Principles of Metal Forming Technology
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Lecture – 13
Yield criteria for ductile materials

Welcome to the lecture on Yield criteria for ductile materials. So, as we know that in the case of metal forming, you have to visualize the cases where the yielding occurs. Actually when the material is subjected to stresses, then under that you know condition of stress the material yields. And there must be certain relationships, certain correlations which must say that under the action of different type of stresses, how the material will fail or when the material will fail.

So, we will have you know we will have to define certain type of cases or certain; you know the criteria by which we can say that this way the yielding will occur in the material. So, normally in the case of the you know when the material is subjected to many stress is combined stress conditions or so; in that case the yielding may occur because of all the presence of all these stress conditions. Otherwise, you can say that when we talk about the yielding in you know uniaxial direction, then we say that it in this yielding will occur when the stress value will reach at the stress value or so.

There are certain basically criterion which must be you know fulfilled and there are certain conditions. One is that the hydrostatic pressure or hydrostatic stress is does not cause yielding. We have already seen that the hydrostatic component of the stress they are responsible for not responsible for the basically yielding or the change in the shape or deformation. So, we are getting one conclusion from here that among the stress part the; if the hydrostatic part is not responsible for causing the yielding, then there must be certain part by the stress deviator part.

Because we know that the total stress is the summation of the at strategic stress and the deviator stress. So, if the hydrostatic stresses are not causing the yielding; in those cases certainly the stress deviator the deviatory component of the stress, they must be responsible for finding for you know yielding actually.

So, basically we will see that based on this there are some you know criteria which are one criteria, which is proposed and that is proposed by the Von Mises and that is why it is known as the Von Mises criteria.

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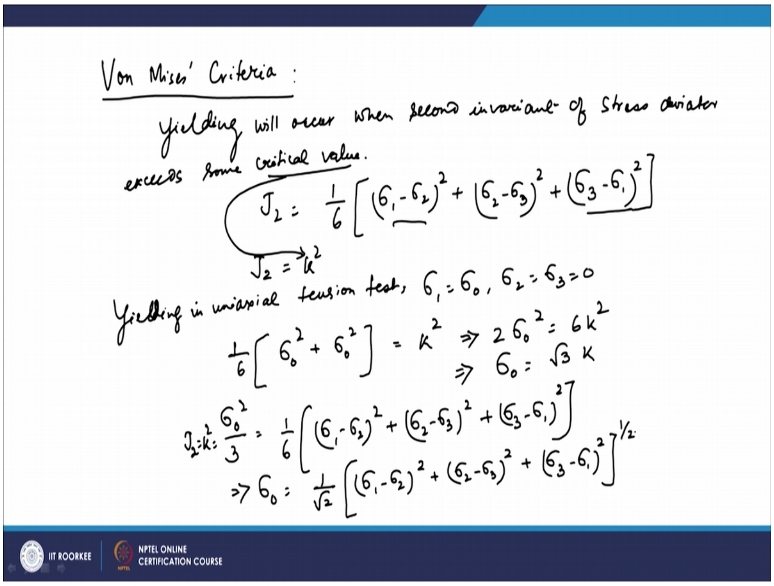

So, first criteria is Von Mises criteria; now Von Mises criteria tells that you know yielding will occur because we have when we discussed about this deviator stresses. So, we found the; we had found that deviators stress deviators, first deviator, second and third deviator in the invariants.

So, what it tells that the yielding will occur when second in variant of the stress deviator that is basically exceeds some critical value. So, in the earlier lectures we have already talked about this stress deviators; J 1, J 2 and J 3. So J 1 we know that it is a summation of the stress values and then J 2 is you know 1 by 6 of the sigma 1 minus sigma 2 x square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square.

So, basically and then J 3 also determinant of that, determinant value that was J 3. Now this values at this J 2; so J 2 must reach certain critical value. So, this J 2 is defined as; so, we had also defined in terms of sigma x, sigma y and sigma z or else we can also define it in terms of sigma 1 sigma 2 and sigma 3. So, this will be sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square.

Now, this value must reach a certain critical value when it is reaching a certain critical value; then yielding will occur that is what it is proposed by the Von Mises. And it is basically related to the stresses of deviatoric nature because we know that the hydrostatic component does not cause yielding. So, in this basically now this J 2 must reach a critical value that is K square. So, J 2 must reach this critical value K square. So, when this J 2 is reaching this critical value K square in that case the yielding will occur.

So, we have to find this critical value constant that is that constant K we have to find and for that we can have you know we can find this K by relating it to a yielding in the tension test. So, suppose we are going for yielding; so for yielding in uniaxial tension test, now if you take the uniaxial tension test case in that case you have sigma 1 will be sigma naught and sigma 2 and sigma 3 will be 0.

So in the case of uniaxial tension test they have you will have the failure when they. So, you will point it reached and in that case sigma 1 will be sigma naught that is yield stress value and sigma 2 and sigma 3 will be 0. Now in this case if you put this value into this expression. So, this will be sigma naught square and then further you have sigma naught square here also. So, what will be happening 1 by 6 of sigma naught square plus sigma naught a square, it will be coming from these 2 these 2 terms and in this term sigma 2 and sigma 3 both are 0; so, that term is anyway cancelled.

So, sigma naught square, plus sigma naught square and that will be actually K square. So, so that will lead to 2 sigma naught square will be 6 K square and in that case you can get sigma naught will be root 3 into K. So, we get sigma naught as the root 3 K now from there you can have. So, K value will be sigma naught by root 3; so, if you place this value into this equation. So, what you see you can have this equation; so what you see is J 2 that is K square K square will be sigma naught square by 3.

So, sigma naught square by 3 that is your J 2 that is K square. So, this will be equal to 1 by 6 of sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square. So, this is what you get from there and so, sigma naught if you keep this side. So, this 3 will go here and 1 by it will come at 3 by 6. So, 1 by 2 and if you take that; so which will be 1 by root if you are taking the square root of it and in this bracket, you will have again sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square and whole raised to the power 1 by 2.

So, this is how you can get this condition if because of the conditions of stresses; if this sigma 1 because of the presence of sigma 1 sigma 2 sigma and sigma 3. If you find this value; now this sigma naught will be being. So, this condition you have got and this condition is known as the Von Mises theorem. So, you can further you can express it in the form of sigma x, sigma y and sigma z.

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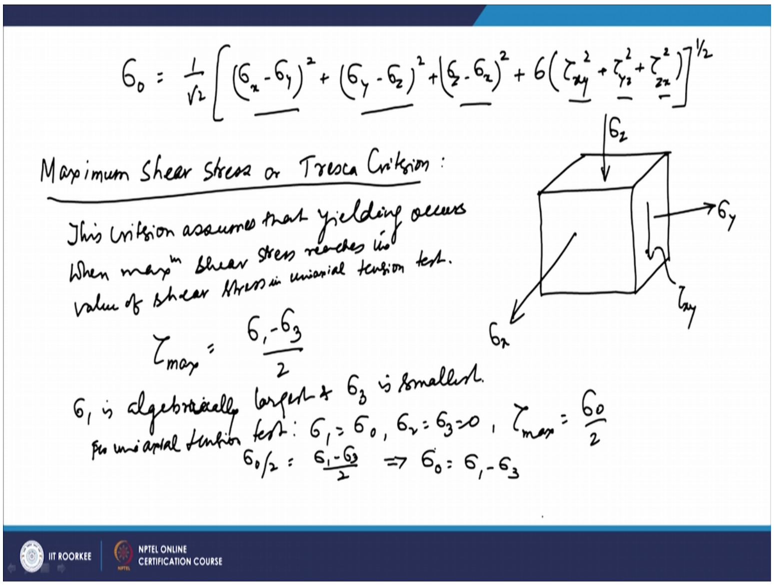

So, if you put that you can write. So, this we can be write written as sigma x minus sigma y square plus sigma y minus sigma z square plus sigma z minus sigma x square. And then you have 6 times tau x y square plus tau y z square plus tau z x square and then whole raised to the power half.

So, you can further write the expression in terms of a sigma x sigma y and the sigma z. Now the condition tells that the yielding will occur when the difference of the stresses on the right hand side that will be exceeding the yield stress in the uniaxial tension test; this is a limiting condition. If this value if the right hand side value is increasing its value is exceeding this yield stress value in uniaxial tension test in those cases the yielding will occur. So, this is known as the Von Mises criteria.

Now, criteria now when this criteria has you know many you know values or many interpretations what you see is that it involves the use of the all these 3 principal stresses or or the all the 3 components of a stresses in the 3 directions. So, so that way you can you can we will say that in most of the cases wherever you have the use of you have been given this with the problem; you can use this formula and come to its you know the prediction of the yield stress.

Now, I mean prediction of the failure conditions now you can have many of the problems in such cases. For example, if you are having a problem of a suppose you know a cube is there and in the cube you may be given certain set of stresses to which this cube is acting, you may be given sum and normal stress, you may be given some conditions like this. So, if suppose this is your sigma x is given then similarly you have sigma z is given and suppose you have a sigma y also given and some shear stress part also tau xy is given.

Now, in this cases what you can do is you can do the analysis of these conditions; you may have the shear stress components on this side or that side also. So, so in those cases you can use this formula and what will be happening in this case is the sigma naught value will be given to you and once you find this side. So, suppose you are given the values of sigma x sigma y and sigma z as well as tau xy you may have tau yz tau xz.

So, what you will do is you will find this you know sigma this right hand side value from this side. And if this is exceeding the value of the yield stress you know for that you know component; it means in that case the yielding will occur otherwise the yielding will not occur. So, so that way you try to calculate to try to analyze those situations wherever you have such conditions given.

Now, what we discussed that this criteria they are they are they involved all the you know the J 3 values of the principal shearing stresses and that way this has a lot of implication. So, as we see that we have seen in the expression that this is this is a term sigma 1 minus sigma 2 or sigma 2 minus sigma 3 or sigma minus 3 minus sigma r 1. So, they are basically the values of the values of the principal shearing stress.

So, what we can make out from this analysis is that it involves all the 3 principal shearing stresses. And this all this principal shearing stresses are taken into account for the analysis for the definition of such yielding criteria. Also you see that there is no hydrostatic component involved; so, we have already seen that you know that we have defined that hydrostatic stress is does not I mean give the yielding or they are responsible for this the formation of changing shapes.

So, that also is clear from these pictures also the another thing is that all the terms are squared one sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square. Even here you have all these terms sigma x minus sigma y square, then you have sigma y minus sigma z square all these terms are square.

So, basically they are plus or minus sign does not matter because they are all basically squared terms. So, this is also that also tells us that and especially to that earlier expression where we see the you know the values of the stresses which does not a matter which is the largest or which is the smallest type of principal stresses because that may be required in another case of another type of criteria for the failure analysis. So, this is the you know condition for the you know yielding of the trial materials based on the stress deviator second invariant of the stress deviator that is Von Mises theorem; this is also known as the distortion energy theorem.

This is known as distortion energy theorem because the distortion energy is defined as 1 by 6 j and then sigma 1 minus. So, sigma 1 minus; so, square 1 plus square plus sigma 2 square plus sigma 3 square minus sigma 1 sigma 2 minus sigma 2 sigma 3 and minus sigma 3 sigma 1. So, using that distortion energy theorem also you can come to the same expression that is sigma naught will be equal to 1 by root 2 and then in the bracket you will have sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 squared plus sigma 3 minus sigma 1 square and whole raised to the power half.

So, this way basically you can have; so, similar expression and also known as the distortion energy theorem. The next type of the criteria which is defined as the Von Mises cri I mean the stress criteria. So, we have seen that in the Von Mises criteria we are taking all the 3 principal stresses into account; shearing stresses into account and this is why the second invariant invariant of the stress deviator is brought into consideration and that is being equated to one critical value that will define the yielding condition.

Now, coming to the second you know criteria that is known as the maximum shear stress criteria or this is also known as Tresca criteria. So, this again we know that yield you know occurs because of the, you know shear stresses. So, this criteria is based on the assumption that yielding will occur when the maximum shear stress which is you know generated that reaches the value of the shear stress in uniaxial test.

So, basically we have the torsion test which we do there from you get the yielding in this torsion because of the shear. And then the maximum shear stress which is you know calculated based on the sigma 1 and sigma 2 and sigma 3 we have seen that the maximum value of the shear stress is sigma 1 minus sigma 2 by 2 or sigma 2 minus sigma 3 by 3 2 or sigma 1 minus sigma 3 by 2.

So, depending upon the value of sigma 1 or sigma 2 or sigma 3 which one is the largest and which one is the smallest you can have the maximum value of the principle shearing stress that will be sigma 1 minus sigma 3 by 2 if sigma 1 is the largest and sigma 3 is the smallest. So, so the criterion assumes; so this criterion assumes that yielding occurs when maximum shear stress reaches the value of shear stress in uniaxial tension test. So, this is nothing, but the yield shear stress. So, we are talking about the yield shear stress when it is we are undergoing the, we are putting the specimen under the torsion test. So, in that uniaxial it is not uniaxial is tension test.

So, in that basically the shear yield stress which is achieved which is calculated which is found that should be equal to the maximum shear stress which is found out. Now if you look at the condition it tells that tau max; so, that is must be equal to sigma 1 minus sigma 3 by 2. So, if you assume the sigma 1 to be the largest one and sigma 3 to be the smallest one in that case. So, here sigma 1 is algebraically largest and sigma 3 is smallest. So, in that case sigma 1 minus sigma 3 by 2 will be the maximum of all the shearing stresses. So, sigma this must be reaching to that and that should be the tau max.

Now, if you take for example, for uniaxial tension test; now in the uniaxial torsion test we have already seen that you have sigma 1 as sigma naught and sigma 2 and sigma 3 as 0. So, in this case the tau max will be equal to; so if you see that tau max is sigma 1 minus sigma 3 by 2. So, that will be equal to sigma naught by 2; so, you can write sigma naught by 2 equal to sigma 1 minus sigma 3. So, that yield; so sigma naught by 2 will be sigma 1 minus sigma 3 by 2. So, this gives sigma naught will be sigma 1 minus sigma 3.

So, this condition you know is this is known as the this is given by the maximum shear stress or the Tresca criteria; now if you talk about the condition of pure shear.

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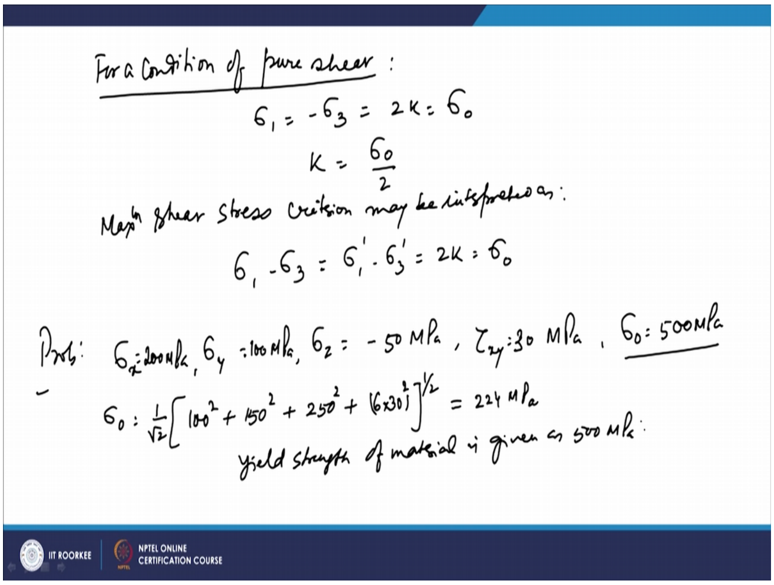

So, far a condition of pure shear trial in that case basically sigma 1 is equal to minus sigma 3. So, it will be 2 K that is sigma naught; so, what you see is K will be sigma naught by 2. So, what we see that we can interpret the maximum shear stress criteria. So, maximum shear stress criteria can be interpreted or can be written as; so, we can write as sigma 1 minus sigma 3 or else you can have sigma 1 prime minus sigma 3 prime that will be 2 K.

So, 2 K that is sigma naught; so, what we see that if you compare these 2 criterion that is maximum shear stress criterion and the Von Mises criterion; you see that this is quite simple type of a relationship where which involves only these 2 stress value sigma 1 and the sigma 3 does not involve the sigma 2.

So, this is basically the major difficulty that you know here the problem is that you have to know that what is the maximum and what is the minimum value of the principle stresses? Then only you can find this you know expression sigma 1 and sigma 3; if you do not know then you cannot use this theorem. So, this criterion that way lacks or it has that you know demerit that you must know which of the fun I mean stress is intermediate in nature I mean it is not either the largest or the smallest.

So, you know that way it has that and also that it does not involved 3 stresses you know. So, that is why although it is you know moreover self less complicated, but most commonly if we go and use the Von Mises criterion because they are always have seen that it takes into account, it has more meaning also it takes into account the square term signs, signs do not that way you know play that much in that case. So, this is about the 2 type of the criterion.

Now, if you if you are given certain problems as we discussed we were discussing about some problem suppose in the earlier case. So, suppose in this case if suppose for this problem if you are given some suppose example to solve such problems. And it is given that suppose for a problem you are given that sigma x and sigma y and sigma z is given. So, suppose sigma x is given as 200 Mega Pascal and sigma now y is given as suppose sigma y is again given as 100 Mega Pascal and sigma z is negative that is minus 50 Mega Pascal. And suppose tau xy is given as you know you can say tau xy taken as 30 Mega Pascal.

Now, if you are told that this for a particular structural member this is the stress to which it is subjected to and it has the sigma naught value of 500 Mega Pascal. Now suppose you are told that whether you tell us whether this material will yield or not. So, if you look at that you will see that from the, you can use the since this 3 component of stresses are involved you have to use the you know condition of Von Mises criteria.

So, if you use the Von Mises criteria using these 3 conditions what you get is sigma naught and then further you can use 1 by root 2 and then sigma x minus sigma y square. So, this will be 200 minus 100 square; so, this will be 100 square plus sigma y minus sigma z square. So, 100 minus 50; so, 150 square and then sigma x minus sigma z square; so 200 plus 50 to 250 square and then you have 6 times tau xy square plus tau by z square plus tau zx square.

So, 6 times 30 square; now this is you know squared square root is calculated. If you find this value this comes out to be 224 Mega Pascal. Now this yield strength of material is given now yield strength of material is given as 500 makes Mega Pascal. Now, what you see is that under the combination of these stresses using the Von Mises criteria you got 244 minimum 4 Mega Pascal and it is your this sigma naught value is given.

Whereas, the materials yield strength is 500 Mega Pascal; it means the material will not failed under these conditions. So, so when we may have a problem where only sigma 1 and sigma 2 or sigma or 1 of the 2 stresses are given or when the 3 stresses are given. So, you can have even the and the calculation by the shearing maximum shearing stress criteria or Tresca criteria and you can come to a conclusion whether the material will fail or not. So, this is about the yield criteria of ductile materials.

Thank you very much.