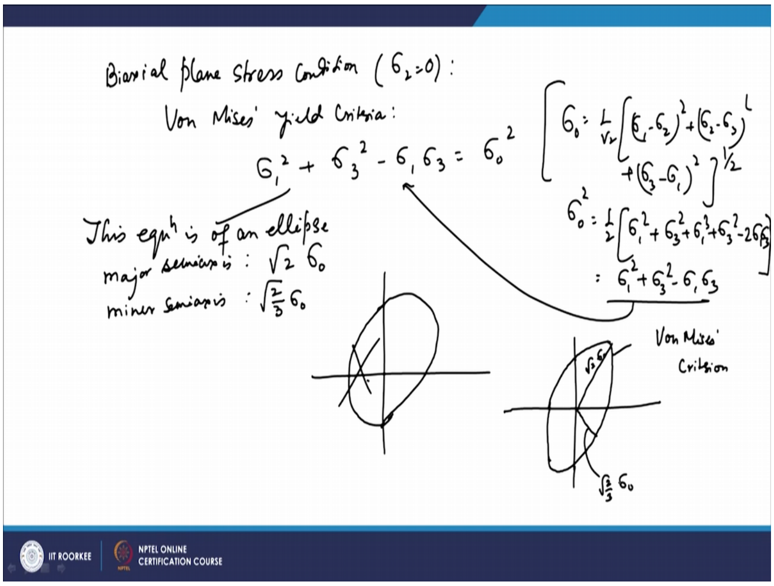
Principles of Metal Forming Technology
Dr. Pradeep K. Jha
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 14
Yield locus, Octahedral shear stress and strain

Welcome to the lecture on Yield locus and octahedral shear stress and strain. So, in this lecture we are going to discuss more about the yielding conditions and more about it is terminology, which may come to us, which may face, which we may face during our subsequent discussions, in the forming process analysis and so. So one of the important terms is the yield locus so, as you at the term indicates it is basically locus, it stocks about the yield points how the yielding will occur it will talk about it. And that can be understood by referring to a condition. So, if you are talking about a biaxial plane stress conditions.

(Refer Slide Time: 01:15)



So, suppose we are talking about a biaxial plane stress condition. So, you have only sigma once a plane stress condition means you have only sigma 1, and sigma 3 sigma 2 1 of the stress 1 of the principal stress is 0 in that case and you have biaxial we are going to draw on the xy axis.

So, you are assuming sigma 2 as 0 in this case. Now, if you talk about such cases then if you try to see the one misses yield criteria. Now, one misses yield criteria for this condition will result in to you have seen the how the Von Mises yield criteria one gives. So, it will give us the sigma 1 square plus sigma 3 square minus sigma 1 sigma 3, that will be equal to sigma naught square. So, actually you do that sigma naught and then this side you will have sigma 1 square plus sigma 3 square minus sigma 1 sigma 3, because this 2 term 2 will be divided 1 by root 2 and that will be square.

So, 1 by 2 and 2 so, that will be going. So, you will have this term and other side you have you can see you have sigma naught will be 1 by root 2 and then sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square and plus sigma 3 minus sigma 1 square. So, so this way on this so, this will be sigma naught square will be 1 by 2 and this way sigma 1 and sigma 1 and 2 is 0.

So, you will have sigma 1 square plus again sigma 3 square plus again sigma 1 square plus sigma 3 square minus 2 sigma 1 sigma 3, that is what we get. So, you have 2 sigma 1 square plus 2 sigma 3 square minus 2 sigma 1 sigma 3. So, this 2 and that 2 will come will we cancelled. So, it will be get sigma 1 square plus sigma 3 square minus sigma 1 sigma 3.

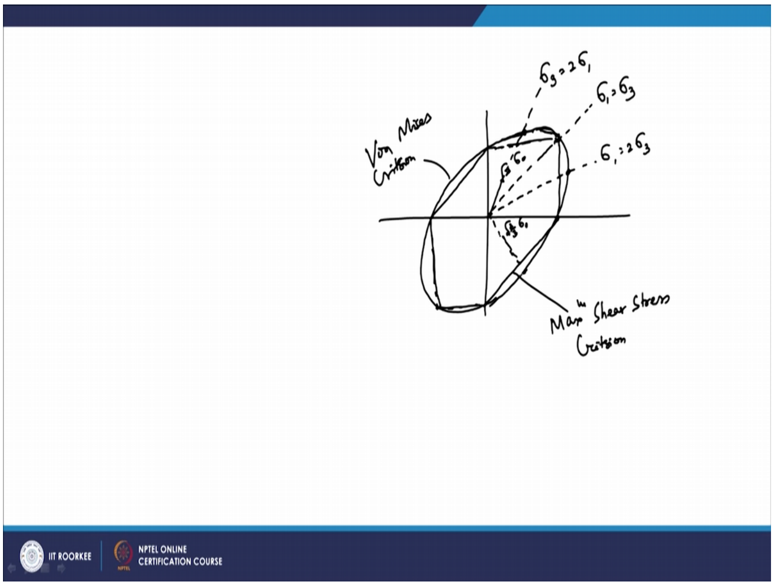
So, that is what we get from here? So, for this biaxial plane stress condition, if you look at the Von Mises criteria tells about it if you look at this equation basically this is an equation of an ellipse and the ellipse has some major axis and the minor axis. So, this is this equation this equation is of an ellipse. So, the ellipse has some measure semi axis you have major semi axis, this is a coming as root 2 sigma 0 and you have minus semi axis.

So, that comes as root 2 by 3 sigma naught. So, this way you get 1 locus this talks about the yield conditions and this plot which you get for this sigma 1 and sigma 3 with sigma naught, that is known as a yield locus. So, if you draw if you try to draw this you know graph. So, what you get is you get in an ellipse you get similar you know ellipse is not you know very accurately drawn, you can have a better, you can try further, you know in a better way.

So, you will have around this way. So, this way I mean it is it is not better anyway you can you can leave it. So, now, in this case is what you see that now here this outer line this talks about the Von Mises criteria. The equation which we have seen this sigma 1 square plus sigma 3 square minus sigma 1 sigma 3, this is the equation of that ellipse and you have the semi major axis as well as semi minor axis, and semi major axis is root 2 and this sigma naught. So, this will be root 2 of sigma naught and similarly this is semi minor axis and this will be root 2 by 3 and this will be root 2 by 3 sigma naught.

So, this is how you get the semi major axis and minor axis now the thing is that this is this equation is the yield locus and this is basically the yield point which is predicted by the Von Mises criteria.

(Refer Slide Time: 06:41)



Now, if you try to further draw it and try to draw for the Von Mises criteria. So, what you see is. So, you get such curve now in this is the Von Mises curve and if you try to see the you know Von Mises Tresca criteria. So, Tresca criteria will be coming like this. So, the inner lines which we see. So, you have this is the Von Mises criteria. And this line is basically indicated for the maximum shear stress criteria or the Tresca criteria. Now what we see this you have the different stress ratio, if you have the sigma 1 and sigma 3 that is what we are taking sigma 2 is anyways 0.

So, if the sigma 1 and sigma 3 have the different ratios for different stress ratio you will have different points. And, if you try to see the conditions like you suppose this is your; we have seen that this is root 2 sigma 0. And, this is minor axis and this is root 2 by 3 sigma 0.

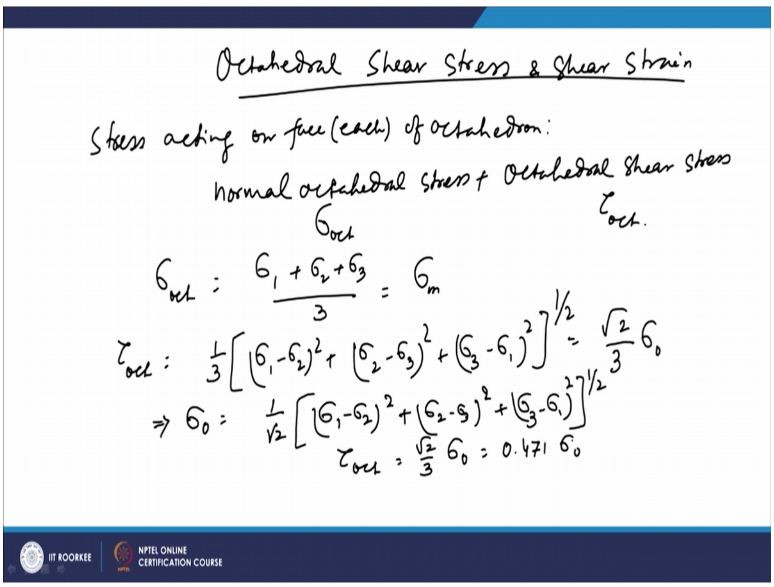
Now, if you look at the different conditions, now if you see this will be a case for the sigma 3 as 2 sigma 1. So, this is how you get this point on the you know Von Mises using Von Mises criteria. Similarly, you will have this graph which you will get and that we get by for the condition sigma 1 as 2 sigma 3. So, based on the these you know stress ratios, you get such curves you see that suppose for this case this is the case which is represents sigma 1 as sigma 3.

So, here you are getting the similar prediction by the both the criteria; however, you see that you have certain you know differences, you see that the prediction by the Von Mises criteria in this reason it is more if here also it is more than the that of the maximum shear stress criteria. Here also it is more than the maximum shear stress criteria, here it is same that of the maximum shear stress criteria.

And, it is basically there is maximum difference here and it looks out to be close to it is said to be close to the 15.5 percent of the stress, which is a predicted then the yield stress predicted by the maximum shear stress criteria. So, what we predict here that will 15.5 percent more than that predicted by the maximum shear stress criteria.

So, that talks about the comparison of the yield criteria for the plane stress conditions and this yields locus will tell you comparatively, that how the both these criterias differ. Now, we will discuss about another term, which normally comes in the case of the plastic deformation and that is octahedral shear stress and shear strain.

(Refer Slide Time: 10:46)



So, octahedral shear stress and shear strain. Now, if you look at this term octahedral. So, certainly octahedral stresses they are basically stress function they are particular set of stress functions which are very important in the theory of plasticity and they are the stresses which act on the faces of a 3 dimensional octahedral. And it has certain property it has certain geometrical property and you know for these octahedral their faces you know faces of these their planes. So, they make equal angle with each of the principle you know the directions of a stress.

So, that angle is fixed and that angle is 56 degree and 44 so, 54 degree and 44 minute. So, that way we define these octahedral you know stresses. So, they these are stresses which are acting on those planes. Now, for them actually as we as we discussed that the angle between the normal to the face to one of the face and the nearest principle you know axis, that is fixed that is 54 degree and 44 minute. And, if you look at that cosine of this angle this cosine of this angle comes out to be 1 by root 3.

So, so that is what the regular you know feature of such terminologies octahedral or octahedron. Now, this is nothing, but this is same as so, 1 by root 3 when it comes basically it is equivalent to the you know fcc crystals 1 1 you know plane. So, as we know that this is the close packed plane which is responsible for the you know very good ductility for the fcc crystals, where they you have and that is why you have a fcc crystals have a good you know ductility or they have good malleability or so.

So, what we try to see that how this octahedral shear stresses are you know treated how they are important when we take in the context of the plasticity theory. So, what is there; that when we talk about the stress which is acting on these faces of the octahedral. So, again these stresses are composed of 2 types of stresses. So, one will be your means stress or that is your normal octahedral stress and then you have the shear stress octahedral shear stress.

So, if you talk about the stress acting on face off each face of octahedron. So, it will be basically you will have the normal octahedral stress and you have octahedral shear stress. So, that is shear stress will be lying in that you know octahedral plane that is act octahedron, we normally define it as and this will be sigma octahedral.

So, you have normal of a octahedral stresses and this is your shear octahedral stresses. Now, this normal octahedral stress is nothing, but the hydrostatic component of the total stress. So, the normal octahedral stress sigma oct that will be the hydrostatic component.

So, it will be sigma 1 plus sigma 2 plus sigma 3 and by 3. So, we also call it as the mean octahedral stresses. So, that is the normal part if you come to the shear octahedral stress now shear octahedral is given as 1 by 3 and this will be sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square, and then whole raise to the power half. So, now, I have already discuss that one we talk about the hydrostatic part of the stresses hydrostatic called mean part of the octahedral stress.

So, I do not cause the yielding, yielding is caused because of the shear component. So, that is shear octahedral shear stresses. Now, if you look at this is again same as somewhat analogs to the stress deviator. So, talk to octahedral is nothing, but it is then I was 2 that is stress deviator which we have discussed. Now, if you see that the critical octahedral shear stress will cause yielding. Now, if you go for further the yield criterion what you see you can see go for the yielding criterion what you can see for uniaxial tension test again if you put it.

So, for uniaxial tension test if that sigma 1 will be sigma naught and sigma 2 and sigma 3 is 0. So, if you do that you will certainly see this 1 by 3 sigma 1 minus sigma 2 square plus these 1 that will be equal to root 2 by 3 sigma naught. So, that you will get because this will be sigma 1 square so, that way you will have 2 here and so, root 2 will come you know out. So, it will be 2 that will be root 2. So, ultimately it will come as root 2 by 3 sigma naught, because 2 sigma 1 square 2 sigma naught square will be there. So, it is root will be root to sigma naught and then by 3.

So, what we see that you get sigma naught as so, this 3 will come and this 1 by root 2 will come this side and what you get is 1 by root 2 and sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square, and plus sigma 3 minus sigma 1 square and then whole raise to the power half.

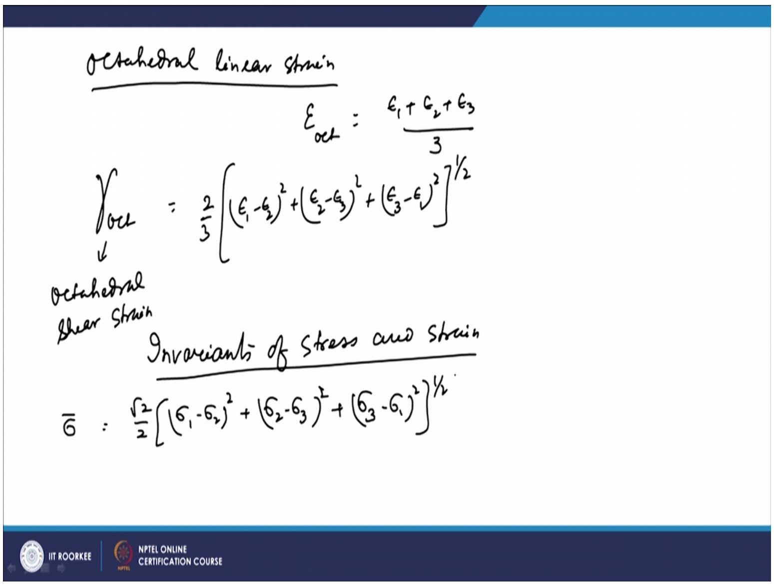
Now, what you see that you are getting the similar expression which you got, when we discuss the Von Mises theory of yielding or distortion energy theorem. So, what we can say. So, we say that these 2 theories the octahedral theory as well as that Von Mises theory they are those these 2 yield theories they are giving you the same value, and that is why what we say that these octahedral theory is considered to be the stress equivalent of the distortion energy theory. Because, the if you look at the stress which is predicted this sigma naught, which is predicted using this octahedral shear stress component, that comes to the same this gives you the same expression the failed criterion failure criterion as that was predicted by the Von Mises or distortion energy theorem.

So, that is why they are said to be so, octahedral shear stress theorem is considered to be the theory is considered to be the same or equivalent of the stress equivalent, I mean equivalent in terms of stress for the Von Mises or the distortion energy theorem and what we see that if you see the octahedral.

So, octa octahedral will be root 2 by 3 and sigma naught. So, that will be something close to 0.471 sigma naught. So, this is how you can find the value of these octahedral shear stress. And that is that has it is implication that has it is meaning the similar to that being you know projected when we discuss the Von Mises criteria. Now, we have also the strain so, when we talk about the octahedral strains.

So, again octahedral strains you have linear strain as well as you have the shear strain octahedral shear strain, and if you look at the octahedral shear normal strain.

(Refer Slide Time: 20:09)



So, octahedral linear strain; so again this octahedral linear strain will be sigma octahedral and that will be epsilon 1 plus epsilon 2 plus epsilon 3 by 3 and then again you have octahedral shear strain. So, that is different as sigma oct that is octahedral shear strain, and this is defined as basically 2 by 3 of sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square and whole raised to the power 1 by 2.

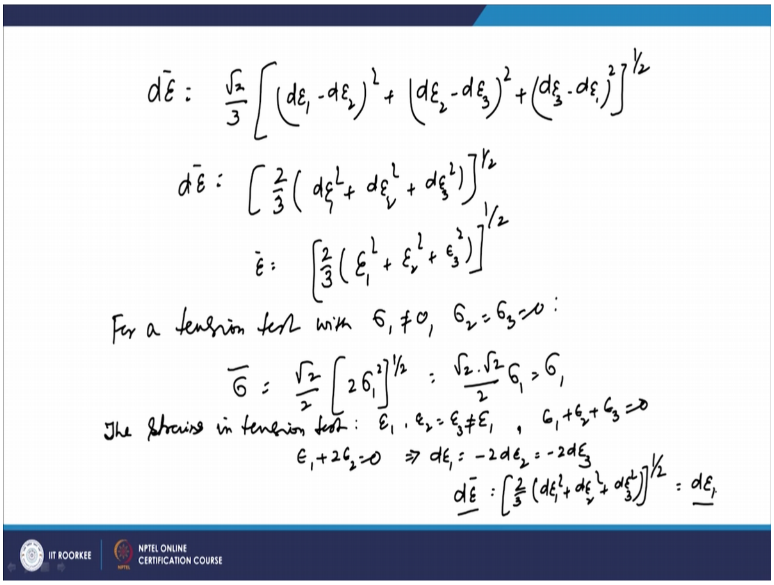
So, this is the definition of the octahedral shear stress and octahedral shear strain you have octahedral normal stress or octahedral mean stress and you have octahedral linear strain these are defined in this situation. There is another term which is normally encountered, when we deal with the theory of plasticity and that is the in variant you know stress or invariant you know in variants of strain.

So, we have already got these invariants we have seen earlier, but then sometimes it is very useful to simplify this complex state of stress or strain by means of the invariant functions. So, invariant functions of the stress and strain are used to represent these cases and it becomes easy and if you are plotting these stress strain curve or plastic flow curve in this in the terms of invariants of the stress and strain. So, then in that case you get the same type of curve regardless of the state of stresses. So, that is why these in variants of stress and strain they are used. Now, we discussed about the octahedral shear stress and shear strain and basically this octahedral shear stress, and shear strain they are the invariant functions and which describe the flow curve is independent of the any type of test.

So, if you see that most commonly these invariantly of stress and strain and in that what is defined is that, we talk normally we describe these term in the formation as the effective stress or effective strain. So, so the effective stress that is this way it is represented as that and this is basically equal to root 2 by 2. And, then sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square and plus sigma 3 minus sigma 1 square and then whole raised to the power half.

So, this way we define the effective stress. So, that we can see that this is a invariant function and this is the expression for the effective stress, if you try to see the expression for the effective strain.

(Refer Slide Time: 24:13)



So, we get the d epsilon bar and this is basically we are taking as d epsilon 1 minus d epsilon 2 square plus d epsilon 2 minus d epsilon 3 bar plus d epsilon 3 minus d epsilon bar 1 square, and then whole raise to the power half. So, this is the expression for the effective stress and effective strain. And, they are further simplified and you can simplify them as and we can further right, as in bracket of 2 by 3 into d epsilon square plus d epsilon 2 square plus d epsilon 3 square and then whole raised to the power half so, that way to comes outside.

So, this way we get it and if you try to see in terms of the total plastic strain, this plastic strain total plastic strain will be like 2 by 3 of epsilon 1 square plus epsilon 2 square plus epsilon 3 square and whole raise to the power half. So, this strain which are used in these equations they are actually the plastic portion of the strain.

So, they are normally denoted by epsilon p. So, that that it denotes the plastic component of the strain and that will be basically the total strain minus the elastic strain. So, when we are talking about the metal working theories normally what we see that the strain, which is there in the elastic part elastic reason is quite small negligible as compared to the plastic strain, because normally the plastic strain is quite larger. So, normally we neglect them and certainly we cannot when we talk about the strains at notch or in the pressure results in where there is over stressing. So, they are actually these cannot be neglected. So, at these places certainly you have to take part take into consideration the value of these you know elastic strains also.

So, that way you try to see that these invariant functions of the stress and strain they are mostly used for you know for representing the complex state of stress and strain. So, suppose you are going to have a for a tensile test if you look at, you know you can have a simple case where the you have if you do for the uniaxial tension test where there is sigma 1, which is not equal to 0 for a tension test and sigma 2 sigma 3 in that case is 0.

So, in that case if you calculate the effective stress. So, for a uniaxial tension test for a tension test with sigma 1 which is not equal to 0 and sigma 2 and sigma 3 as 0; so if you try to find the effective stress in those cases what you see is root 2 by 2 and then sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square and all that. So, you will have sigma square 2 times so, 2 times sigma 1 square and that being half.

So, so, what you see is root 2 into root 2 by 2 and then it is sigma 1. So, it will be sigma 1. So, what you see is that effective stress becomes the same as the stress principle stress, which is their sigma 1. So, similarly if you take the strain in the tensile test in that case epsilon 2 and epsilon 3 will be same and it will not be equal to epsilon 1. So, and also if you talk about the plastic deformation so, in that case you will have. So, if you can further analyze so, for the strains in tension test.

So, we have strain which we get epsilon 1 and epsilon 2 and epsilon 3 is there is equal, but is not equal epsilon 1. And, you have we have already seen that in the case of the plastic deformation the constancy of volume that leads to that condition of epsilon 1 plus epsilon 2 plus epsilon 3 will be equal to 0. So, what we see is that since epsilon 2 is equal to epsilon 3 is you will have a epsilon 1 plus 2 epsilon 2 that will be equal to 0.

So, we see that d epsilon 1 will be minus 2 of d epsilon to and that will be again as minus 2 of d epsilon 3. So, we can further find the value of the invariant strain and if you see the invariant strain value, you can calculate these the invariant strain, and that will be equal to in 2 by 3 of d epsilon 1 square plus d epsilon 2 square plus d epsilon 3 square and then raised to the power half.

So, if you should we do the analysis you will see that it comes out to be d epsilon 1. So, say same thing is coming the case of tension test you get the effective strain as the strain in the one of the direction. And, similarly the stress (Refer Time: 30:31) stress is coming as sigma 1. So, the you know you have what we do is normally the expression for this yielding that sigma equal to k epsilon and that can also be written as sigma bar that is effective stress will be k epsilon bar raised to the power n. So, that is also predicting the behavior of the stress and strain.

So, this is about these invariants quantities in stress and strain.

Thank you very much.