$  is given as 0.09 and die angle is given as suppose bar 12 degree. Now the thing is that based on this you can have so first of all you will find the B and we know that the B is  $\mu \cot \alpha$ , so  $\mu$  is 0.09 and then  $\cot \alpha$  this is basically  $2 \alpha$ , so  $\tan 6 \cot 6$  degree this way you can get the value of B.

Similarly, you can find the so now, this  $\epsilon_1$  now  $\epsilon_1$  will be actually  $\ln$  of 1 by 1 minus  $r$ . So we know that there is a 20 percent reduction so it will be  $\ln$  of 1 by 1 minus 0.2, so it will be 1 by 0.8 and that that value may be calculated. So  $r$  is basically your 0.2 so this way you can get this value also then now you can get the value of every stress value and that will be that is given by this value.

So this will be normally  $k$  and  $\epsilon_1$  divided by  $n$  plus 1,  $\epsilon_1$   $\ln$  so once you know this  $k$  is given 1300 and then this  $\epsilon_1$  is taken from here and then that raised to the power you know  $n$ , so  $n$  is given as 0.3 and then you have 1.0 plus 0.3 so 1.3.

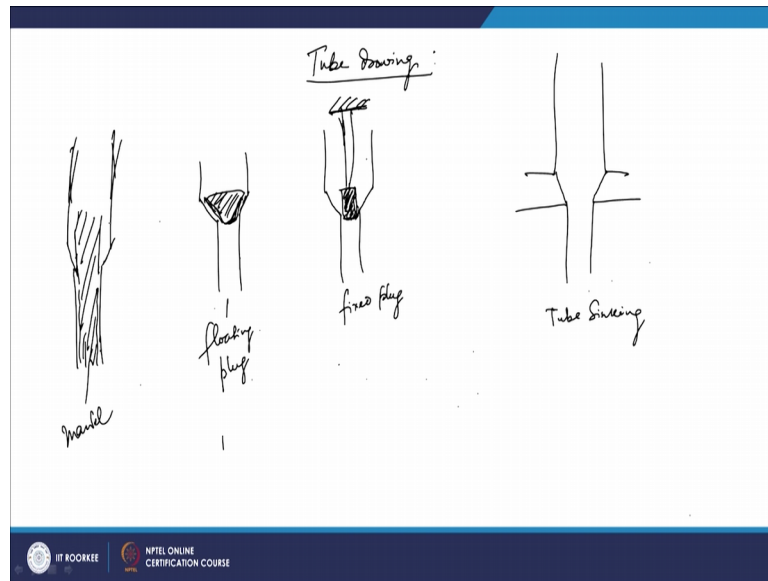
So, this way you can get the sigma you know average stress value and then you have the value of A b and A a is as given so based on that A b is 10 mm, so similarly A a will be 8 mm and based on that you can have the expression for sigma x a as 1 plus B by B and then 1 minus A a by A b raised to the power B.

So, you know every term and you can get this value, similarly you can use the row analysis and from there you can get the expression sigma x a as sigma average value and then  $\ln$  A b by A a and then in bracket 1 plus B. So from here also you can get the values and also you can see that how much they are approaching each other.

So, you can do the work you can do the analysis and in this case it comes 240 and this comes as 264 megapascal. So you can check this by solving and you will get the confidence that how to you know get these values in such cases.



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Now, we will come to analyze the tube drawing process. So, as we discuss that we do the tube drawing process either without using the plug or mandrel or we can use the plug or mandrel, now when we do not use this you know plug and mandrel and then it is known as tube sinking and other wise when we use the plug you have the fixed plug or the floating plug or you have the mandrel. So what we do is normally when you have you do not use when we do not use the you know this any kind of plug or mandrel then it is known as tube sinking. Then further when we use the we use the plug which is fixed or may we use the plug which is floating, so in that case if suppose you have this as the die.

Now, in that case if you are using a plug, so this plug will be like is and this plug will come it will be coming here and then this may be fixed. So if it is fixed at these point and if you are doing that you know tube drawing known as fixed plug process of you know this is a case of fixed plug.

You may have a floating plug also, so the floating plug comes like this you have this situation and then you have the floating plug which will be the in between there will be clearance and then you will have this as the floating plug. So this is a case of floating plug otherwise you may have the use of the mandrel, so in that case you have see the use of the whole mandrel is there.

So this mandrel will go like this and based on that so this is your whole mandrel. So, you can use the mandrel itself directly and that is the use of mandrel to get, so these are the

different methods of using for making the tubes in the you know in case of the tube drawing operations now when we talk about the analysis of this tube drawing processes, now what happens that in the in the case of this tube drawing you have either use of the plug or the mandrel.

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Analysis of Tube Drawing Process

$$\sigma_{xa} = \sigma_0 \frac{1+B'}{B'} \left[ 1 - \left( \frac{h_a}{h_b} \right)^{B'} \right]$$

$$B' = \frac{\mu_1 + \mu_2}{\tan \alpha - \tan \beta}$$

$\mu_1$  : Coeff. of friction between tube & die wall  
 $\mu_2$  : Coeff. of friction between tube & plug.  
 $\alpha$  : Semi cone angle of die  
 $\beta$  : Semi cone angle of plug.

And when we are drawing with a plug in that case similar to the equation which we have done earlier, we can have the sigma x a. So here you have at the tube so you have you know sigma x you will be basically sigma naught prime and you have 1 plus B naught by B naught and it will 1 minus h a by h b and raise to the power B naught.

So, what happens that now in this case you have mu 1 and mu 2 both are you know working. Now in this case what happens that your B prime another parameter is defined and this parameter is mu 1 plus mu 2 upon tan alpha minus tan beta. Now in this case the mu 1 is the coefficient of friction between tube and die wall and that the mu 2 will be your coefficient of friction between tube and plug.

Now, what we see you can see from here you would have 2 places where you have coefficient of friction between and between this and the you know tube and then again you have the coefficient of friction between the tube as well as the plug 2 you know interfaces which is there and you will have the friction acting there and that is why you will have you know 2 friction terms in such cases. So you will have and that is why the B

prime is defined by taking into this  $\mu_1$  and  $\mu_2$  otherwise you had only one interface or the friction was working.

When we use this power mandrel in those cases you have this  $B \mu_1 + \mu_2 \tan \alpha - \tan \beta$  and  $\alpha$  is the semi cone angle of die and similarly you have  $\beta$  as semi cone angle of plug. So if you have a cylindrical plug then that case that  $\beta$  will be 0 and based on that you will have the you know value of this  $\sigma \times a$  can be calculated.

So, that happens normally in the case of tube drawing and find all these parameters you can find the value of  $\sigma \times a$ . Now this  $B$  prime that may take the different value may be you may have depending upon the mandrel or the which is moving many times then in that case it will be  $\mu_1 - \mu_2 \tan \alpha - \tan \beta$  and in that case you will find the  $B$  dash to be different and then accordingly you will find the value of  $\sigma \times a$ , so that is how so the thing is that you must know that why this term is coming this term is coming.

Because there is friction at 2 places one is at these you know there are 2 places where this pipe is facing the friction one is on the left side and another is toward the right from the plug or mandrel side. So because of that this is coming, so there are many analysis by different authors.

And basically the concept is the same depending upon that either uniform deformation energy approach or by is tube drawing approach process approach you can or using that slab method you can have the calculation of the  $\sigma \times a$  and so this is how this analysis should be carried out you can solve problems based on that you have more clarity on these topics.

Thank you very much.