Principles of Metal Forming Technology

Dr. Pradeep K. Jha
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 15
Plastic Stress Strain Relationships

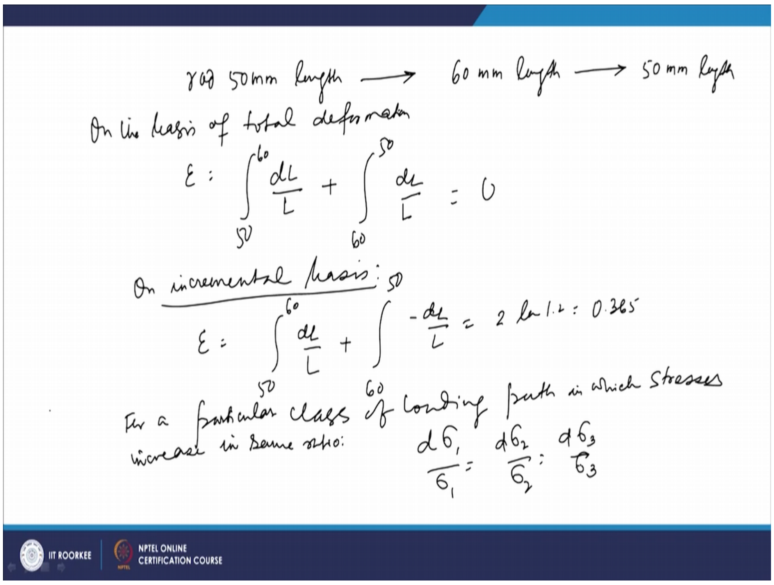
Welcome to the lecture on Plastic Stress Strain Relationships. So, we have discussed about many kind of relationship in the earlier lectures, now we will go to discuss about the stress strain relationship in the case of plastic deformations; so in the case of plastic deformations. So, in the case of elastic deformation as we a nu that you have certainly up to certain limit the out slow valid, and then you have the posies ratio is there and that way you have many type of relationships by which you can relates the strain to stresses and can find the value of stresses.

But we where we go into the plastic reason, then this linear relationship is no longer valid and you will have to find basically the find the value of stresses. And basically in this case the stresses will be depending upon the entire history of loading.

In this case may be you know that when something suppose any specimen of suppose 50 mm length is you know its length is increase to 60 and then further coming down to 50. In that case if you look at the otherwise there is no change it needs length. But in the case of plastic deformation basically where its go in increments, you will have to take into a account the strains in every increment. And then that way the total you know strain will be found by summing up the all the strain which have taken place.

So, this way you can take what very simple examples, suppose if you look at the case where suppose a rod of 50 mm is there; so rod of 50 mm length.

(Refer Slide Time: 02:34)



And suppose you are converting this rod to 60 mm length. So, if you look at this rod which is 50 mm length extended to 60 mm length and further this 60 mm length is compressed.

So, this is longer to 60 mm length, and then it is further compress to 50 mm length. Now you can see that if you talk about the total deformation, if you take the analysis based on the total deformation; in that case a on the basis of total deformation what you can find, a we will find like a 50 to 60 ln of; so dL by L. So, that is ln and then that that comes. And then further you have from 60 to 50 and then again you have the dL by L and this time you have 60 to 50.

So, if you do in the based on total deformation case, you are epsilon or in the strain a that comes as 0. Whereas, if you talk about the strain which is occurring in the increment, in that case a on incremental basis. Non incremental basis (Refer Time: 04:17) see a such kind of you know find in the strain. So, what we see is, you have 50 to 60 dL by L and then you have 60 to 50 and introduced compression. So, it will be minus dL by L and that is why it will be 2 times ln 1.2.

So, it will be 0.365 which is basically the material has subjected to such is strain conditions. So, what is say it is say it is no practical, and in the case of plastic deformation basically we are concern with the whole history of all you know increment of inviser deformation take place and we are taking into a account all these increments.

Again many a times what we see that, in certain types of loading what we see that the stresses are increases in the same ratio; so for a particular class of loading. So, you have many a times you deal with this loading path in which the stresses increase in the same ratio. So, in which stress increase stresses increase in same ratio.

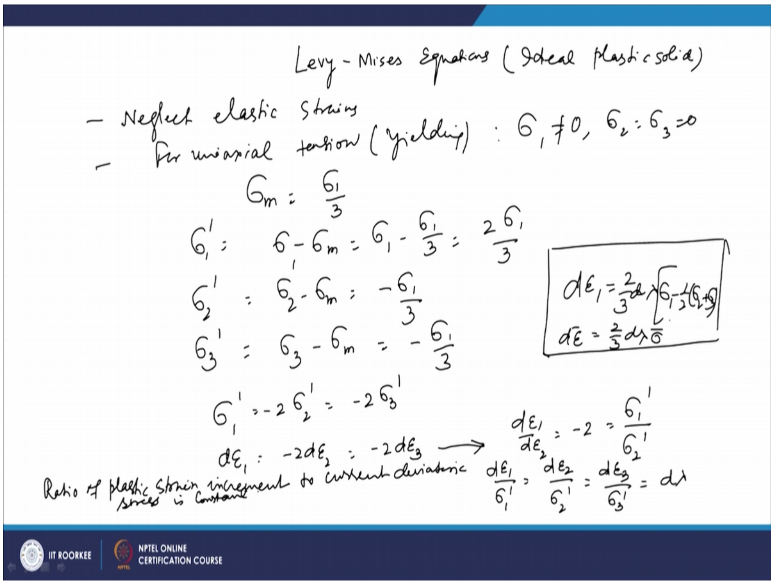
So, this is also known as the proposed the loading sort of case, and in those cases what we see the d sigma 1 by d 1 basically that usually becomes constant that is, d sigma 2 by sigma 2 and same as d sigma 3 upon sigma 3. So, in this case say basically the plastic strains will be independent of the loading path. So, in such case is the plastic. So, final it will be depending upon only the final state of the stress. So, in such situations; otherwise what we will see that you have to go for the every increment and it will depend upon the all these you know incremental behaviors.

So, normally what we when we talk about the plastic stress strain equations, normally we deal with 2 types of or 2 categories of the relations. One is that the incremental or flow theories that is that will be basically relative the stress to the plastic strain increments. So, in that case normally we are neglecting the elastic strain components and in one case we are basically talking taking into account even the total plastic strain basically. So, so in that case we also take in to account the elastic part also.

So, based on that we have 2 two theories, which analyze these stress strain; I mean relation between the stress and strain in the case of plastic deformation. So, we will deal with that. Now we will deal with the first one that is known as a Levy-Mises equation which is for the ideal plastic solid.

So, when we talk about this Levy-Mises you know equations that is for ideal plastic solid.

(Refer Slide Time: 07:59)



So, that equation that is known as Levy Mises equations and it is for ideal plastic solid. Now in this case we normally neglect the elastic strain and so, we are neglecting neglect elastic strains.

So, this is for the ideal plastic solid where the elastic strain is neglected and they are known as the floor rooms because we are dealing only with the plastic strain, and that is known as the Levy-Mises equation now in this case. So, what we do suppose we are assuming for a uni axial tension, where you have only sigma 1 is there which is not equal to 0 which is non zero quantity, and sigma 2 and sigma 3 is a 0. So, if suppose for uni axial tension case, if you some specimen subjected to the uniaxial tension and in that there is yielding. So, uniaxial means you have sigma 1 is non zero and sigma 2 and sigma 3 is 0.

So, for this have we know that you have sigma m is sigma 1 plus sigma 2 plus sigma 3 by 3. So, that will be sigma 1 by 3. Now if you see that then you have the deviatory history where stresses can be computed, and if you find the deviatory histories sigma 1 prime. So, that will be basically sigma 1 minus sigma m, so sigma 1 minus sigma m that is sigma 1 minus sigma 1 by 3. So, it will be 2 sigma 1 by 3. If you find the sigma 2 prime that deviatory component for the sigma 2 is the principle stress in the second you know direction. So, that will be again this will be sigma 2 minus sigma m, and sigma 2 be in 0.

So, it will be minus sigma m and sigma m m is minus. So, it will be minus sigma 1 by 3 similarly that will be same as sigma 3 primes. So, it will be again sigma 3 minus sigma m. So, that will be minus sigma 1 by 3. So, what we get in such case is in the such case of uniaxial tension, when specimen subject to yield in what do you see that, sigma 1 prime you get it as 2 times sigma 2 prime minus and then minus of 2 times sigma 3 prime.

Now, in the case of plastic deformation we also know that there is constancy of volume, which is not the applicable thing in the case of elastic deformation. Now in the case of in the plastic deformation what you get is, you will get d epsilon one it will be minus of 2 times d epsilon 2, that will be again minus of 2 times d epsilon 3. So, what you see from these 2 equations you get now; now from this equation you will further get d epsilon 1 by d epsilon 2 that will be minus 2.

So,. So, this will be nothing, but if would see, it will same as sigma 1 prime upon sigma 2 prime. So, so what we see? You can have this equation in a generalize way and that gives you the Levy-Mises equation and so, the equation comes you can have from here you can get that d epsilon 1 by sigma 1 prime, it is seen to be same as d epsilon 2 by sigma 2 prime and that will be same as d epsilon 3 by sigma 3 prime and that we take as this d lambda. So, what it shows this expression d epsilon 1 by sigma 1 prime equal to d epsilon 2 by sigma 2 prime divided by equal to d epsilon 3 by sigma 3 prime, now what we see that at any instant of deformation the ratio of this is the plastic strain increment ah.

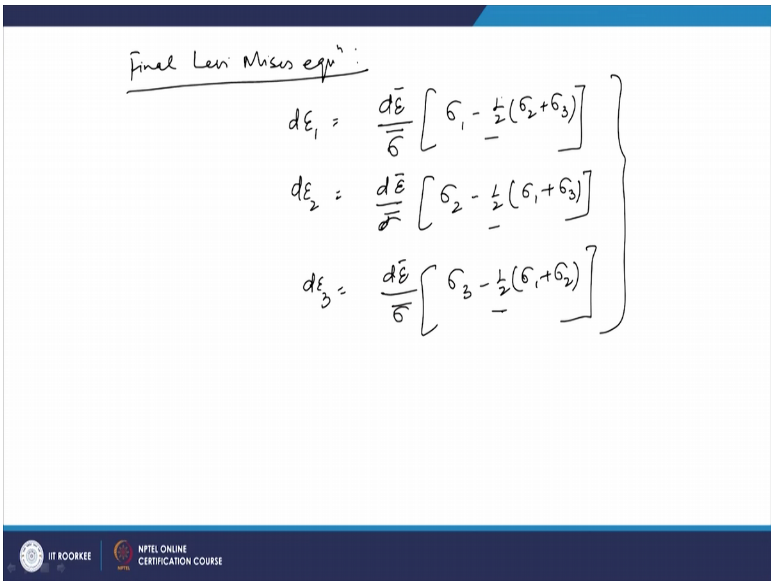
So, ratio of the plastic strain increment at any instant and the this sigma 1 prime is nothing, but the deviatory stress component. So, ratio of this plastic strain you know increment to the current value of the deviatory stress, that remains constant that is what we there is a stemmed out from the this Levy-Mises equations.

So, ratio of plastic strain increment to current deviateric stress is constant. So, this is true for at any instant of deformation, and this is known as the Levy-Mises equation and you can further we have already seen the equation in the last chapter and from there, we can generalize this equations like you have d epsilon 1 will be 2 by 3 d lambda, and then it will be sigma 1 minus half of sigma 2 plus sigma 3.

So, this is what. So, this similarly you will have d epsilon 2 will be 2 by 3, d lambda sigma 2 minus half of sigma 1 plus as sigma 3. Similarly d epsilon 3 will be 2 by 3 d lambda into sigma 3 minus half of sigma 1 plus sigma 2. So, what we see that we try to get this effective strain values and what we get is that if you talk in terms of effective strain d epsilon bar will be 2 by 3 and then d lambda and effective same. We can also write in terms of these effective is strain and effective stress. So, 2 d epsilon bar will be 2 by 3 d lambda, and this will be sigma bar. So, this way we get this expression for the plastic strain increment and the effective strain in such cases.

So, if you try to see the final Levy-Mises equation, the final Levy-Mises equation becomes.

(Refer Slide Time: 15:14)

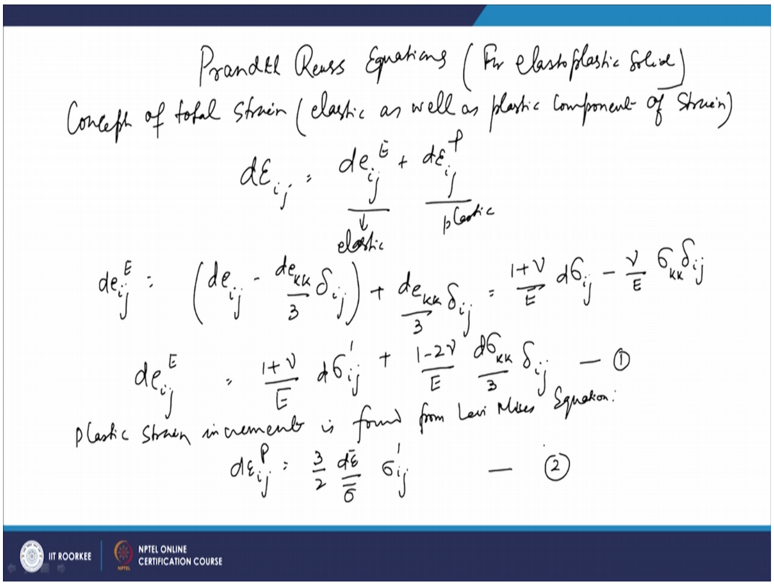


So, your final Levy-Mises equation will be coming if you look at, this will be d epsilon one it will be d epsilon bar by sigma prime that is what if you this place this 2 by 3 d lambda term and then you get sigma 1 minus half of sigma 2 plus sigma 3. Similarly d epsilon 2 will be d epsilon prime by sigma prime. And then this will be sigma 2 minus half of sigma 1 plus sigma 3 and d epsilon 3 will be again similarly the effective value of the strain divided by the effective stress, and then sigma 3 minus half of sigma 1 plus sigma 2.

So, if you try to see this equation with the equation which we got in the case of elastic strain, now you can see that in place of here in place of this d epsilon prime by sigma prime, we got the 1 by E. So, that this is this displaced by you know 1 by E and then so, because as you know that stress by strain component was in that case the e values. So, it is strain by stress components it will be 1 by e that was there in that case earlier which we discussed; and also here we are you were getting the nu term and you also found that in the case of plastic deformation, the nu was because of the constancy of volume and all other conditions involved, the nu was coming out to be 1 by 2.

So, that is nu is coming as they 1 by 2. So, this is the equations for a typical plastic you know stresses strain, which talks about these parameters, which are used in such cases. Now the next story which talks about these plastic stress strain equations, that is the Prandtl Reuss equation.

(Refer Slide Time: 17:39)



So, that is known as Prandtl Reuss equations, this is for normally elasto plastic solid.

Now, what we have seen in the earlier derivation was Levy-Mises equation, now that is basically for such case where the plastic strain component is very large, and basically we are neglecting the elastic strains. But when we are talking we are in the range of this elasto plastic reason, then it is you know more important not to neglect these deformations which occurred or strain which occurred in the case of the elastic deformation or in that rang of elastic deformation.

So, for that the Prandtl Reuss they have proposed the concept of total strain and so the concept of total strain was proposed. Total strain that is you will composing of elastic as well as plastic component of strain. So, what we doing that, in that basically we are dealing with it is elastic strain part also and the plastic strain part also, elastic part we have already discussed in our earlier lectures. So, they are from we get elastic strain part and the plastic strain parts we are getting from the Levy-Mises equation, which basically deals only for the plastic strain and which neglects the elastic strain part.

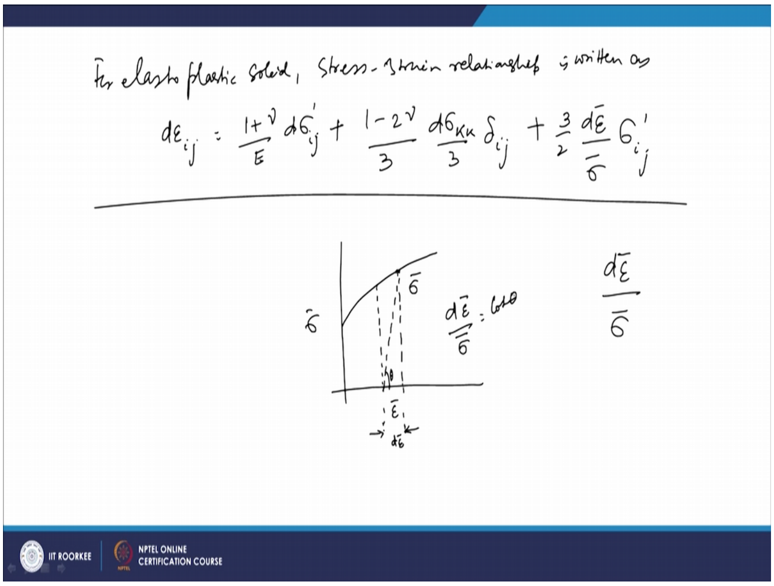
So, we can write we write d epsilon, i j will be de ij that is elastic plus d epsilon ij that is plastic. So, this is this part is the elastic part and this part is the plastic part. Now if you recall we can convert this elastic part, now if you see the equation for the elastic parts. So, we had got the equation for the elastic part and that can we further return in a in the form that is de ij minus de kk by 3 delta ij, and then plus de kk by 3 delta ij. So, we can write certainly this because this part and this part will be cancelled, where we know that this is the Kronecker delta. So, you have that component and this is the main strain part. So, certainly this is the main part.

So, that is why we know that now this will be coming as the deviator. So, now, we can write this as 1 plus nu by E and d sigma ij, and then minus nu by E delta kk and sigma kk and delta ij. Now again you try to look at it, this part is suppose total strain. So, we are basically subtracting form there this part. So, this is the main part will strain part, and we get this 1 plus nu by E and d sigma ij and then this part has we know. So, this part will be as you know that this is sigma kk and delta ij into nu by E. So, this way you define this.

So, you can write de ij E so that can be return as 1 plus nu by E and there it will be d sigma prime ij plus 1 minus 2 nu by E d sigma kk upon 3 and delta ij. So, this can be return further which talks about this total elastic part of the strain. Now form the Levy-Mises equation you can find the expression for the plastic strain. Now for the plastic strain we have already what certain expression; so the plastic strain. So, increment is found from Levy-Mises equation.

So, we write these Levy-Mises equation as d epsilon ij that is plastic, and it will be 3 by 2 and then d epsilon by sigma bar, that we had seem that expression and then income it as sigma prime ij. So, if you add the expression for the total is strain, which is the addition of this strain that is elastic strain and the plastic strain. So, this will be 1 plus 2 will give you in the total strain. So, if you look at the taking into account to both the components then for elasto plastic solid.

(Refer Slide Time: 24:03)



So, stress strain relationships can be retained as now here you can write d epsilon ij. So, this will be the sum 1 plus 2. So, you have. So, this 1 and this 2 we are adding, and that we getting the expression for the total strain. So, this will be get a 1 plus nu by E and that will be coming as d sigma prime ij, which is what we got 1 plus nu by E by d sigma ij prime, then will you have again you have one plus. So, this is coming 1 minus 2 nu by E d sigma kk by 3 and that is sigma ij.

So,. So, that is what will write. So, this expression fin further be return there. So, it will be 1 minus. So, we have this acquit 1 minus 2 nu by 3 2 nu by E d sigma kk by 3 delta ij. So, 1 minus 2 nu by 3 d sigma kk upon 3 and delta ij and then, you have the summation of the plastic strain part and that comes as 3 by 2 d epsilon bar prime upon plastic stress, and then that will be sigma ij prime.

So, this way, this is what we got in this and then from here we get the expression form the total a strain and that is the you know the j known as the Prandtl Reuss equation which basically involves the you know a calculates both these strains other be plastic or be the elastic, both are taken into account when we are dealing with. So, elasto plastic solid and we do not have the you know liberty to neglect these elastic you know component; because when that is very very large the plastic strain part is very very large as compared to elastic part then we can have, but otherwise they are cannot be neglect.

So, in that cases indorse cases we try to use this formula for finding the strain and then further you can have the calculation of the stresses using this. Now there is Levy-Mises and the Prandtl Reuss equations, they basically are providing the relationship between the increment of the plastic strain and the stresses. So, you have one side you have the equation for the elastic part that we have already earlier discussed, and in this case these 2 equations are basically given you the relationship between the increments of the plastic strain and the stresses. But the challenge which is their when we deal with certain situations such situation that, what will be the next increment of the plastic strain for a given you state of stress when the loads are increased basically incrementally.

So, that is how they are the basically essence is they are do to fine the you know increment in the you strain, because of the increment in the a stress. And once you find the increment in the stress in every stage, in those case then what you can do is that, you can simply sum them and you can find the total plastic strain. That is what we have seen that in the case of plastic strain analysis, we are basically we cannot neglect or that that we have seen that we did for total stress analysis.

So, there in we saw that when we are between from 50 to 60 and coming back from 60 to 50, based on the total strain concept it was coming as 0. Whereas the actually when we do the analysis on the incremental basis, in that case it tells you that what is the strain which is subjected too. So, this way you use these plastic stress strain equations and also the yield criteria and other flow behavior of the material, we are like we use also the concept of the effective stress and effective strain you know concepts, to find basically the stresses and strains in the plastic flow analysis now.

So, this is about these 2 methods and normally what we have seen that in the Levy-Mises equation you get these. So, here you get these a d epsilon prime upon sigma prime now if you look at this how to establish this. So, if you try to see in the in the equation for you suppose you have this sigma prime and this is the epsilon prime. So, basically being the you know you are talking about your plastic ideal plastic solid. So, in those cases what happens that you are if you look at this point, now if you see from here, now this suppose this becomes.

So, this is your sigma prime. And now in this case you when a it is loaded. So, that way now this part theta, now and this will be your part that is d epsilon prime and that is why this d epsilon prime upon the sigma prime if you look, at this is from this curve if you look at this theta. So, that will be basically the cot theta. So, what happens that; we have seen that this we have the expression for the effective stress, effective strain curve for the increment of the strain that is a d epsilon bar which is there in the case of the plastic strain analysis or plastic stress strain analysis. And this way we try to stabilize this component that is d epsilon prime by sigma prime.

So, depending upon so, we can deal with the different types of problems which we come across in the case of the plastic stress and plastic strain, and these two Levy-Mises equation as well as the Prandtl Reuss can be applied depending upon the type of the soli, whether it is real plastic or the elasto plastic solid.

Thank you very much.