Principles of Metal Forming Technology

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Lecture - 22
Mechanics of metalworking and analysis methods

Welcome to the lecture on Mechanics of Metalworking and Analysis Methods. So, we will try to be aware about some of the terminology is in this what are the mechanics for the metal working processes; which and an some of the terminologies, will we will be acquainted with which will be required to analyze the operations in metalworking. Now when we try to analyze that, we know that we have studied about the deformation theories. We have studied about the plasticity theories in that we have this seen many points like we have discuss many points like, what are the conditions in the case of plastic deformation like the value of this Poisson ratio or also the other, you know, constancy of volume conditions and all that.

So, we will discuss about certain things because in most of the cases, you your concerned with the reduction of the, you know, or change in the dimension of the product or the cross section of the material. And in that case you need to you find the amount of stress which is required to do that, and what will be the associated things which will be occurring with that so, that will be discussed.

So, when we talk about the mechanics. In that case, the first thing which we come across in those cases is the constancy of volume approach.

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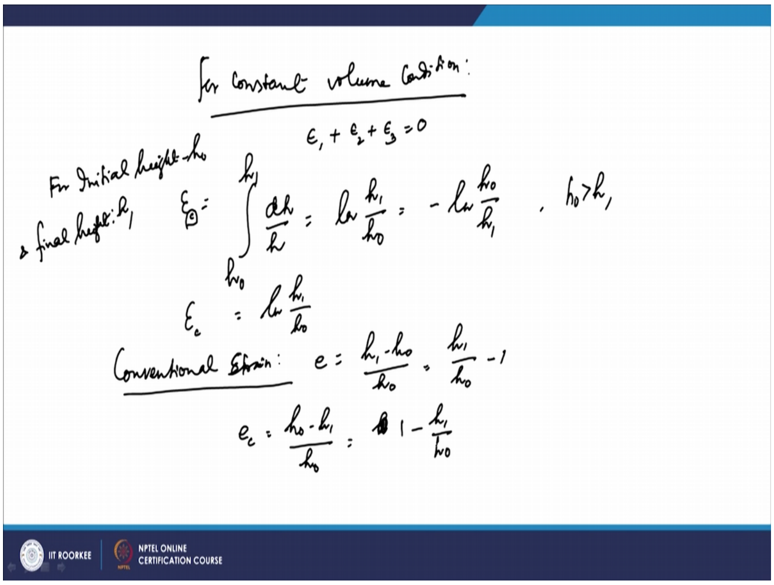

So, for constant volume condition so, in that case, what we get is that epsilon 1 plus epsilon 2 plus epsilon 3 is 0; because we know that if the strain is there in one direction positive, then certainly you have in transfers direction you have the strains negative directions. So, that way this epsilon 1 plus epsilon 2 plus epsilon 3 is coming out to be 0.

Now, when we talk about the metal working processes, then what we see that normally when we talk about the stresses, and we know that the stresses are of primarily 2 nature either tensile or compressive. So, we take tensile stresses as the projective one and the compressive stresses as the negative one. So, but when we talk about the analyses of metalworking. So, normally we deal with the compressive force only because normally, the compressive force is have applied.

So, in such cases in the cases of this metal working analysis, normally these compressive directions they are not taken as negative because every time you have to do it in the negative manner. So, so normally since they predominate in the analyses. We normally take it as the positive one, and for that we have the different convention. So, suppose we are compressive the material, in that case, if from the height h naught to h 1 we are compressing. So, if suppose for initial height h naught and final height h 1.

So, if you look at the compressive strain which is developed, so as we know that compressive strain will be so, if you write it like this a compressive strain or so. Now, in this case we will have the definition h naught to h 1 and this will be dh by h so, it will be ln of h 1 by h naught. Now since the h 1 is smaller than h naught. So, it will be we will write it like minus of ln h naught by h 1; so because the h h naught is more than h 1.

So, normally what we do is, we we write these in this strain in the case of this plastic deformation as minus of ln h naught by h 1. So, basically this is a this is the strain, but we write when we put this subscript c, that is for compressive, you know, strain when we do the metal working analysis. In those cases, we write we remove this negative sign, and we write ln of h 1 by h naught. Similarly, if you try to find the conventionally engineering strain; so for conventional strain, now in the case of conventionally strain, what we see is it will be h 1 minus h naught by h naught.

So, again it will be h 1 by h naught minus 1. So, what we see that normally you have the negative values, but when we talk about compressive, you know, values. So, we will write the engineering strain that is in compressive. So, which we it will be h naught minus h 1 by h naught. So, when we talk about the compressive value it will be like h 1 minus then h 1 by h naught. So, this is how we normally have the convention, when we talk about the deformation in the case of metalworking.

Now, further we know that when you are try to increase or decrease the length associated with that there will be the change in the reduction of the area or so.

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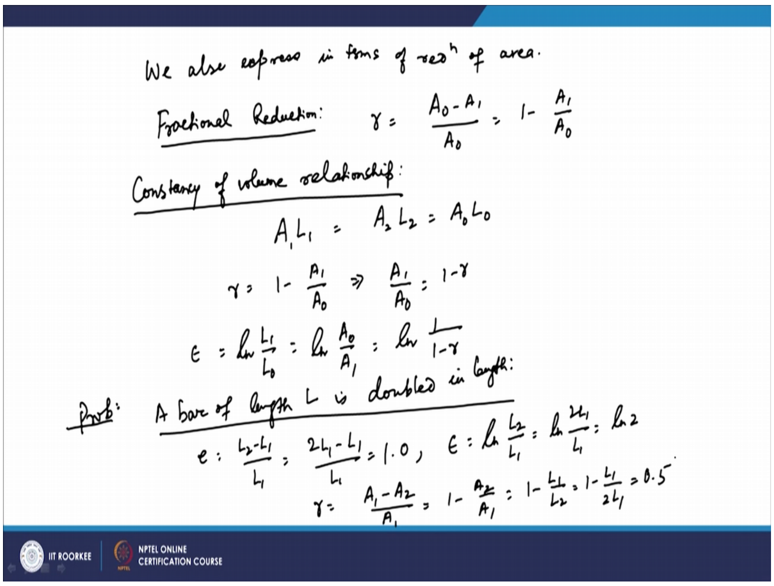

So normally we also express we also express in terms of reduction of area. So, in that the commonly used term is the fractional reduction.

So, the fractional reduction will be defined as r. And this will be A naught minus A 1 divided by A naught. So, this is the original area which was they cross sectional area which was there earlier, then what is the final area, and then divided by the original area and that is known as the fractional reduction of the area. So, if you take the constancy of volume relationship, now as we know that you have the change in length, and associated with that you have the change in the cross sectional area, and if the volume is constant then the, you know, area multiplied by length it will be always same.

So, you will have A 1 into L 1 will be A 2 into L 2 that is your A naught into L naught. So, what we see is that r you can we are writing it as so here we write it as 1 minus A 1 by A naught. So, it will be r will be 1 minus A 1 by A naught, or A 1 by A naught which return as 1 minus r. Now if you see the definition for the strain to strain value, it is ln of L 1 by L naught, so, it will be ln of A naught by A 1. Because L 1 by L naught will be A naught by A 1 because A naught L naught will be equal to L 1 into A 1 into L 1.

So, you can write this as 1 by 1 minus r, because it will be 1 by 1 minus r; so ln of 1 by 1 minus r. So, this way you can find so, this are these are the terminology is which are mostly use fractional reduction in area or the compressive strains or so. And they can be use for the analysis of the processes; you may have suppose, if you take the example certain example like if suppose you have a problem, and it is said that you have a bar which is a a bar of length L is basically doubled in length.

So, you may be told that you find these different, you know, value of the engineering strain or true strain or the, you know, reduction. So, suppose a bar of length L is doubled in length that is becomes L from L to 2 L. In those case if you try to find the e it will be L 2 minus L 1 by L 1, so, L 2 L 2 is nothing but 2 L 1. So, it will be 2 L 1 minus L 1 by L 1, so, it will be 1. Similarly, if you try to find the true strain there, in that case it will be ln of L 2 by L 1. So, it will be ln of 2 L 1 by L 1, and in that case it will be ln of 2. So, ln of 2 value is 0 point 6 9 3 so, that will be found out; similarly, if you try to find the r.

So, r will be suppose so, r you can get is as A 1 minus A 2 by A 1. So, so that way you can find so, it will be 1 minus A 2 by A 1. Now A 2 by A 1 is nothing but L 1 by L 2 so, it will be 1 minus L 1 by L 2, and L 2 is 2 L 1. So, 1 minus L 1 by 2 L 1; so it will 1 minus

1 by 2 so, 0.5. So, this way when L bar of length L is doubled in length, in that case it is basically you can calculate this true strain or engineering strain or the fractional reduction values in such fashion.

You can also calculate if it is halved length, and that way you will have the values coming to the negative side in the case of engineering strain. And, further you can calculate other values like fractional reduction or in 2 strain values and in compression whatever we have discussed. So, that way you can find those values.

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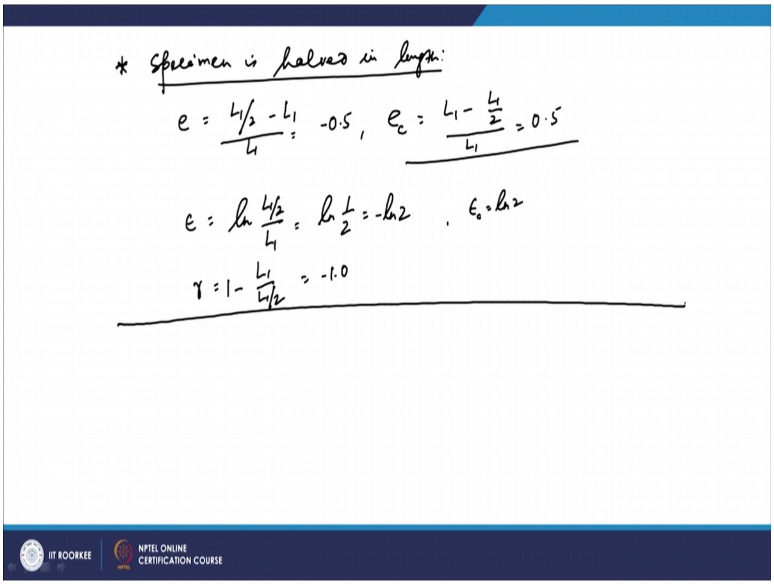

So, let us take for example, that if the specimen is halves in length. Now in this case, L 1 becomes L 1 by 2. So, we can find the engineering strain and engineering strain will be L 1 by 2 minus L 1 by basically L 1. So, it will be minus of 0.5, but if you take the engineering strain in compressive if direction compressive strain engineering compressive strain; in that case we will do the reverse L 1 minus L 1 by 2 and divided by L 1 so it will be basically 0.5.

So, this is what they convention is used in the case of the metal forming analysis where you largely you are trying with the you are dealing with the compressive stresses. Similarly, if you go for the true strain values so, true strain value also ln of final length where original length so, L 1 by 2 by L 1. So, it is ln of 1 by 2 so, ln 1 minus ln 2. So, it will be minus of ln 2, but if you talk about the true compressive stain it will be ln 2, because it will be L 1 by L 1 by 2 so it will be so, that way it will be ln 2,.

And if you look at the reduction of area so, it will be L 1 by L 1 by 2, and then 1 minus that so, it will be something like minus 1. So, that is how you tried to find these parameters whenever required in the, you know, metalworking analysis. Coming to the analysis part of the, you know, metal working operation we discuss about the zone in which we are going to confine and do the study about the metal working processes.

Now what is there in that basically you are applying the stresses, then you have many conditions you have the equations you are getting equations of equilibrium you have the 2 adjust the forces which are being applied and you have to make them, you know, a balance in certain directions, and accordingly you try to get the values you try to find these stress and strain values and different points. So, that is what the aim is there in the case of metal forming in every point of the deformation zone. In the deformed reason you try to find the velocity you try to find the stresses or the strain.

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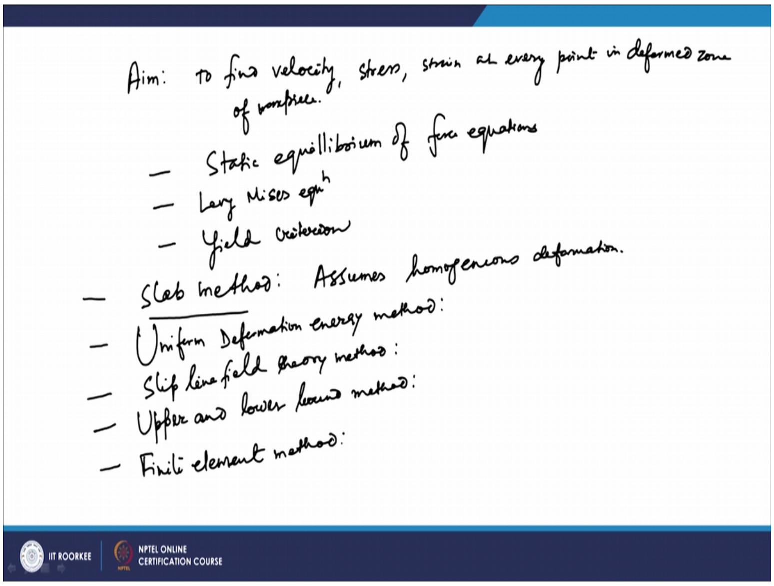

So, it is basically your aim is to find velocity stress strain at every point in deformation zone in deformed zone of work piece. Now you have many ways for approaching the problem in that case, because ultimately we what we are interested in that we want to know that with what velocity the material is deforming or the stresses or strain at every point in that. So, we are in the interested into it and you have many ways. And basically you have 3 sets of equation which will be coming up which has to be solved, which are to be solved to find all these values. Now these 3 sets of equations are that you have first the static equilibrium of force equations.

So, you have this static equilibrium of force equations which are to be, you know, solved. Then you have the levy mises equation ; with so, this levy mises equation will express the relation between the stress and the strain rates. So, based on that, you will have the equation and you will find you will used these equations for further finding, and then you have the yield criterion.

So, you have basically 9 independent equations, and they are to be solved and you have the 9 also the unknowns you have 6 stress components, and 3 the velocity component or the strain component, and these are to be solved and you have the 3, is this way you have the 9 equations and you have 9 unknowns and they are to be, you know, solved.

So, normally the solution is certainly tedious, you analytically equation I mean solution is not that way easy to solved them. And you have many methods which are used, and the different methods which are used are the, you know, slab method. So, this slab method it assumes homogeneous deformation. So, in this case when we are doing an analysis, and you have some element of, you know, certain shape; suppose, you have a the square element is there. So, once you deform it then it will be converted into rectangular elements.

So, that way you have a homogeneous kind of deformation that is assumed in the case of slab method. Similarly, you have another approach which is used is uniform deformation energy method. Now a in these cases what we is, then these that your applying you are giving the energy I mean for deforming the work piece. Now from these, you know, work of plastic deformation you are trying to calculate the average stress value. So, so that way that is why it is known as the uniform deformation energy method.

So, in this case is you must know that what is the work done on the machine, on the material and based on that you find you predict these average forming stresses. The second the third method is the slip line field theory method. So, it is basically calculating the point by point these stress values are, you know, calculated. And normally we assume the plane strain conditions in such case, and for every point by point, the value of the stresses are calculated. Then another method is upper and lower bound method. Now this is done based on the limit analysis, and basically it will be using, the so this upper and lower bound method.

You will have the limit, and you will use the reasonable, you know, stress value and the velocity field. So, that you calculate basically the bound with in which these actual forming load should be, you know, lying. So, based on that you calculate this parameter values, then the last method is the finite element method. Now this is basically also known as the matrix method.

And it will be a lying the large increment of deformation for the rigid plastic materials, and basically lot of computational time is reduced in this is the new approach is when latest approach which is used and in this case is the lot amount of these competition time is saved; however, the you need to have the proper understanding for, you know, making the matrix which is to be solved. So, basically all these methods are in the order of increasing complexity, and the slab method is the most easy one which is basically normally used for the analysis of the metal forming processes.

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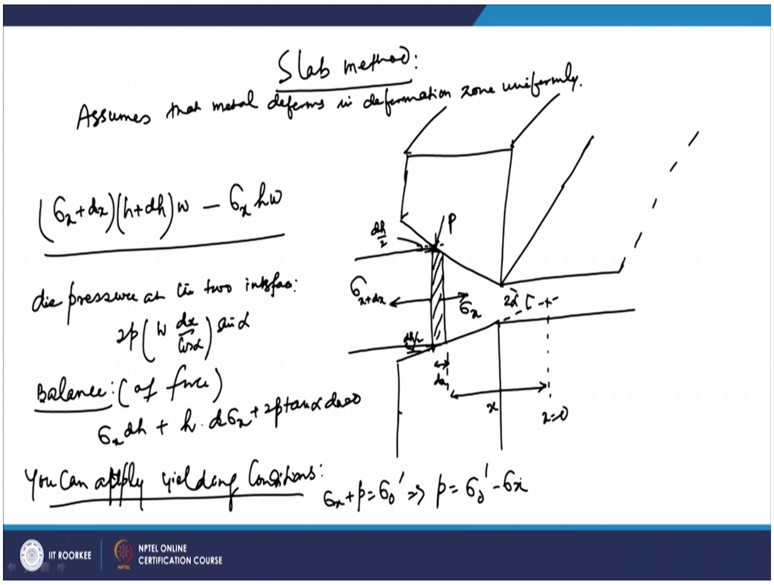

So, if you try to see that we try to analyze this slab method. So, this slab method it assumes that the metal is deforming; so, it assumes that metal deforms in deformation zone uniformly. The thing is the meaning is that if you have a square grid element, and if it is going under the application of stresses.

So, it will be converted it will be, you know, having a rectangular type of element it will be converted into rectangular type of element when it is, you know, uniformly basically, you know, deformed. So, that is what this approach is and this approach is used for. So, you have all these conditions we apply we apply these, you know, force balance equations you apply the you apply also apply the; you will criteria an based on that we try to calculate the stress values.

So, suppose if we when we do for certain analysis, suppose we do for the strip drawing of a plate. So, suppose you have die which is going like this and you have this, die has certain width, and similarly you have this as another die. On the top as well as on the bottom, and then if suppose you have one strip is there the strip is basically, you know, we are this is strip basically we are applying this is a this middle point.

And this is suppose you one a strip is take an here. Now if you look at this is coming to suppose meeting at this point so, this angle is alpha that is 2 alpha, this is included angle, now you want to a change the thickness of this is strip, and later on this strip will come out this thickness. Now in this case this is you are this is this way you have the width of this strips.

So, you are assuming the width to be constant, in that case you have constraint from both the sides, and this length is basically going in this particular direction. Now what is sorry that you have certain height on this side, and this is reduced to this is small size, and how you will analyze this process now if you look at this. So, here you apply this pressure p. And similarly you have this is strip subjected to a stress of sigma x in this direction and if this is taken as the origin.

So, if suppose this is the point which is at x direction x distance, and if suppose this is the length d x. So, this is x at express g x so, you will have stress value of sigma x plus d x at this point. Now we take this height as h, and the reduction in this height is dh. So, if this height is h so, in that case you will have this is the halve of the distance with it is will be halve d h, similarly you will have half dh here. So, so h this will be h and this will be h plus d h. So, this height is h and this side it is h plus d h. So, this halve will be dh by 2, and here also you will have halve will be dh by 2, here and similarly you will have halve will be dh by 2.

So, that will be so, this if this is height h and this is h plus dx, this is x equal to 0, other things are like when you are. So, this is strip which is there, you will have it is, you know, width along this. Now in this case we have to find, what will be the how you will find the stress which is at the, you know, when it is leaving the die; so in those case so, suppose you do the analyses of the forces balanced.

Now what you see is that you have sigma x at this point and this is sigma x plus tx. At this points so, if you look at that way. So, you have sigma x plus d x, at this point at this point, and your height is h and width is we are taking as constant so, this will at h plus dh.

Similarly, so, this as this is one direction, and this will be the a w that will be the, you know, if you try to find the, you know, force which is applied because of the so, you are the height with height you are multiplying with the width. So, you will have the area and that multiplied by the stress. Similarly, and on this surface you will have sigma x h into w.

So, this way you have the, you know, force balance equation in the x direction. You may have the force balance equation in the y direction, and if you take the y direction so, basically this is 2 alphas. So, you will have similarly you have alpha here. Now this case if you look at the die pressure at the 2 interface, now in this case you take because of this die pressure and if you take the, you know, force which is the it is component in the x direction. So, that will be basically 2 p into w dx by cues alpha into sin alpha.

So, what we see is that if you look at this and this, the addition of them must be equal to 0. So, basically that should be in the equilibrium condition that should be equal to 0. So, sigma x h and w so, and then sigma x h dx w sigma x dx dh w and sigma dx h w. Like that, so, if you dx and dw term is neglected, because they will be the small, you know, smaller values.

So, based on that if you take the balance of the forces of force equation if you see you will see, in the x direction you will get sigma x dh plus h into d sigma x plus 2 p tan alpha d x will be equal to 0. Now similarly you get the equation also you have the force balance equation, may be in the y direction or z direction depending upon the other cases.

So, you will have such equation then, what you will do is you will also apply for the wider excellent from there you will get certain condition. Then you apply the condition of, you know, yielding and you can apply you can apply yielding conditions. And from there you will apply suppose the test of criteria so, in that case you will have the sigma 1 minus sigma 3. So, maximum is sigma 1 and minimum sigma 3.

So, sigma minus sigma 3 has to we 2 k. Now in this case when we do the balance in for the y direction (Refer Time: 30:41) that this is sigma y is basically coming as p. So, you have basically sigma x, and then that is coming out to be minus p. So, sigma x plus p that will be coming out to be sigma naught dot so, p is coming as sigma naught prime minus sigma x. So, this way what we get is now one we integrate that you will get certain equation and on integration, you will get sigma x by sigma naught prime, it will be ln of h plus constant.

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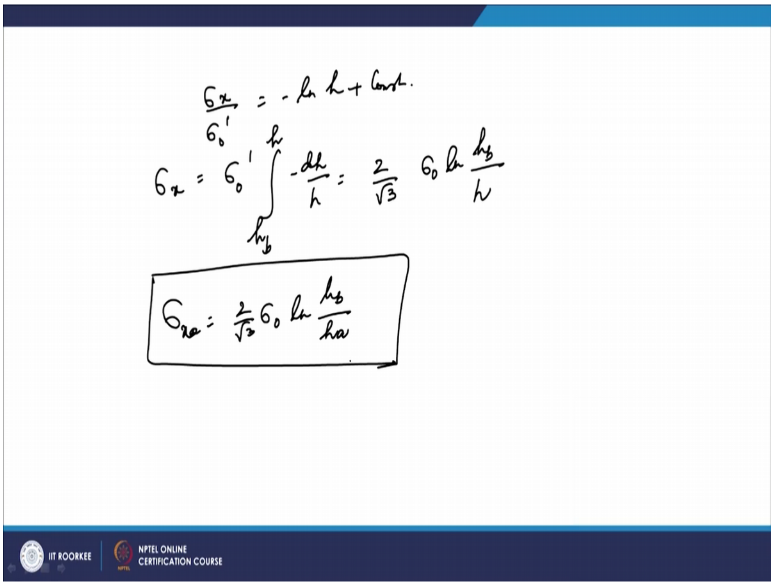

And then you can have the values of these, you know, a limits because the h will be varying from h 2 h b.

So, you will have the on one side we have one height another side you have another height. So, if you do the analyses further so, that is why we do this analyses based on these methods, what we get is you get the equations. Finally, you can we can further see we will finally get the equation like h b to h and then we get minus dh by h. So, what we get is you finally, get 2 by root 3 on sigma naught ln hb by h. So, this way you can find, now in this case you find if you find at the exit where h is h a. So, in that case, sigma x a can be found as 2 by root 3 sigma naught ln h b by. So, this way you can also do it in terms of the reduction of area that terminology also.

So, what we see that this is a the simplest of the approach which is used in the case of the deformation analysis, and this is known as the slab method you have other methods also, and most of time we will deal with the calculation of the stresses using this slab method. So, that is about this slab method, you have other methods, also you can have more understanding about the other methods and we will see how this slab method is applied for finding, expressions for the stresses calculation and all that in the different forming processes.

Thank you very much.