Principles of Mental Forming Technology
Dr. Pradeep K. Jha
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 17
Instability in tension

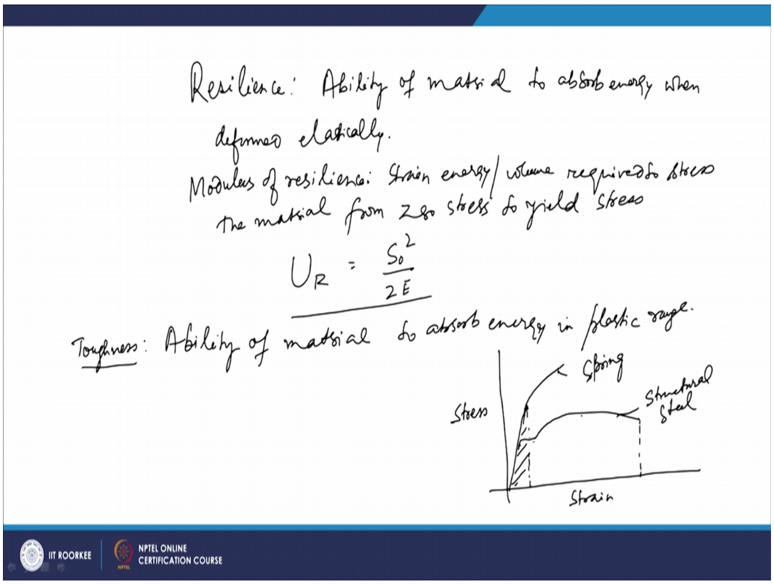
Welcome to the lecture on Instability in tension. So, in the last lecture we discussed about the flow terminologies, in the stress strain diagram. And in this lecture we are going to discuss few more about it and then also we are going to discuss about the stability criteria's; not criteria's basically that making which occurs in tension and mostly we will talk about the true stress true strain curve and its related terms.

So, in the last lecture we discussed about you know the terminologies in the stress strain diagram. You we saw the elastic limit. We had seen the proof stress, yield stress and then the specimen goes to a maximum load, then is further decreases and then finally, you have a fracture point.

So, in the initial reason the line which is there, the linear reason the, which is there its slope is nothing, but the elastic modulus of the material and as far as the modulus of elasticity is there, it is intrinsic property of the material. And actually this is because of the binding forces between the atoms. So, you can change it by only doing undergoing those treatments which basically changes the force of the attraction between the atoms of molecules.

And so, there are certainly certain methods by which you can somewhat change, but otherwise it is a structure insensitive property. So, that way you can do some changes by the heat treatment or alloying or so. So then apart form that, we discussed also about certain terminologies like resilience and toughness.

(Refer Slide Time: 02:36)



So, as we discussed that the resilience basically, this resilience which we talk about; now this resilience is the ability to absorb energy when deformed elastically. So, ability of material to absorb energy when deformed elastically; normally we measure it in the terms of a modulus of resilience which is nothing, but the strain energy per unit volume that is required for the material to stress the material from zero stress to yield stress. So, that is known as the modulus of resilience. So, if you try to define that this is basically the strain energy per unit volume required to stress the material from zero stress to yield stress. So, this property is known as the modulus of resilience and then this is basically defined as you know S naught square upon 2 E.

So, basically what we mean by this equation? This equation what we see you are that is your modulus of resilience. And this is basically the you know resisting energy load in application when the material must not undergo permanent distortion. So, that is basically we are talking about the material being deformed in the elastic region. So, mostly it is used typically we when we try to define the property of the springs where the deformation has to be elastic range, the when springs has to deform and then it has first further to gain to come to its own you know position. You cannot to afford to have the permanent deformation in the case of springs.

So, in those cases we define this property and you known for the springs, the springs which will have the higher yield stress and low modulus of elasticity; certainly they will have the values of the higher value of the modulus of resilience. So, that way this resilience is defined, but then another property which is further having the importance is toughness

So, we often come across this term toughness. Now, this also the same ability of material to absorb energy; but this is defined when you are in the plastic range. So, this is ability of material to absorb energy in plastic range. So, in when the material goes beyond the elastic range and it reaches in the plastic range is there, its ability to absorb energy that is known as the toughness. And many a times you know it has the material has to undergo the stresses value which are more than the yield stress and that time the it is ability to absorb the that energy without fracture that is this property.

And normally many a times this is encountered in many applications like mostly for the gears or chains so, or the crane hooks. So, in those cases you required this property that is a toughness. So, you require this toughness of the material. Another way you can also think of the toughness is by finding the area under the curve that is the stress strain curve. So, basically you have two areas one is; the area under the zone which is the elastic zone, another will be the area under that zone which is the plastic zone.

So, if you topically look at a stress strain diagram if you see, now you may have a stress strain diagram which may go like this and then it may go here. And you know have another material which may go and which may go like this and then that way it will be it will be showing the, you know behavior like this. You may have two kind of material. Now if you look at this material, here you have the zone which is the you know elastic zone.

Now, if you look at these two curves what you see is that in this is the area under this curve. So, basically if you look at the work done per unit volume in this cases now, the total area under this stress strain curve; this is the amount of work that is per unit volume which can be done on the material without causing into fractures. So, that is this is how this area basically they are indicative off. So, this area will talk about how much energy this material, I mean the how much work per unit volume can be done on this material so, that before fracture. So, that is what the area is.

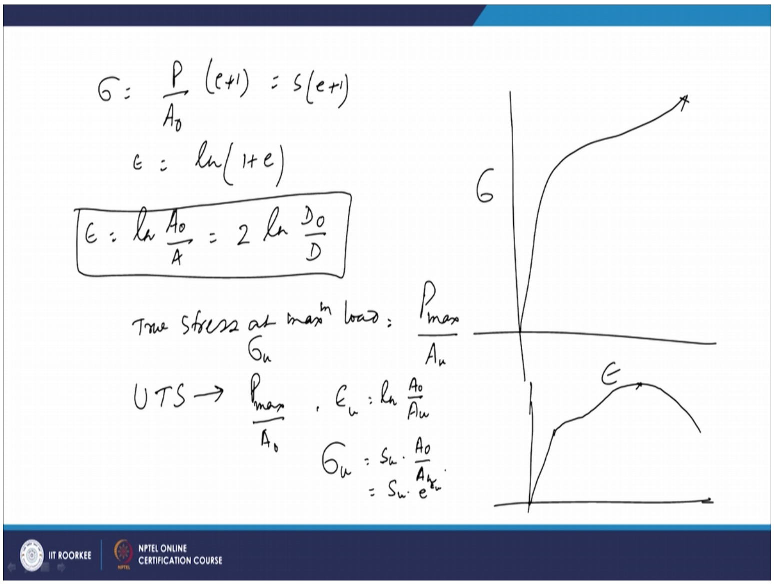
Now if you look at this curve, now in this you have two this is for two materials. Suppose this is for material which is you know for a spring. This is for the material this is for this normally the high carbon steels are used for making this steel and what you see is that it has this is indicative of the resilience. So, they have the higher resilience whereas, if you have the structural material structural steels are there. So, this is for that material. Now if you can see for this is spring material, it has basically the high yield strength because here your yielding occurs here from and in this case the yielding occurs here.

Now if they have the higher yield strength and tensile strength, then this structural steel which is having lower carbon you know material that is medium carbon steel. But what you see is that it has more resilience which it is more resilience. It has what you see is that this structural steel is having more ductility. It goes up to this point whereas; this does not go here itself. After that it does not go, so it fractures; but it is going to the larger value of the strain.

So, basically when it is comprising; so here you have the larger ductility as well as you have strength also, but it has a larger ductility and that is why since in the plastic zone, so this area whole area will be basically indicative of the energy work which can be worked on this before fracture and that is why this material is said to be the tougher material. Whereas the spring steel this steel is having a more resilient is said to be more resilient. It is you know modulus of resilience is higher for this spring steel because of its higher yield strength.

So, this is something which is required when we try to deform the material. When we try to use the material for different applications in those cases, this is very much required.

(Refer Slide Time: 11:33)



Now you can further try to refer to the, we can our we can recall our studies on the true stress true strain curve. So, as you see that when you have the true stress true strain curve so, you go to that and then this way it goes. So, this your true stress and true strain curve and here it fractures

Now, in this case the stress which is required I mean to be for the material to flow plastically that is known as the flow curve and here the difference as already discussed between them is that. In this case you calculate the stress based on the instantaneous area whereas, in the case of engineering stress calculation we find the load divided by the original area. So, that is how it is a found and what we see that in this case stress will be this will be P by A A naught into E plus 1. So, that is how we find. So, that will be E plus 1.

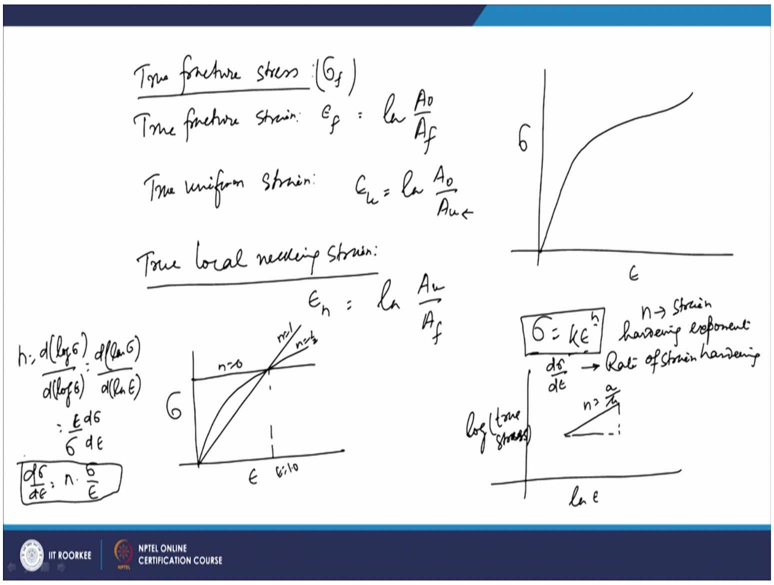
So, also we get the true strain as l n 1 plus e. So, this way we get the value of the true stress as well as the true strain, also when you have because there is constancy of volume in the case of plastic deformation. So, we you also call this as the ratio of the area original to origin of area final and that is why we call it as 2 of l n D naught by D.

So, basically when you have the change in length, at the same time you have the decrease in the you know this is the diameter of the specimen. So, so that way you also define this true strain as this also. So, this is known about the true stress true strain curve. And if you see the true stress at maximum load, so true stress at maximum load also can be calculated and true stress at maximum loads anyway; in all these cases what you do is so, maximum load will be if someone maximum load is P max and then that will be you have to derived by the area at that point.

So, basically you have sigma u. Now before that you have to see the ultimate tensile strength of the material. When we decide about the ultimate tensile strength of the material, U T S at that time when you have the engineering stress strain curve that time we so, we normally go to this point. So, at this at this point you have the maximum load and this maximum load divided by the original area that is your ultimate tensile strength. But the true stress at maximum load that is sigma u that will be your P max by A u and your epsilon u that is ultimate strain at that point that will be again ln of A naught by A u.

So, if you try to find the expression, you will find that the ultimate true stress. The true stress at the maximum load sigma u it will be basically S u into A naught by A u. So, so u can write that it will be S u times E to the power epsilon u. So, this way you find the ultimate you know a true stress at the maximum load and this will be required when we analyze this curves in the further you know analysis.

(Refer Slide Time: 15:47)



You must also know about the certain terms like you have the true fracture stress so, if you talk about those terminologies like true fracture stress. So, you go to the fracture point and at that time the true stress value which is calculated that is your true fracture stress and again that will be basically glowed at load which is there at the facture divided by the area of cross section which is at the fracture. So, that is true fracture stress. Similarly you have true fracture strain.

So, again true fracture strain will be so, that will be denoted by epsilon f, this will be sigma f. So, true fracture is true stress this will be true fracture strain. So, that will be again l n of a naught by A f. So, that is area which is there I mean that is reduced area at the fracture point. So, that will be l n of a naught by A f. So, that is your true fracture strain. So, you can further converted in terms of the reduction of area so, that you can further do it, in terms of l n of 1 by 1 minus q. So, that way true fracture stress can be found. So, this is A naught by A f or you can also write it as l n of 1 by 1 minus q.

Similarly, you have a the other conditions like you have the true uniform strain. So, if you talk about this terminology, it is basically based on the strain up to the maximum load. So, it is basically that and it means it may be calculated based on the cross sectional area A u and and that is why this epsilon u is defined as l n of a naught by A u. So, it is basically based on this a u value what you get is that is known as true uniform strain and you have also the true local necking the strain.

So, true local necking strain can be calculated and in this case what we feel is that this is a for the strain which is required to deform the specimen from maximum load to the fracture point. So, so in the case of engineering strain, you have this from there actually the the the value of stress normally dips down whereas, it that does not happen in the case of the true stress true strain curve.

So, in this case the local necking strain it will be defined as l n of A u upon A f. So, a u will be the area and for when this necking starts and then it will be going up to so at the maximum point and then from so, in the in the case of engineering stress curve and then from at the fracture where you have the final area which is occurring and that will be the epsilon f. So, this way you try to find the true fracture strain.

We already discussed about the true stress true strain curve and what we saw in that is that you have that can be represented by the equation that is sigma equal to E epsilon n. So, you have 1, stress 1 curve which goes like this and we know that this is the strength coefficient and this is the you know n is the strain hardening exponent and this is the true strain and this is the true stress value.

So, you can find this n by finding ah; so that will be nothing, but if you find the graph between the log sigma and log epsilon, then slope of that curve which can that will be a linear curve and the slope of that curve basically will give you the n. So, basically if it is the log of true stress and this is by log of true strain, then what you see you get one line and basically the slope of this line, basically this n; that is the known as the strain hardening exponent in those cases.

So, basically depending upon the value of this n, you will have different type of the curves you have sigma equal to K epsilon n. So, n may vary n n may different values; n may be 0, n may be half or so. So n may be 1. So, if the n is 1, in that case you have a linear curve. So, you may have different kind of curves. So, if you have sigma and epsilon. So, if you look at n equal to 0 so if n equal to 0, so you will have sigma equal to K. So, you will have this value for n equal to 0.

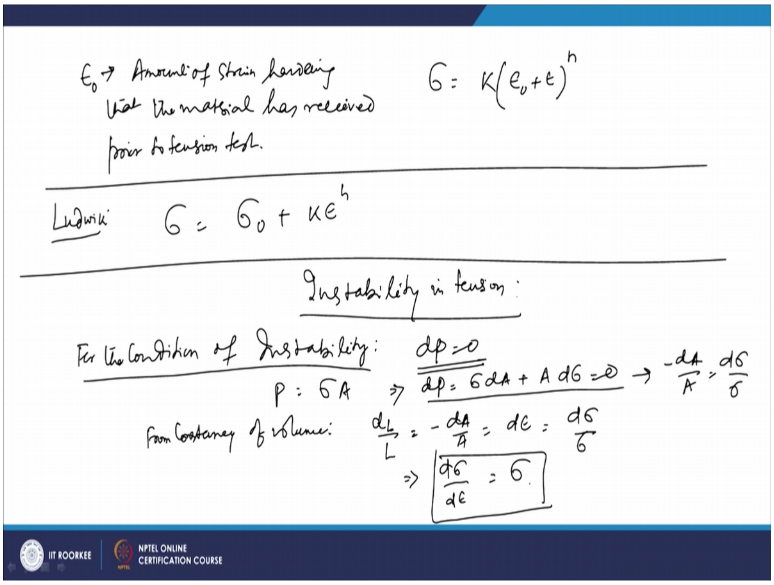
Now if you there is n equal to 1; so again n equal to 1 means this will be sigma equal to K epsilon. So, you will have a liner equation. So, it will go like this. So, this will n equal to 1. You may have n as half and in those cases, if your n becomes half. So, n will go like this and this will be n equal to half. So, this way this is epsilon equal to 1. So, this way this power like equation that is this sigma equal to K epsilon n. This is known as power law equation and that basically changes in shape for the different materials.

There is another terminology which is required to be known to you and that is basically the rate of strain hardening. So, the rate of strain hardening and the strain hardening exponent, this n is known as strain hardening exponent. And there is a terms known as rate of a strain hardening. The rate of a strain hardening is basically different, then this so this is basically d sigma by d epsilon. So, the curve which is there so, d sigma by d epsilon is known as the rate of strain hardening.

So, you can use this curve analyze further to we find the rate of strain hardening and if you look at the value of n, if you try to find. So, value of n is nothing, but is d of log sigma upon d of log epsilon that is what we know. This is the slope of the log log curve log true stress by log true strain. Now this can be written as d of l n sigma upon d of l n star I mean epsilon and that can be written as; so this will be log sigma will be 1 by sigma d sigma. So, this will 1 by sigma d sigma and this will be 1 by epsilon d epsilon. So, it will be d sigma by d epsilon and this will be epsilon. So, it will be if basically epsilon by sigma d sigma by d epsilon. So, what you see is that d sigma by d epsilon. So, which is nothing, but the rate of strain hardening that will be basically, if you look at this will be n into sigma by epsilon

So, this is if certainly a function there is a correlation, which exists between the rate of strain hardening and the strain hardening exponent. Further there has been many kind of these you known relationship between the true stress and true strain proposed by the different types of researchers.

(Refer Slide Time: 24:11)



So, the another kind of relationship which has been suggested by (Refer Time: 24:09) and he tells that the sigma is basically can be expressed as epsilon naught plus epsilon raised to the power n.

Now, in this case this is also the value of true strain and here this epsilon naught. So, this is basically the amount of strain hardening that the material has received a prior to the tension test. So, material might have got certain strain hardening prior to the tension test which is, it is subjected to now. So, that the material has received prior to tension test.

So, in this case that is represented by sigma equal to K into epsilon naught plus epsilon raise to the power n. There is further one variation in this and that is proposed by Ludwik and Ludwik equation tells that sigma will be sigma naught plus K epsilon n. Now in this case this sigma naught is the yield stress and K and n are the you know strength coefficient. This is and then this is n is the same. They are same as what we saw earlier in those cases. So, this way they have given different types of expressions and from there you can have the value of you know you can find this slopes you can find the value of n or so.

Now, we will try to discus about the instability in tension. Now what happens that so, if we talk about the instability in tension. So, by this time we know that in the true stress true strain curve, it will come to a point and then from the there the plastic deformation starts and then ultimately the material starts you know necking down and finally, it fractures. So, if you look at the engineering stress strain curve at the point of necking, what you see is that the curve starts coming down.

Basically so, what you see that at this is the point maximum point and from there it starts coming down. So, that point can be found out by letting the derivative b equal to 0. So, basically the necking will be obtained necking. Necking will start at the maximum load during the tensile deformation of the ductile material. And the ideal plastic material is one where the no strain hardening occurs in those cases which would become unstable in the tension. So, once you apply it will become unstable and it will be start necking as soon as the yield you know starts.

So, as soon as the yielding will start, there basically the you know it will there will be no strain hardening and then there will be necking started. Whereas, in the normal cases what we saw that after yielding you have a reason of strain hardening and then once you reach at is as a different point, there the necking starts. So, basically in ideal plastic material from there actually necking will start. Now so, this is normally for the case of the real metal where the strain hardening goes on. Now necking or the localized this you know deformation, we think about this start at the maximum load and where the increase in the stress so, you as we have already discussed that in one case when it is increasing.

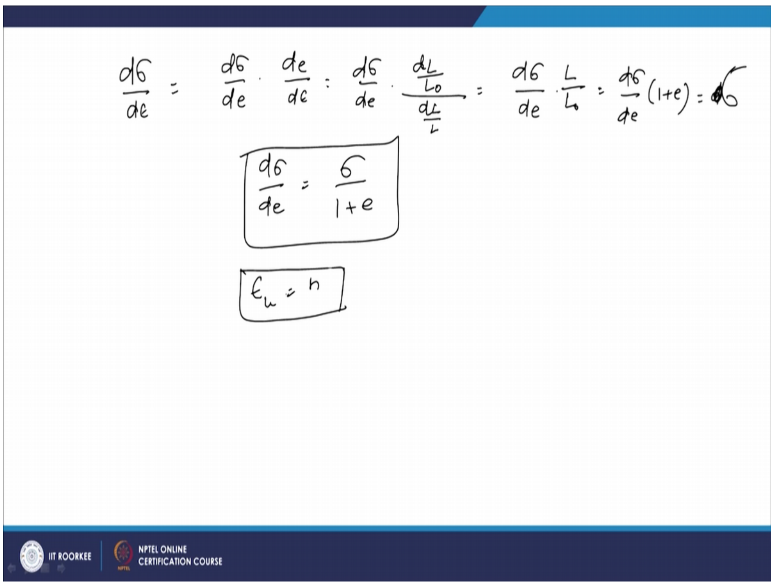
So, there one of the parameter is dominant whereas, the another parameters you have increase in the strain hardening, then you have the decrease in the cross section. So, increase in the stress due to decrease in the cross section area of the specimen is larger. So, that is that is why there will be you know the increasing strain hardening that is observed. And so, what we can say is that the condition of instability.

Now in this case the condition is that d p should be 0. So, the maximum load is there. So, at that point when the d p becomes zero that time the necking will start. Now what we see is P is sigma A. So, once you have P equal to sigma A, so, that from here you can find d p will be sigma d A plus A d A d sigma. So, that is basically equal to 0. So, this is the condition for the instability.

Now what you see that from since in the plastic deformation? You have constancy of volume. So, in the constancy of volume, you we see that d l by l is minus of d A by A and which is nothing, but d epsilon. So, what we see that if you use this minus d A by A, it will be d sigma by sigma. So, that is why so, from here you get minus d A by A equal to d sigma by sigma. So, this minus d A d A by A d sigma by sigma. So, that will be equal to this. So, what you see is d sigma by d epsilon that will be equal to sigma. So, this is the condition of instability.

So, what we see that in the case of the necking point where the necking starts, you see that the in the true stress true strain curve. It is a slope becomes equal to the true stress value. And at that point where the slope becomes equal to the true stress value that is you know this point that will be at that point there will be necking started. So, that is how you try to find the point of necking. You can further have alternate expressions for this.

(Refer Slide Time: 31:26)



So, what we see that you get d sigma by d epsilon and you can write further that as d sigma by d e into de by d epsilon. Now you can write this d sigma by d e that is your engineering strain and then the d e by d epsilon. So, d e we known that d e is d L by d naught L naught and then d epsilon will be d L by L

So, what we can write is that d sigma by d e and to it will be same as L by L naught. It will be left as L by L naught L by L naught is 1 plus e. So, it will be d as sigma by d e into 1 plus e. So, that can be written as d. So, that is that we have already seen that this sigma by d e has to be equal to sigma for the condition of instability. So, what we see is that d sigma by d e will be sigma by 1 plus e because d e we know that d sigma by. So, e is epsilon is 1 plus e. So, we can write d sigma by d e as sigma by 1 plus e. So, these are the few conditions which we get to know where about the condition of instability in the case of the tension testing when the necking starts.

We also get some other you know the relationship values and from there what we see is that we get these two uniform strain that becomes equal to n also in the case of the instability in tension. So, what we have seen in this lecture that you have you must be quite conversion all these terminologies and then you have the condition instability where we try to see that how with under the different conditions. How we can have the different expressions, which express those conditions of the necking and those when the stability occurs and there is necking and then you have fracture. So, these expressions will be of importance when we discussed about the failure criteria or not failure criteria; in fact, when we discussed about finding the stress and strain.

Thank you.