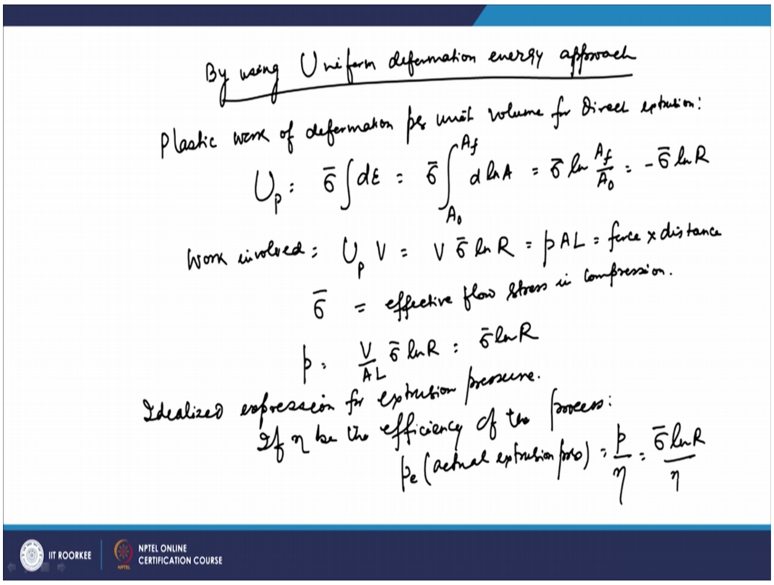
Principles of Metal Forming Technology
Dr. Pradeep K. Jha
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 33
Analysis of extrusion processes

Welcome to the lecture on Analysis of extrusion processes. So, in this lecture we are going to discuss about the expressions, which are required to find the extrusion load calculations, and all that by the different methods of analysis. So, if you try to see that you know analysis of this process now, if we use the so, by using uniform deformation approach deformation energy approach.

(Refer Slide Time: 00:54)



So, we have got some idea about uniform deformation energy approach, where we talk about the plastic work of deformation. So, the plastic work of deformation per unit volume if you look at for direct extrusion. Now that can be taken as U P and it will be nothing but you take this average value of a stress, and then d epsilon. So, that is the plastic work of deformation per unit volume, and this can be taken as further you have d epsilon. So, it will be you are doing from A naught to Af and it will be d ln A.

So, that will be coming out as epsilon bar, sigma bar that is average stress and then you have ln of Af by A naught. So, it is nothing but minus of sigma bar and ln R. So, that is what if you do it through the uniform deformation energy approach you get this value U P.

Now, if you take the work involved, now what is work involved? So, it will be if this is the plastic work of deformation per unit volume. So, this multiplied by volume, it will be the work involved. So, it will be U P times V so, V is the volume of the work piece. Now it will be so, so you have v, and then you have sigma bar into ln R. So, that will be your work involved, and you can that will be basically the pressure into A into L.

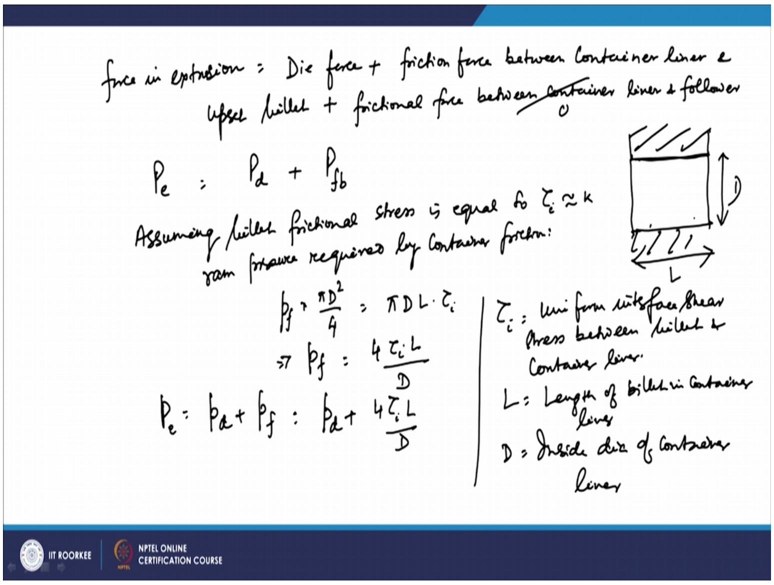
So, this will be in a force into distance is nothing but the force into distance. So, p into A will be the pressure. And then you have the distance so, it will be force into distance that is what the work done it is. So, as we know that you define this sigma bar, it is as you defined as effective flow stress in compression. So, from this expression V sigma bar ln R equal to p AL, from here we get p as the V by AL and then sigma bar ln R. So, since AL is V so, it will be sigma bar ln R. So, what we see is that this is the extrusion pressure p is the extrusion pressure, and this is nothing but sigma bar into ln R. Now in this case this is basically the idealized expression for the extrusion pressure.

And so, this is idealized expression for extrusion pressure ah, because it is not considering either the frictional you know effects or the redundant deformation. So, that is why it is the expression for the idealized expression for the extrusion pressure. Now if we incorporate the efficiency of the process, so, if the if eta is the be the efficiency of the process, now in this case the pressure which will be required will be nothing but p by eta.

So, so the the you can take actual extrusion pressure. So, Pe that is your actual extrusion pressure. That will be this p divided by the efficiency eta, and that will be your sigma bar ln R upon you know eta.

So, this is how you try to find the you know expression for or calculate the value of actual you know extrusion pressure. Now the analysis by or by once you the measurement or the forces has been done by one of the you know scientist dpr. So, he has shown that he has shown that the force which is an extrusion it has many components. So, he has shown so the dpr has shown that the force in extrusion so, it consists of many you know components.

(Refer Slide Time: 06:44)



And it will be a sum of the die force, then you will have the friction between the billet and the container wall.

So, container liner and the upset billet. So, friction force between container liner and upset billet. Then there is another friction and that was between the container liner and the you know follower. So, the frictional force between container liner and follower; which is following from the back so, that basically is normally considered to be 0. So, you are taking basically the 2 types of you know pressure, force that is die force so, you are taking this force in extrusion, it will be the die force we call it as the Pd and the friction force between the you know container liner and the upset billet, and that we are basically representing by the fb.

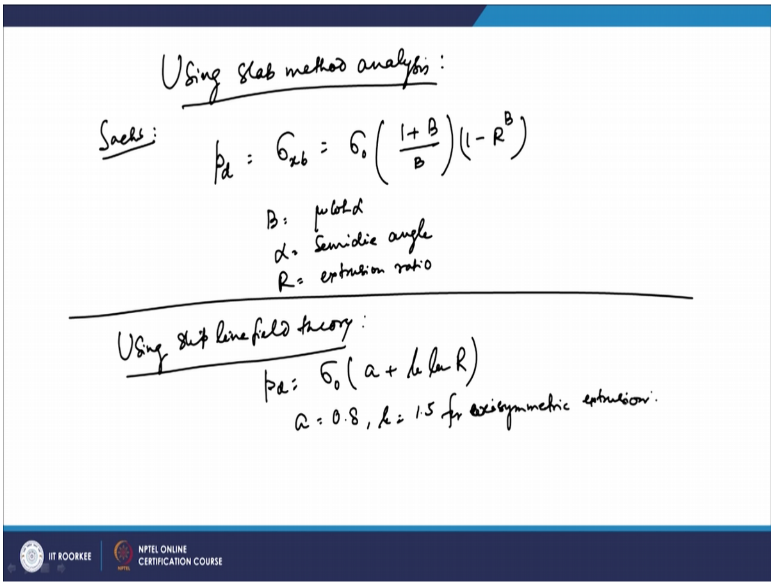
So, this is f for the friction, and between the container liner and the billet. Now if you assume that this billet frictional you know stress, that is tau i because and that is equal to k. So, assuming billet frictional stress is equal to tau i, that is similar to the k, in that case if you talk about the ram pressure. So, if you look at the ram pressure so, the ram pressure required by container friction. So, in that case if you look at this ram pressure which you calculate now if that is pf so, that will be multiplied by pi d square by 4.

So, this is been applied on that area pi d square by 4. Now that will be basically you are if you talk about this friction which is acting on that you know in between that, cylinder that is liner curve the container and the moving a billet. So, in that you have pi into d and into L and that is multiplied by this ti. So, that will be pi D L into that tau i so, that is what you can have from here, and if you try to find this pf. So, this pf will be 4 into. So, you have 4 go will go from here and D and D will cut on one side.

So, it will be 4 into tau i and pi and pi will cut D and D will cut so, 4 tau i L upon d. So, what we see is that if you look at this expression, what is being found further. So, you will have if you to log of the pressure. So, this is Pd plus pf and that is basically Pd plus 4 tau i L upon d. So, this terminology is tau i it is the uniform interfacial stress between billet and container liner. And similarly you have L which is they have which is the length of these container liner billet, that which is the container with that container liner. So, length of billet and container liner, and D is the inside diameter of the container liner.

So, that is what I mean you know that you have these. So, so this becomes your L and this becomes D. So, this how this is how it because it has to move so, this is container. So, so that way this is your L and this is your D. So, once you measure this Pe and Pd, once you know that, then you can you know find the appropriate value of this fictional stresses. A simple approach by the slab method analysis has also been carried out.

(Refer Slide Time: 12:32)



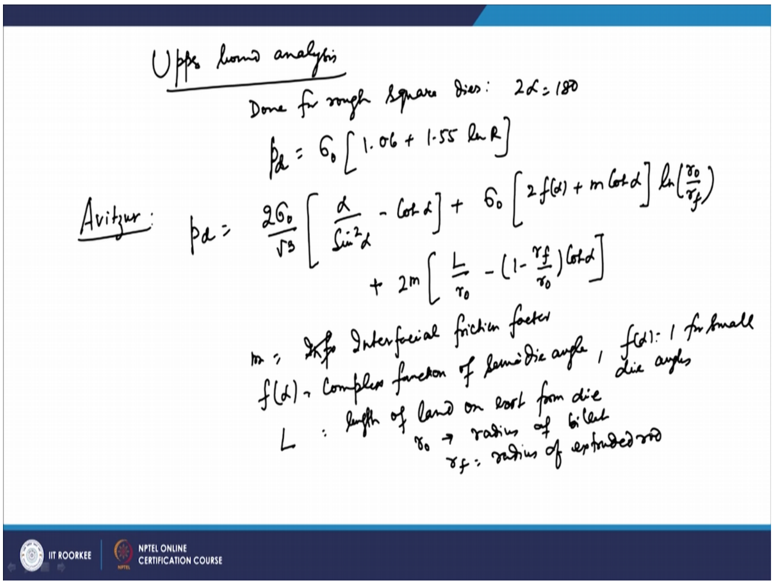
So, using that slab method analysis.

Now, in that basically that has been expression for this die pressure, and in that you know you have taken that friction into account, and you have the there is conical die, and the material is extruded through that conical die. On for that sacks has performed you know the analysis. And they have done the analysis for using that coulomb friction. And coulomb sliding friction basically and this has been soon out. So, similar to that which is stand for the wired drawing case also, now in this case Pd is said to be Pd as sigma x be that is sigma naught into 1 plus B by B and that is 1 minus R raised to the power B.

So, this is the expression for the die pressure which is required. In this case the b parameter b is the mu cot alpha, alpha is the semi die angle in this case. So, alpha is semi die angle, and R is extrusion ratio. So, basically this work involve the friction, but does not involve taking into account the redundant force ah, which is there in the case of extrusion. There has also been analysis by using this slip line field theory, for the plain strain deformation cases and for these using (Refer Time: 14:26) slip line field theory. So, most of these expression which used to come will be of the form p d will be actually equal to sigma naught into a plus b ln R.

So, that has that work with this analysis in that which is coming to of; let us say a and b are said to be the constant, and a normally is taken as 0.8 and b as 1.5 for the axisymmetric type of you know extrusion. So, there has been reported by many researchers, like the hill has Robert hill R hill has given this expression, Pd has sigma naught a plus b ln R a is taken as 0.8 and b as 1.5 and this is for axisymmetric rotation for axisymmetric extrusion. Now even there has been analysis in the upper and lower bound you know upper bound analysis, and when the analysis has been done using the upper bound process.

(Refer Slide Time: 15:57)



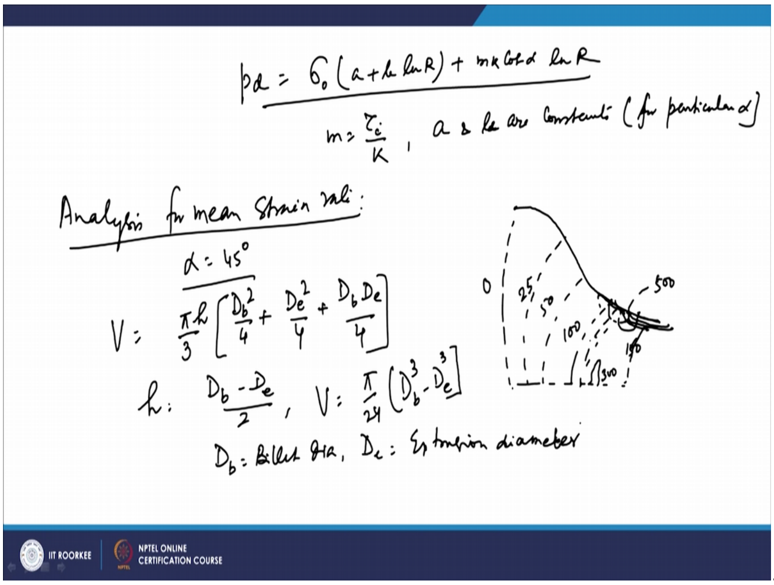
So, upper bound analysis has been performed by researchers and kudo has done this analysis using this upper bound analysis. And they have done for the rough square dies so, they have done for rough square dies. So, in this case 2 alpha is basically 180. So, in that case they have found Pd as sigma naught into 1.06 plus 1.5 ln R. So, in that case the A, which was there in the A in that axisymmetric type of case suggested by hill. Now here kudo has suggested, Pd as sigma naught into 1 plus 0 6, a will be 0 1.06 and b will be 1.55 times the ln R.

Now, there has been you know other expression which has been suggested by avitzur. And avitzur has given a more generalized expression using the spherical you now velocity field. So, avitzur so, he has given more general type of generalized expression ah, using that spherical velocity field. And it will be applied for the lubricated extrusion through a die or semi die angle alpha. And we has shown the expression as Pd equal to 2 sigma naught by root 3, and then it will be alpha by sin square alpha minus cot alpha, then plus again sigma naught multiplied by 2 f alpha plus m cot alpha. And that will be multiplied again with ln R naught by rf. So, and then further you have another term 2 m into L by R naught minus 1 minus rf by R naught into cot alpha.

So, this is the generalized expression for the that is done by avitzus. And in this case you have m as interfacial friction factor. Then you have f alpha it is complex function of semi die angle. So, very for very small angle say for will be normally one. So, f alpha is one for a small die angles. Then L is L which we get in this case it is length of land on exit from die. And then you have this L is coming here, than another parameter is R naught so, R naught is the radius of billet and R f is radius of the extruded rod.

So, if you know all these parameters you can find the expression for Pd which has been suggested by avitzur. And further there has been other analysis using the upper bound analysis further, and there has been good agreement for the experimentations done with hydrostatic extrusion.

(Refer Slide Time: 20:44)



And dpr further shown that if you look at the expression by dpr, then he has shown that die pressure sigma naught a plus b ln R plus n k cot alpha ln R. So, in in this case m is coming out to be tau i upon k and a and b are the constants.

So, depending upon the die angle semi die angle a and b are constants for particular alpha that is semi die angle, and for the die angle as it is increasing the value of a and b basically will be slightly increasing. And basically in the value of a will be more increasing as compared to b. There has been further the analysis by studying the velocity field, in the case of this extrusion process.

And what has been seen that if you have a mean you know temperature of extrusion which is there inside the domain. Then strain rates are actually calculated, and they are seen to vary inside you know that domain. And it is seen that the strain rates are maximum at certain positions. And average rate is then computed for that particular you know truncated conical volume region.

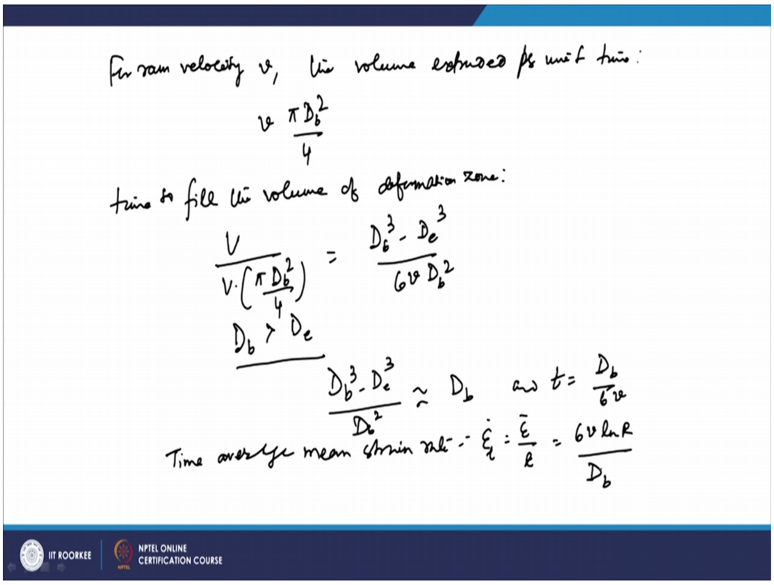
So, if you are taking one semi die angle so, you have analysis for mean strain rate, and in that basically the flow field analysis has been carried out you have distribution of stress and strain rate and all that has been calculated, and you have the variation of temperature has also been recorded. And what has been seen that near the exit you have the maximum you know strain rate values. So, it was seen that it goes like this value so, what was seen that, this is the point of 0. Then as it goes it will you will have a you know strain rate value of 25, then you have 50, and what was seen that it was further increasing to 100, and then what was see in that here.

So, this is basically coming down so in this locality where there is further there is drop, now in this locality you have maximum, and that comes out to be around to about 500 maximum strain rate is recorded. Now here also it is recorded 300 something like. So, this way you have calculation of there further it is decreasing. So, this come comes out to be 150 or so. You have increase it to 200 or 250 or so. So basically if you do for these you know alpha, when it is taken as 45 degree, in that case, the how to calculate these mean you know strain rate or average mean strain rate.

So, for that what we do is we can define this V, V as this should be having conical say taking into account. So, you know that for conical shape you have pi by 3 you know. So, pi by 3 is taken, and then so, you will your h will be taken and then you have. So, d square term so, that is R square term so, d square by 4. Now you have Db square by 4, this is the first term, then you have De square by 4. And then you have Db into De by 4. So, you know that we have been the volume if you calculate it is 1 by 3 pi R square h.

So, 1 by 3 pi h is there and R square that is dv square by 4. So, you have taken all these you know diameters into account dv square by 4 De square by 4 plus dv into De by 4. So, h basically you can take average one and that you can take as Db minus De and by 2. So, that you can take it as h. So, your V becomes pi by 24 into Db q minus De q so, Db is the below diameter and De is the extrusion diameters. So, that Db is the billet diameter, and De is the extrusion diameter. Now if you look at the ram velocity which is V so, volume extruded per unit time so, for ram velocity V ah, the volume extruded per unit time if you try to calculate so, that will be V into pi and Db square by 4.

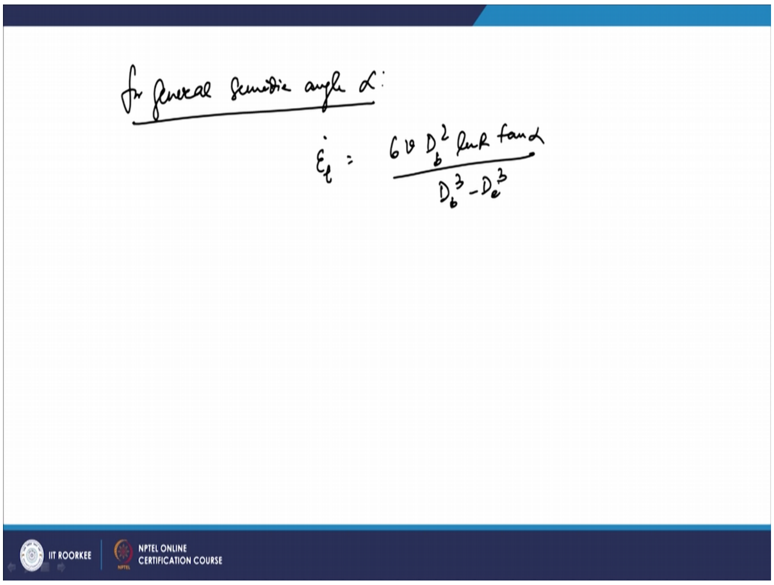
(Refer Slide Time: 26:06)



So, that will be the volume extruded per unit time. And if you see that time to fill, the volume of the deformation zone so, it is nothing but volume divided by they this area multiplied by this. So, you have you know so, area multiplied velocity so, this is a same thing. So, volume by volumetric you know expansion per unit time so, that will be the time. So, it will be V by V into pi Db square by 4. So, and that is has to be equal to Db cube minus De cube and divided by 6 V Db square.

So, if this since Db is quite larger than de so, Db being quite larger than De, in that case you can write that Db cube minus De cube by Db square, you can take as Db because this De term can be neglected, and you also you get t as dv by 6 V. So, from this expression, you can have the expression for the mean strain rate. So, time average mean strain rate if you calculate. So, that will be basically epsilon t dot, and it will be epsilon bar by t, and it will be 6 V ln R by D b. So, this is basically the computed value of the time average mean strain rate.

(Refer Slide Time: 28:45)



And if you lock c for the general semi die angle alpha so, for general semi die angle so, for general semi die angle alpha, if you try to calculate so, this epsilon t this will be 6 V Db square ln R tan alpha divided by Db cube minus De cube.

So, this basically expression can be used to calculate this strain rate, these mean average means strain rate, and that can be used further for calculating the flow stress values once you are given any problem in such type of you know analysis. So, it all can be seen from here how we use that you know flow stress related these when the analysis has been carried out. And using that mean strain rate you can calculate these average means strain rate values. And for that that can be used for solving the different type of problems.

Thank you very much.