Principles of Metal Forming Technology
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Lecture - 12
True stress and true strain

Welcome to the lecture on True Stress and True Strain. So, we already know that when we talk about the forming technology, in that case the engineering stress and engineering strain are not the very relevant, you know, terms we have actually to refer to the you know, stress and strain term in terms of true value that is true stress and true strain. So, we will try to see more about this true stress and true strain how they are related with the engineering stress and strain values in this lecture.

So, when we talk about you know, in case of, you know, linear strain when we what is the strain basically? So, normally when we talk about the strain, so, you define the strain as change in length upon original length.

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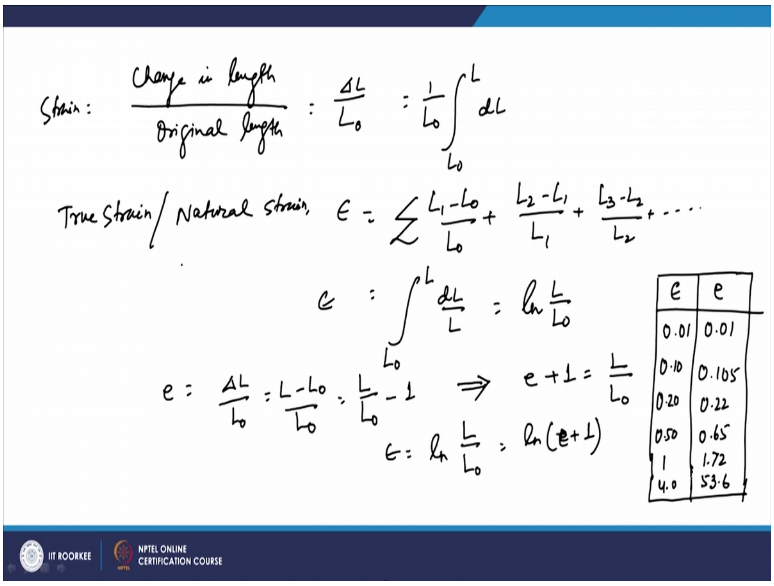

So, this is what the strain is defined as in conventional waste. Now the thing is that when this dell L is very very small so, we call it as delta L upon L naught. So, if L naught is the original length, in that case this delta L upon the L naught that will be giving you the, you know, actually the strain, and if you look at it so, you what do you see that in the case of conventional strain it is nothing but L naught to L and it will be dL.

So, normally we find the conventional strain the conventional concept of train is defined in this fashion. Now this is valid only and this dell L is very small. Now in the case of plastic deformation the strings are very very large, and also we have already discussed that when there is plastic deformation going on, then the gauge length is change considerably. So, what is happening? So, the (Refer Time: 02:45) had proposed the concept of the true strain or natural strain.

So, you have the value of true strain or natural strain. So, true strain is also known as the natural strain, and it is represented by the epsilon, and this removes this difficulty in expressing in the case of the large plastic strains. Now it is basically it is with respect to the instantaneous gauge length. Now in this case we are taking the gauge length as the fixed one.

Now, in the in such cases basically, in this definition of the true strain basically the change in length is the referred to as the instantaneous gauge length rather to the original gauge length. So, your this is defined as so, summation of L 1 minus L naught by L naught plus then L 2 minus L 1 upon L 1 plus L 3 minus L 2 upon L 2 plus.

So, it will go like this, and it can be further expressed like it will be L naught to L and this will be dL by L. So, in this case this gauge length is now in this case that is fixed 1 by L naught, but that is not fixed; so from every increment if there is increment in steps.

In those cases, you have L naught here L 1. So, earlier you had L 1 before that it was L 2 or so. So, so that way so, that is from L naught to L 1 then L 1 L 2 or L 2 to L 3, in those cases every time you have this way so, that is the definition.

So, you get the true strain value calculated in this fashion, and if you calculate that if you see that it will be ln of L by L naught. So, basically this ln by L naught, it is the true strain value. And you have engineering strain as delta L by L naught. Now if you see how can you define the relationship how can you maintain the relationship between the engineering strain and the true strain.

So, what we found that engineering strain is delta L by L naught. So, delta L is nothing but L minus L naught upon L naught which will be L by L naught minus 1. Now if you look at so, what will happen? You can see that from here you get e plus 1 it will be L by L naught. So, what you see is, you see that epsilon is coming as Ellen of L by L naught.

So, L by L naught e plus 1 so, it will be ln of e plus 1. So, you see that once, you know, the is conventional engineering strain, then add it to 1 and then you can get the true strain value and that can be compared. So, what you see is that when you have suppose engineering strain of very low value, like you have 0.01 at that point of time if you take this true strain, it will becoming as point 0, when it is low value of the engineering strain you will have the true strain as the similar one.

Similarly, so, if you have suppose if you temporarily you can calculate and you can compare the values of the true strain and the engineering strain. So, true strain is this is an engineering conventional strain is like this. So, for smaller values like if you have true strain value as 0.01. So, if you take ln of 1 plus is equal to 0, once it will be coming as same as 0.01. So, similarly if you take 0.10, it will be 0.105. Similarly, if you increase further 0.20 when it will be coming as 0.22, and you have 0.50, in that case it will be 0.65 1 1.72 and 2 and 4 as 53.6, what you see is that for this values very large increase in this value.

So, this way once you have the known value of e you can find epsilon once, you know, the epsilon, in that case you can find the value of e. Now the thing is that why we use this true strain, now the use of true strain basically has many, you know, in many way you can interpret that it has more realistic, you know, interpretation.

So, suppose if you if you talk about the strain suppose you have a cylinder, and which is extended to twice it is length. Now what will happen so if you if you if the cylinder is if a specimen if any cylindrical as specimen is, you know, extended to twice it is length, 2 times it is length.

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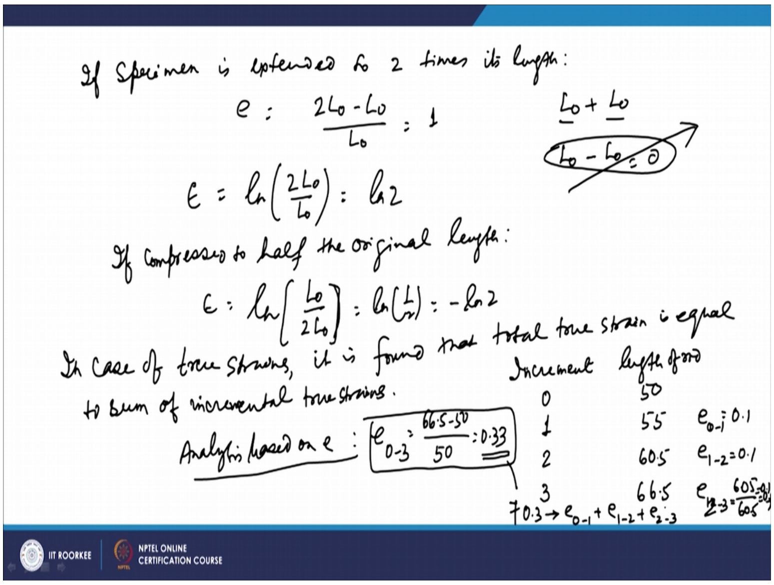

In that case if you find you know, engineering strain. So, it will be engineering strain L naught is the actual length. So, the final length will be 2 L naught minus L naught by L naught so, it will be one. So, engineering strain of 1 means it is increased to double it is length, now if you that is strain of 100 percent. Now suppose if you are thinking of the same strain in the compression side. So, in that case what will be happening? So, L naught has gone so, this is nothing but L naught plus L naught.

So, that is 100 percent of train. Now similarly if you try to interpret this in terms of compression, it means virtually it should be coming from L naught to L naught, if it is the reduction in it is length to talk about so, it is coming to 0. And this basically, you know, you cannot interpret it anyway, because anyway you are strain in the material, in the compression side there must be the value of strain, but then if you look at the engineering that way so, you cannot go from for this 100 percent compression like that.

So, there will be no length, but that is that is, you know, that you cannot say. So, intuitively by this seems to be correct. So, but if you try to use the concept of, you know, the true strain what is true strain. So, if you look at the true strain if you see in this case L from L naught it has gone to 2 L naught; so it will be ln of 2 L naught by L naught ln L by L naught we have seen that. So, L become 2 L naught so, that is how it became ln 2. Now what does it mean? Now if you are compressing to half the length; similarly, so, if you are compressing so, if compressed to half the original length, now if you look at this, if you try to find the, you know, true strain value. So, ln of final length will be L naught by 2 and then that is divided by L naught.

So, it will be ln of 1 by 2 and that is why it will be minus of ln 2. Now what you see is that this is the same value, but it is positive and it is come negative. So, basically that the amount of strain which you see normally is the same and it makes sense that if you are increasing to twice it is length of decreasing to half it is length. Rather than if you take the convention of engineering strain value, you see that it should go to the 0 length which is not possible.

So, that is why this true strain concept is basically used in the case of plastic forming. The next advantage what you see in the case of the use of this is a true strain is that that total true strain basically is the sum of the incremental true strains.

So, in case of true strain analysis, it is found that total true strain is equal to sum of incremental true strains. Now this can be found because the normally what we do in plastic forming. You never strain in one space from one height or one length to the final length. So, you are doing in the case of, you know, increments. Now the thing is that in the case of true strain it is basically found, that the total true strain will be equal to the sum of the incremental true strains. So, if you see that if you have a rod of suppose certain length suppose you have a rod of 50 mm length, now, in the first increment you are increasing to 55 mm length.

So, you will have the, you know, engineering strain as 0.1, similarly that time again you have to go for the, you know, length of 2 60.5, then it will have again engineering strain of 0.1. And you will have further to from 60.5 you go to 66.5 and what way you get the another engineering strain value of 0.1. So, that is what you see when you see the concept of engineering strain.

Now, if you go to the true strain concept, then if you look at the true strain concept, you will see that true strain for 1 0 2 1 to 1 to 2 and 2 to 3. You can find as you can find it is value, and what you see that you will see that the true strain for the total will be some of the engineering strain in the incremental part.

So, that can be checked suppose you have one increment and then you have the length of rod. So, suppose you have increment 0 at that time you have the actual length is 50. Now you are in the first in increment it goes to 55. So, basically the value of e from 0 to 1 will be 55 minus 50 by 50 so, 5 by 50 that is 0 0.1.

Similarly, if you go to the second increment; so for getting e 0 e 1 to 2 for 0.1 it will come as 60.5, because from 55 it will go to 60.5. So, 5.5 is the increment change in length divided by 55 second it will come to 0.1 and in the third, you know, situation it has to go to 66.5. And then in that case your engineering strain from 2 to 3 will be further 6.05 by 60.5 so, again it is 0.1; so this again coming out to be 0.1. So, in this cases, you get this sum as 0.1 plus 0.1 plus 0.1; so if you do the analysis based on the engineering strain. So, analysis based on conventional strain, what you see is, if you see the finals train, it will be 66.5 minus 50 divided by the 50.

So, basically e 0 to 3 up to 3 total, it will be 66.5 minus 50 and divided by 50. So, it will be 16.5 by 50 so, it is coming out as 0.33. And what you see is that if you add this 3, it is coming as only 0.3, whereas, the total is coming as 0.33. Now this 0.33 is not equal to 0.3.

0.3 is nothing but the sum of e 0 1 plus e 0 e 1 2 plus e 2 3. Now, if you try to see, if you try to analyze the situation based on the true strain concept.

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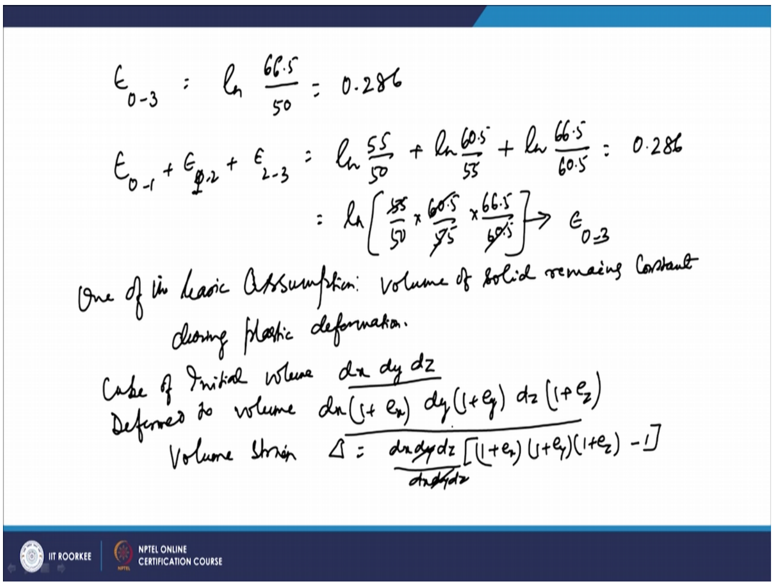

Now, in the case of true strain concept if you look so, epsilon 0 to 3 will be ln of 66.5 divided by 50.

So, this way you get, this is 0.286, now if you look at if you see go for epsilon 0 1 plus epsilon 0 2 plus 1 2 plus epsilon 2 3. So, this is the, you know, increment in steps now this will be; so, basically you are going from initially from 55. So, from 50, then you are going from 55 to 60.5, then you are going to 66.5 from 60.5.

So, this way you can get it, and this you will also get this will be same as ln of 66.5 as, so, this will be same because once you do the logarithm, it will be nothing but the ln of 55 upon 50 into 60.5 by 55 into 66.5 upon 60.5. So, this will be cutting and ultimately you are getting, same as epsilon 0 3. So, what you see is that in the case of true strain analysis, if you look at incremental strains the total strain will be the sum of the incremental strains which you get in those cases.

Now, we will move to another concept in the case of plastic deformation, when we deal with the plastic deformation. One of the basic characteristic of plastic deformation is that the metal is essentially, you know, in compressible. So, basically density will be whatever density there is changing after large plastic strains they are very very small they are less than point one percent.

So, normally what we consider that the volume of the solid remains constant during plastic deformation. So, one of the basic assumption is that volume is volume of solid remains constant during plastic deformation. So, this way we if we talk about the cube of the initial volume suppose you take a cube of initial volume dx, dy and dz. Now suppose you this is the volume of the cube and it is deformed and it is volume after deformation.

So, deformed to volume dx 1 plus e x then dy 1 plus ei and dz 1 plus e z if e x e y and e z, they are the strain. So, in that case this will be it is volume. So, if you try to find the volumetric strain which we have done in our earlier lectures. So, if you find the volumetric strain volume strain and that is represented by delta. Now this will be nothing but the final volume minus so, this is the final volume this is the original volume divided by original volume.

So, it will be dx, dy, dz and then that will be 1 plus e x, 1 plus e y, 1 plus e z minus 1 divided by dx dy dz, so, this will be cutting this side. So, it will be 1 plus ex 1 plus ei 1 plus ez minus 1. And that way so, what we see is that normally we have done this analysis for the elastic, you know, materials in the case of elastic strains your this the elastic strain where it is less.

So, in that case we neglected the product of these strains because they are smaller. So, we neglected the product of ex ey or ey ez or ex ey ez. But we cannot neglect this when we are doing the analysis for the plastic strain, because in those cases this strain values are quite high in the last in case of elastic analysis you can neglect them because this values are quite small.

So, and also the volume change is 0 for the plastic deformation. So, what we see is that you can go further you can take this 1 to this side and this way the expiration becomes like this, so, delta plus 1.

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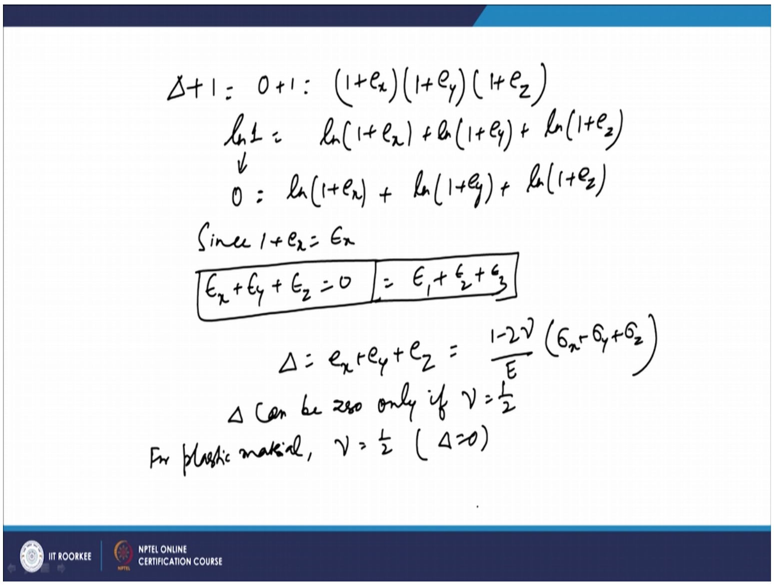

It will be coming as so, since this volume change will be 0. So, it will be 0 plus 1, and it will be 1 plus ex 1 plus ey and 1 plus ez so, this is what we get.

So, this is one in this side and once you take ln to both the sides so, it will be ln 1 will be ln 1 plus ex plus ln 1 plus e y plus ln 1 plus ez. So, this is what is there in the case of plastic deformation, now ln one is basically 0, you get 0 as ln of 1 plus ex, then plus ln of 1 plus ey and plus ln of 1 plus ez. So, we have seen that 1 plus ex is nothing but epsilon x the true strain value of the x; so since 1 plus ex is coming out as epsilon x. So, what we get from here that epsilon x plus epsilon y plus epsilon z is equal to 0. So, this is what a relation we get in the case of plastic deformation and this can also further we written as epsilon 1 plus epsilon 2 plus epsilon 3.

So, this way this is what you get in the case of the plastic deformation, and this is nothing but the first invariant of the strain tensor we have already seen that, and we have already proved that. So, this is the sum of all this value epsilon xy and z they are to be, you know, their sum is 0.

So, what we see that this equation is not valid for the elastic reason, because, you know, there is in the case of ballistic strain there is appraisable volume change. Whereas, in the case the, you know, plastic analysis, the volume change is normally neglected it is 0. So, what we see that if you use if you try to use this, you know, if you try to add the hookes law, 3 hooks law we get we have already seen that we get this ex plus ey ez.

So, we have seen that it is coming as 2 mu by e and sigma x plus sigma y plus sigma z. So, what we see that in the case of, you know, plastic deformation the delta has to be 0. So, if the delta has to be 0 in the case of plastic deformation the volume we know that there is no change in volume. So, if this has to be 0, in those cases this has to be 0, and that is why the nu will come as 1 by 2. So, delta can be 0 for so only if mu is 1 by 2. So, then only the nu, for the nu value of 1 by 2 the this delta will be 0 for the plastic deformation case. So, what we see that the poissons ratio is equal to 1 by 2 for a plastic material, for plastic material nu is equal to 1 by 2.

So, because the delta is 0 for plastic material for which the volume of the strain that is 0. So, for that the nu becomes equal to the 1 by 2. Now another thing which we can we can get from these values is the value of the true strain in another way.

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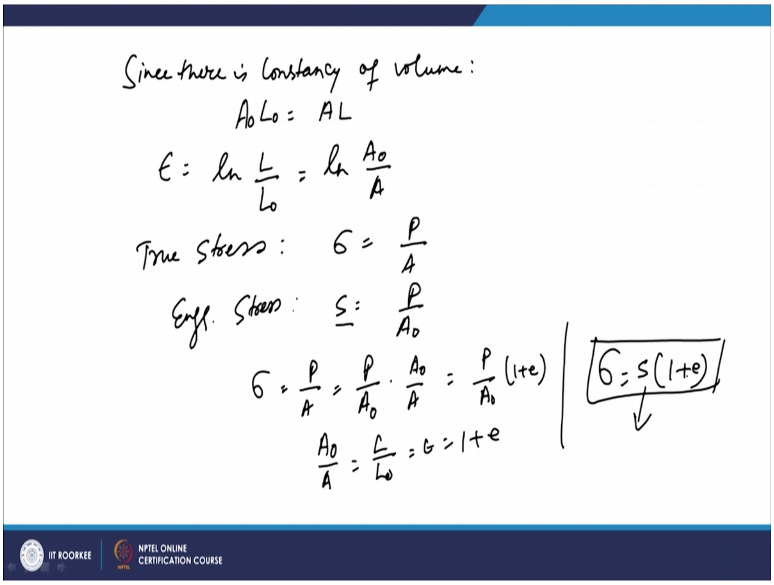

And in another way you can represent so, since there is constancy of volume. So, as we see that there is constancy of volume in the case of the plastic deformation

So, length multiplied by cross section area will be same. So, we see is you can write A naught L naught will be nothing but A into L. So, so, what we have seen that epsilon is ln of L by L naught. And so, you can write it as L by L naught will be A naught by A so, you can write A naught by A. So, ln of A naught by A is also known has the and true strain.

So, either A naught by A or L by L naught you c find these, you know, true strain. Now true stress when we talk about true stress is basically in that case we are dividing the load with the instantaneous area, you know, in that case in the conventional analysis, the stress is by the original area, whereas, in the case of true stress, it is the load divided by the instantaneous area so, true stress when we try to define.

So, sigma is point by A, and engineering stress if you see it is defined as SSP by A naught A naught is the original area. So, what we see again? So, sigma since it is P by A so, we can write p by A naught into A naught by a. So, again A naught by a is if you look at A naught by a is nothing but L by L naught. So, which is nothing but epsilon so, 1 plus e. So, what we see is, you can write p by A naught and then so, and A naught by ea is 1 plus e.

So, what we see is A naught into 1 plus e. So, what we see is, that you get sigma as p by A naught is S so, this is S into 1 plus e. So, since you have the value of engineering stress known, then and if you know the engineering strain in those cases the true stress is I mean related with these 2 values.

So this, we must solved some problems based on this true stress true strain engineering and try to familiarize with these terms. And that will be helpful in our future calculations.

Thank you very much.