Principles of Metal Forming Technology
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Lecture – 28
Forging in Plane Strain

Welcome to the lecture on Forging in Plane Strain. So, in this lecture, we are going to discuss about the analysis of the forging process. So, we discussed about the, forging processes, in which the, RAM from the top will come and being touch with the stock and then it will try to further deform it. So, it is in touch with these RAMs or the dies and then when it is deforming. So, you will have it is spread as the lengthwise or widthwise or thickness wise, but for analysis what we do is, we consider a case of plane strain.

So, in that we take you know a strip and, that is forced and you know we, we take a strip of constant thickness and then under width will be remaining constant and so, it will be spreading in the lengthwise direction and for that we are analyzing the main thing, which is analyzed in this. Process is the forging pressure which is required and, during that process basically, you will have a friction generated.

Because was there will be force by the RAM top, RAM on the, the body. So, it will try to expand and, and. So, there will be a, a frictional you know force generated. So, under that application, how can you find these pressure so, far that we will be analyzing. So, we are analyzing a situation for a plane strain, you know problem, where we will analyze the forging process; so, suppose, we are talking about the surface.

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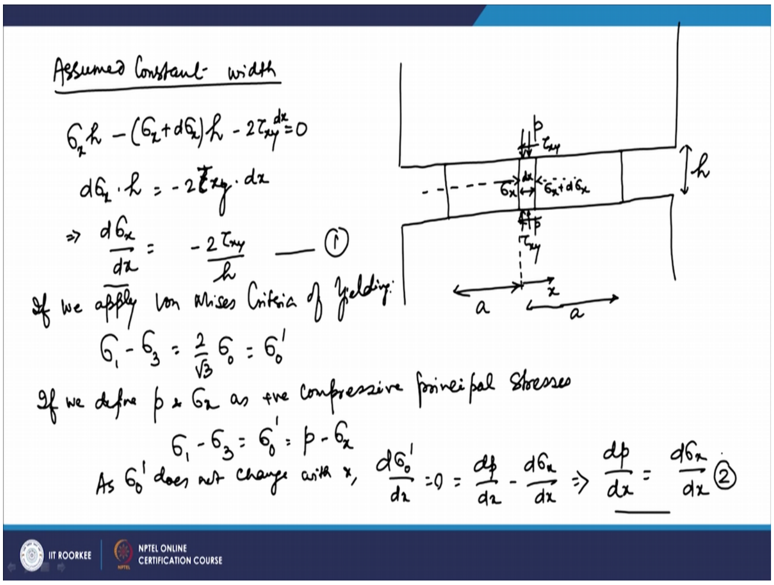

So, we have such is the strip and it has, the thickness is constant all along and it is in touch with basically the dies. So, these are the dies, on both the sides and now, in this basically, we are going to consider a strips. So, suppose you have a strip, which is of certain length. So, that these, this will be taken as of the length d x and we are applying the pressure.

So, if you are considering this element, small element. So, we are considering about the pressure, which is created. So, that is p. So, this is p is created applied on both the sides and we are talking about. So, basically, we have got this as the origins of basically, we take x into this direction and you have d x as the, you know length of this strip. Now, at this point, you need take. So, basically, when you apply, now what happens that there will be, you know friction shear, because there will be lateral flow.

So, perpendicular to the RAM travel direction and that will lead to the frictional shear stresses at, these surfaces and this, shear which is applied they will be directed towards the center line. Because you have the movement in the outward direction or so, their frictional shear will be acting towards the central line.

So, this way you will have the frictional shear acting and this will oppose basically the, the mean flow, which is going to occur. Now, if we see that this, the, the frictional you know forces, the presence of this friction, this will be causing and imbalance in the, you know on it's this force, which is acting on it and that is why you will have the, that will accommodated by the change in the pressure, which is acting on these two, you know sides. So, on this side we assume that the stress which is acting. So, we, we have this side, here the stress acting is sigma x and, and then this on this side. So, you have this stress which is acted.

So, since it is acted the, dx distance of; so, x plus d x distance. So, basically we take it as sigma of x plus d sigma x. So, that will be increment in the, you know stress which is being caused. Now, what we assume that in this case, we assume that the, the your width is unity, in the normal to the plane of the paper and this width remains constant. So, we have assumed the constant width. So, that is why if the, you know, there is no change in the dimension in particular direction. So, it is a so, there is no strain in particular direction.

So, it is a case of the forging in the plane strain. Now, what we do is we have to analyze. So, we are taking this height in between as the height h. Now, if we, take; so, we are assuming that you have, this is from here the x goes. So, this half-length is here a, and this side also you have a. So, whole length is basically, 2 a l is basically equal to 2 a now the thing is that as we discuss. So, if your element is from here in that case your frictional stress will be towards the centre line. So, friction stress will be in these directions.

So, it will be like this and similarly you will have a frictional stresses in these direction. So, the frictional stress is acting in these direction, ah, because it is opposing the metal flow metal will flow in this direction and. So, if frictional flow will be in this direction if you take the element in this in this side then your, frictional you know forces will, stresses will act in the opposite direction. So, that way you have this length as total as two ways. Now, we will take the equilibrium of these forces, if you, if this height is h and since we have taken the width as the unity.

So, if you take this element; now, on this element you have a. This is sigma x into h into width. So, that is sigma x into h. So, that will be and width is unity. So, you have sigma x into h. Similarly, you have the sigma x plus d x in the opposite direction. So, it will be minus of sigma x plus d sigma x and the height is the same. So, you have h here and what we see is that you have the 2 to this is your tau x y. So, that is your frictional shear stress. So, that will be tau xy on this side and you have tau x y on this side

So, this will be two tau x y and. So, it will be two tau x y this will be equal to zero. So, if you do the force balance in the x direction you come to this basically expression and. So, what we get is a sigma x dx sigma x into h sigma x into h will be vanishing. So, you get d sigma x into h will be equal to two tau x y. So, tau tau tau x y this is your subscript and. So, tau x y and here we have basically you know we have missed this term. So, this tau x y and it will be on this length dx.

Because this is the area, there will be d x into width. So, d x into width is 1. So, that will be dx. So, that will be into dx. So, we can write and it will be minus basically, because it will be d sigma x into h plus 2 tau x y dx will be equal to 0. So, it will be minus of the sign. So, we will have d sigma x by dx will be equal to minus of 2 tau x y upon h. So, that is what we get, this expression. Now, we will apply the, Von Mises criteria. So, if we apply so, we if we apply Von Mises criteria of yielding. Now, Von Mises of criteria of yielding, provides us basically, sigma 1 minus sigma 3.

So, that will be basically given as 2 by root 3 into yield stress value. So, that we calculate term is as sigma not prime. Now, in this what we see is that we have this as the p and then you have the stress in the x direction is sigma x. So, what we do is, if we define, if we define p and sigma x. So, if we had defining this as the positive, compressive, principal stresses. So, if you define them as positive compressive, principal stresses. Now, in that case we can write. So, what we can write is that; so, sigma 1 minus sigma 3 basically.

So, that is now, it is sigma not prime and sigma 1 will be your p and sigma 3 will be sigma x so, that we can write. Now, the sigma naught prime, which is nothing, but 2 by root 3 sigma naught that is, that will be not depending upon the x, because that is the stress value.

So, it will not be depending upon the x. So, as sigma not prime does not change with x. So, what we can write. So, since it is not changing with x. So, so we can write that the sigma not prime by dx, it will be equal to 0. Now, from there what we get is d p by dx minus d sigma x by dx. So, that will be equal to 0. So, that is why, what we, get is from there you can get. So, this sigma x by dx, we are getting minus 2 tau x by h. So, so what we get is. So, since it is 0. This will imply that dp by dx will be equal to d sigma x by dx.

Now, d sigma x by d x so, if this is equation 1. So, from this equation d sigma x by d x is coming out to be minus of two tau x y by h. So, if we substitute this d sigma by dx value from here into this equation two. So, if we are substituting, in equation 2. So, we are getting this expression as d p by d x, and that will be equal to minus of 2 tau x y upon h, because d sigma by d sigma x by d x was actually, we got as minus of 2 tau x y by h.

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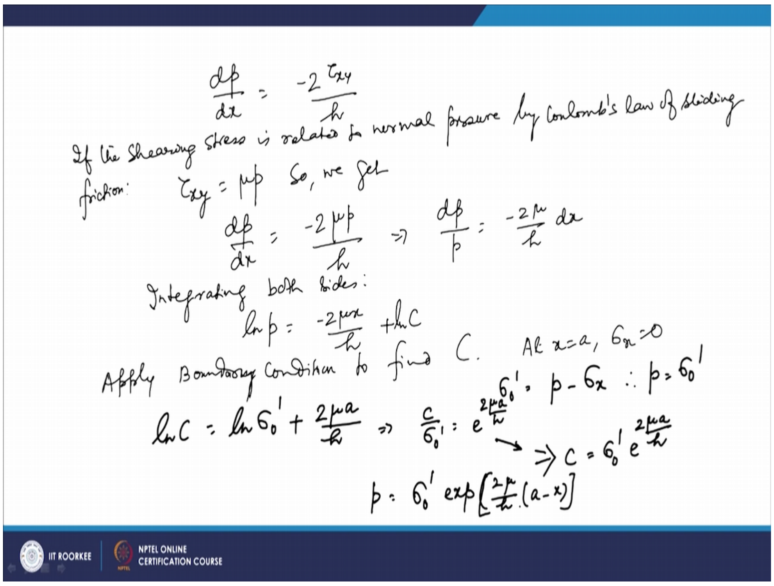

So, we are getting this expression d p by d x will be minus of 2 tau x y by h. Now, if this shearing stress which is applied here, if it is related to the normal pressure by coulombs law of friction. So, basically you, this is nothing, but this is, because of the friction apply. So, if this is because of the normal pressure which is applied, because of the you know normal force and, and because of the sliding friction. So, we can write that if, if the shearing stress is shearing stress is related to normal pressure p by coulombs law of sliding friction ok.

So, what we, you can see that you have tau x y acting and mu is at the surface, you have the coefficient of friction. So, tau x y will be basically, mu times the sliding. So, mu times this pressure now, what we can get from here. So, you will have d p by dx. So, we get now, from here we get d p by dx. So, this minus 2 and tau x will be mu p and then you have here h. So, what we get is, we get d p by p it will be equal to minus 2 mu by h into dx that is what we get, from this equation and if we integrate it then if we integrate both the sides. We are getting l n p will be you know this side you have x. So, minus 2 mu by h into x. So, minus 2 mu x upon h and plus integration constant. now, this constant, we are writing as l n of a c.

Now, this constant of integration c will be evaluated by using the boundary condition and you have as we know that you have on one side, you have a and another side a. So, this constant of integration c will be evaluated by putting the boundary condition and once you go at the, this free surface x equal to a, on this side. So, if you, take you can see at this point basically, here basically, you have this point your, when there is a, at this point you have.

So, at x equal to a you have sigma x is 0 and we had also earlier found that sigma naught prime will be actually equal to p minus sigma x. So, since sigma x will be 0. So, your p will be equal to sigma naught prime. So, as so, that is why at x equal to a you will have p equal to sigma naught prime. Now, once you have this p equal to sigma naught prime then you can put these values into this place. So, you will have l n c on, on this side and you have this side you have l n p.

So, p will be actually l n of sigma naught prime and then you will have a plus, this side you have 2 mu and x is taken as a by h. So, so this way, you will have, these values. Now, you can take this side. So, this will be, c by. So, that if you take c by sigma naught prime will be you know, exponential of 2 mu a by h, because this l n c minus l n sigma naught prime.

So, it will be l n c by sigma naught prime that will be 2 mu a by h. So, then c by sigma naught prime will be nothing, but from here you will get c will be sigma naught prime and then you will have e raised to the power 2 mu a by h. So, that is what you get from, this equation.

So, if you further simplify you have these value p and c. So, ultimately you get p as sigma naught and then x prime then exponential and it will be 2 mu by h into a minus x. So, you will have been get you will be getting this expression for p that is a exponential of 2 mu by h into a minus x, because anyway this will be again l n and then that term will come this side. So, this will be you have to move a by h and you have minus of this. So, ultimately you have a minus x will be coming.

So, this you get the expression for the p. Now, if you try to find the value for this exponential. Now, mu is very-very large I mean small number.

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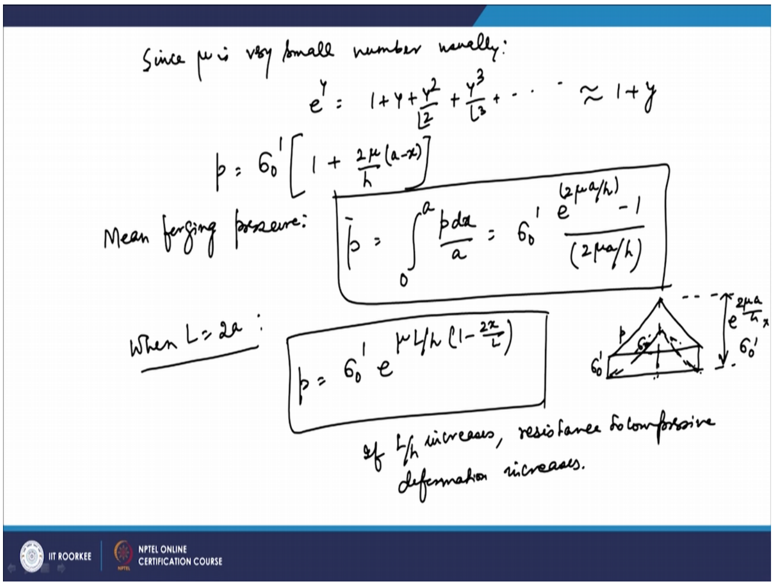

So, since mu is very small usually. So, what is happening that we write e raised to power y as normally y 1 plus y plus y square by factorial 2 plus y cube by factorial 3 or so. So, basically we can neglect the higher order terms. So, basically we can write e raised to power y as 1 plus y. So, so that is why if we take this 2 exponential raised to power 2 mu by h into a minus x so, it is basically taken as 1 plus y when y is very small. So, that is why we can write p as sigma not prime and then exponential raised to the power 2 mu by h into a minus x.

So, it will be 1 plus 2 mu by h into a minus x that is what we get from the value of in the value of p in this expression and we can get the mean forging pressures if you try to get the mean forging pressure. So, mean forging pressure will be basically p mean and that we can again forward the distance of a on one side. So, it will be nothing, but 0 to a and this is pdx divided by a. So, if we do that we will get sigma naught prime and you will have e raised to the power 2 mu a by h.

So, that will be this minus 1 and then divided by 2 mu a by h. So, this will be basically you know the expression for the average you know forging pressure or mean forging pressure being calculated and this can be used to calculate the forging pressure.

Now, many a times we also write in a more convenient manner because a is nothing, but a is l by 2 or l equal to 2 a. So, when l equal to 2 a. So, we can even more conveniently it can be written. So, we can be writing p equal to sigma naught prime and e raised to the power basically it will be mu l. So, because this 2 a will be l. So, it will be mu l by h and then you will have 2 mu a by h on this sides it will be going on the other side. So, it will be 1 minus 2 x by l.

So, that way we based on this, you have the alternate, way of writing this expression also. It is possible, in this fashion; Now, what we see that from here what we see that, if the l by h is increasing if the length, by this height that is increasing then in that case you have the resistance to compressive. You know deformation is increasing the p value will be increasing, because once this is increasing. So, if l by h increases that is length to thickness ratio. Now, that will be increasing the resistance to compressive, resistance to compressive deformation increases.

So, what happens that this concept is basically used, when we talk about the impression die forging and when we talk about the flaxes. So, in those cases your l by h is large. So, you will have the large pressure required for the flow of these flaxes on the, on that neutral plain or on that, you know surface when, where I mean, on that you know, parting plain or so. And in that case, since the pressure requirement for its flow will be more. You will have larger pressure, which will experienced by the fluid, which is inside for the, for the metal, which is inside and you will ensure, it will be ensure that the die is filled completely.

So, that very concept is basically used in, in such cases. Now, the thing is that, this expression also tells that when x will be 0 then its value will be maximum. So, basically that gives you a concept that if you have this strip, which is forged. Now, in this case the pressure, which is found, that will be basically maximum here and, and based on that you can have the variation of, variation you know and finding the, the hill value at this place. So, basically if we try to find this variation of p and this sigma x over this you know length.

So, what we see is that your, you know sigma p and sigma x will be varying something like you know it, it goes like this. So, in, in the centre you have so, this way. Now, here basically, what we see is this, this is the maximum value and that maximum value will be sigma naught prime e raised to the power mu l by h. So, the or 2 mu a by h. So, the maximum value which you get that will be basically e raised to the power, you know here 2 mu sigma naught prime. So, this will be e raised to the power 2 mu a by h. This is multiplied by sigma naught prime.

So, this is the maximum value, which is being achieved at this point that is why it known as friction hill, here you have the maximum value of or sigma x value, which is achieved a in such cases and p minus sigma x certainly. As we have seen that that will be equal to sigma naught prime. So, you have this is sigma naught prime. So, that will be, basically, you will have this way, you have the, you know and the (Refer Time: 26:26). So, that way it will vary and 1 will be p and another will be sigma x. This will be for sigma x and this will be for p. So, p minus sigma x is a sigma naught prime. So, this sigma naught.

So, this way that is known as the friction hill. So, this is, how we calculate these. You know values of the, you know forging pressure, which is required to be, calculated in such cases. Now, there may be cases of the, you know also the open die forging. Now, in the case of open die forging, what happens that you have, you know, you have the production of the simple tips. We have discussed about, these open die forgings and in such cases what happens that you have length width and the, you know height. So, both are getting varied in the case of open die forging. You have certain terminologies like S will be, equal to, width elongation by thickness elongation.

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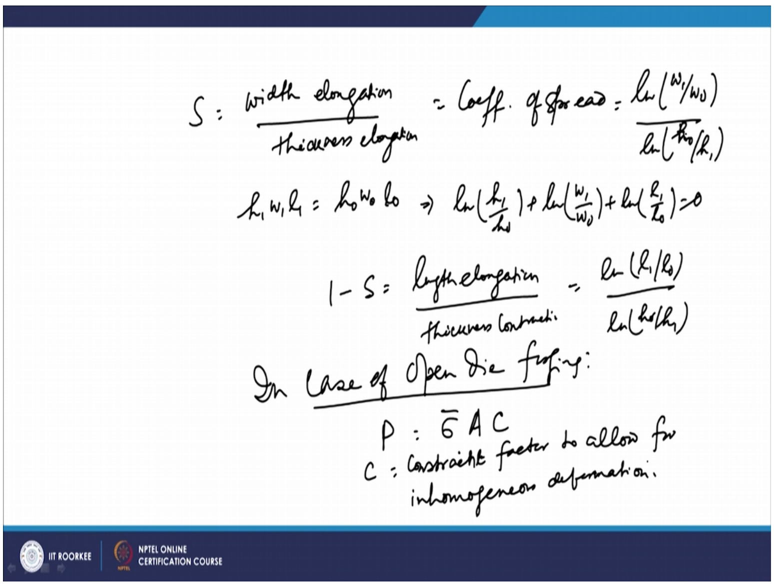

So, that is normally known as the coefficient of spread and in such cases basically, if you look at this value. So, if you take the true stress values. Now, if the width is basically increasing and thickness is decreasing. So, it will be l n of w 1 by w naught divided by l n of t naught by h naught by h 1. So, if the height or thickness that is reducing. So, initial value by the final value. Similarly, you have final value by initial value because in that case it is, increasing and also they are you have some constancy of volume.

So, for constancy of volume you have h 1 w 1 l 1 as h naught w naught l naught and then from there you get l l n of h 1 by h naught plus l n of w 1 by w naught plus l n of l 1 by l naught that will be equal to 0s that is all. It is epsilon 1 plus epsilon 2 plus epsilon 3 equal to 0. So, from there we have another expression, which we get. We get the coefficient 1 minus S will be basically depending upon the change in length you can. I mean as compared to the change in the breadth direction, we easy to compute in the length direction.

So, we also get 1 minus S it will be length elongation divided by thickness contraction. So, we get l n of ln l 1 by l naught divided by l n of h naught by h 1. So, this way, we calculate these parametric values and they also depend upon sometimes, you have other. You know parameters also and you have a terminology know, known as the byte ratio.

So, that is a gain depending upon the b and, and the, you know w naught. So, that way you have other formulas and in; in fact, in such cases in the case these open die forging. Now, in these flat; so, in case of open die forging. Now, in these cases what we do is you calculate these forging loads. Now, in this case we take the average stress and then here you multiplying with a, and you have a, constant that is a known as a constraint factor.

So, this C is known as constraint factor to allow for inhomogeneous deformation. So, this, constraint factor is provided and based on that we calculate these, forging loads. So, this is, you can read more and more you know books for doing the analysis of such processes and get more understanding about the analysis part.

Thank you very much.