**Principles of Metal Forming Technology**
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**Lecture -10**
**Elastic stress strain relationships**

Welcome to the lecture on Elastic stress strain relationships. So, we talked about the stresses and strain and basically we have to relate the stress with the strain and normally you have the constitutive equations basically which basically relate with this strain with the stress.

So, as we know that we have the measurement of strains and from there you find the value of stresses. So, you can find the stresses and the relation between these stress tensor and the strain tensor and they are basically found from the constitutive equations.

Now, we are going to discuss about the elastic solids only in this lecture. So, normally what we see, we have already we know that you have in the elastic reason you have the application of Hooke's law and ah if you talk about the Hooke's law. So, what we see in the Hooke's law basically stress is proportional to the strain.

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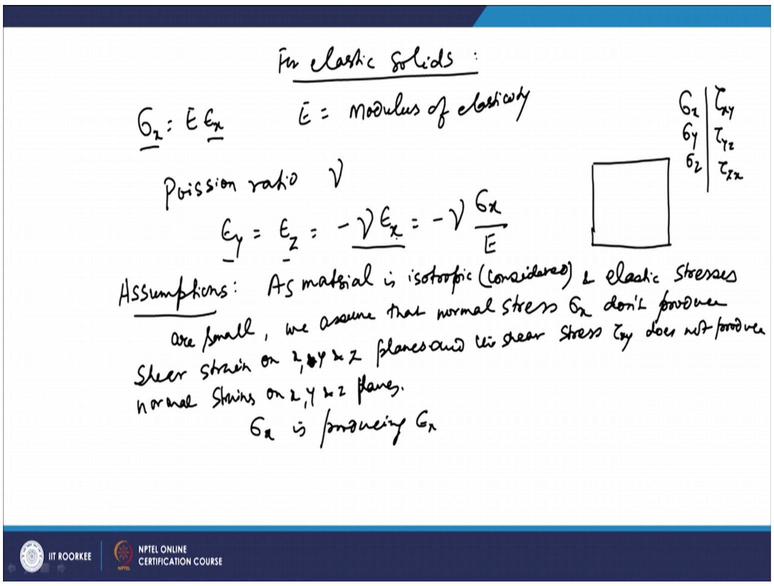


So, for elastic solids, you have the you know we know that the Hooke's law is valid in the elastic zone. So, we say that the sigma x is basically a constant times the strain. So, that is stress is proportional to this is a stress and this is a strain stress is proportional to strain and then we give a proportionality constant and this E as we know this is known as the Modulus of elasticity.

So,. So, this modulus of elasticity, it is either in tension or in compression and as we know that when we provide the you know tensile force, we when we apply a tensile force in the x direction. So, there will be elongation in the x direction. But it will be producing a compaction basically; a contraction in the transverse directions that is in y and z direction.

So, basically this transverse strain which you get you have one is the longitudinal strain. This is a strain in the direction of the stress and the other the strains which are there in the other directions; now they are known as the transverse strain. And it was found that this transverse strain, it is a constant fraction of the longitudinal strain. So, it was seen in most of the solids that this transverse strain which we get it is a constant times the longitudinal strain and basically that way this Poissons ratio that is defined.

So, you have the Poisson ratio. So, that is defined that is nu and this nu basically it is the ratio of ah these the two types of these strains and they are normally found.

So, that is ratio of the transverse strain to the you know this longitudinal strain and it is found to be basically in the range of something like close to 0.33. It is for most of the solids. So, actually what we see is that this strain in either y or in the z directions, if you talk about the x direction as the longitudinal one. So, you will have the transverse directions as y and z.

Now, this strains in the y and z direction, they can be said to be minus nu times epsilon x. So, if there is a you know elongation in the x, you will have the you know contraction in the y and z. So, that is why you have negative sign and then it is multiplied by this epsilon.

So, epsilon will be nothing but the ratio of epsilon y by epsilon x and that of minus sign. Because it is contraction and this is expansion. So, so what you see is that you get minus nu times and epsilon x is sigma x by E. So, this way you have this is the co relationship which is used for finding the epsilon y or epsilon z when you get the epsilon x.

Now, when we are talking about the, you know 3 dimensional state of stress. So, in those cases, so this is for the when we talk about. Now when we talk about the 3 dimensional state of stress and the strain, then in that case suppose you have a cube. Suppose you have a cube. So, it is it is subjected to the stresses sigma x sigma y sigma z and tau x y tau y z and tau z x. So, suppose it is subjected to that in those cases. Now you will have basically the elastic stresses.

So, they are normally smaller and the material being isotropic in nature. So, you have to assume. So, you have to assume that the, this normal stresses that is sigma x, they do not produce any shear strain. So, that is on x y or z plane or the shear stresses that is tau x y, they are not producing any normal strain on the you know in the x y and z planes.

So, this assumption is being made and then, we apply the principle of superposition. So, you have certain assumptions you can think of and we what we assume that we assume that as the material is isotropic. So, we consider the material to be ah isotropic and the elastic stresses are small.

So, what we assume? We assume that the normal stresses basically a sigma x suppose they are not producing any shear strain. So, normal stresses normal stress sigma x donot produce you know the shear strain on x, y and z planes; x, y and z planes.

Similarly, the shear stresses which are; so, you have normal stress has sigma x, sigma y and sigma z. Similarly you have shear stresses at tau x y, tau y z and tau z x. So, and and the shear stresses and the shear stress that is tau x y; now that do not produce does not produce normal strain so, that on x y or z plane.

So, this is the assumption which is made and then, we try to find the these expressions by the principle of superposition and then we try to find the expression for epsilon x, epsilon y and epsilon z for the you know you have sigma x, sigma y and sigma z; so, based on that you can find it.

So, what you can do is that suppose you have the stress sigma x; so, we have the stress sigma x. Now this sigma x is producing epsilon x; but it is also producing epsilon y as well as epsilon z that is why we have seen that epsilon y and epsilon z that will be produced that will be minus of nu sigma x. So, this is further used. So, that can be used by referring to you can try to find the in the tabular form how can we find its a expresses suppose you have the stress, you have individual stresses and now, if you have the stress and if you have the strain in any direction.

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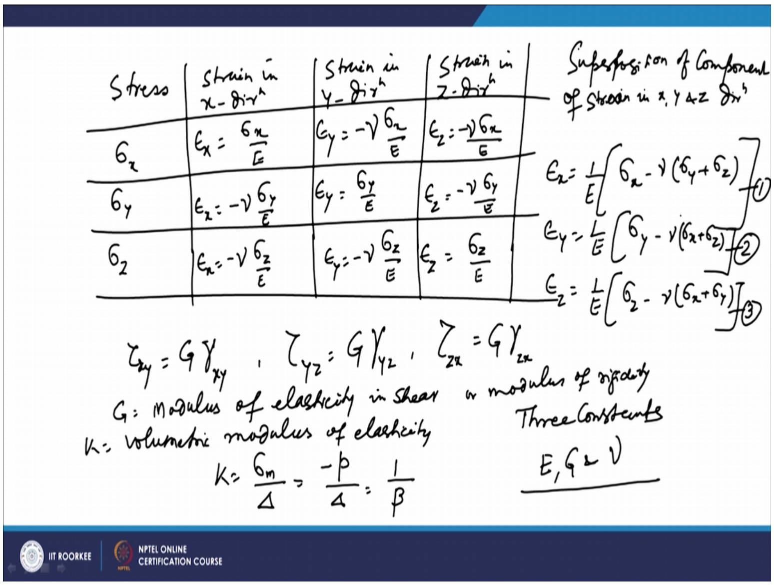

So, if you have a strain in suppose x direction. Similarly, you have a strain in y direction and you have a strain in z direction. Now, if you look at this. So, now, you have you know that you have the normal stresses are sigma x, sigma y and sigma z.

Now if you look at this, we know that sigma x will produce the stress strain in x direction as sigma x; sigma y will produce in y direction as sigma y and sigma z will produce z direction as sigma z and this is ah this you know that if this sigma z is the stress, then sigma z by E will be the strain in the z direction. Similarly, this will be sigma y by E and similarly this will be sigma x by E. So, these are the longitudinal strains the strains in the direction of the stresses.

Now, in the y and z direction; now in the y direction, we will use that property of the material that is Poisson's ratio. Now in the poisons, if the Poisson ratio is used the sigma y will be basically will be minus of nu times and then; that will be sigma x by E. So, that we have already defined this property of the material. Similarly sigma z will be minus of nu times sigma x by E.

So, similarly, here sigma x will be minus of nu times sigma y by E and this will be epsilon z will be minus of nu times sigma y by E; similarly you have sigma x as minus of nu times sigma z by E and epsilon y also will be minus of nu times sigma z by E. That is what we see that if you have these stresses, they are telling you they are giving you these strain components.

Now you can have the superposition of the component of so uses. So, superposition of these you know component of strain in x, y and z direction. This is strain in x, y and z direction; if you see that these three expressions. So, you have sigma x you can find.

So, you can find sigma x as 1 by E and then, it will be sigma x minus nu times sigma y plus sigma z. If you look at this sigma x fine so, sigma x by E minus nu times sigma y by E minus nu times sigma z by E. So, that is what 1 by E will be common sigma x minus nu times. So, the similarly you will have sigma y has 1 by E and you will have sigma y minus nu times sigma x plus sigma z and similarly sigma epsilon z will be 1 by E sigma z minus nu times sigma x plus sigma y.

So, this way we get the expression for epsilon x, epsilon y and epsilon z. Now these are shearing strains will also be generated and for the shearing strains, as we know that shear stresses produced the shear strain. So, that will be for the for the further unit cube, you can write for that tau x y and tau x y will b G times epsilon this gamma x y. So, gamma x y is the shear strain and similarly you will have tau y z. So, that will be G times gamma of y z and similarly tau z x. So, that will be G times gamma z x. So, now here these proportionality constant which we use in the case of shear strain that is G.

So, this G is known as Modulus of elasticity in Shear. So, this you have modulus of elasticity in tensor or compression that is E and you have modulus of elasticity in shear that is defined as the G and we also call it as modulus of rigidity. So, this value of G will be basically derived from the Torsion test we find it. Now the thing is that we got these expressions and for elastic and isotropic solid, you have 3 constants. What we see you have three constants, E G and nu. Now again another elastic constant is there and that is your volumetric modulus of elasticity.

So, if you go to that next type of ah elastic constant that is you know where because of the volume change, we have already seen that the, they are going to change the volume, the stresses. Now this volumetric you know modulus of elasticity is K. So, we defined the volumetric modulus of elasticity as K.

Now, this volumetric modulus of elasticity, basically here we know that these volume change is because of the hydro static component or mean component. So, you have a stress as the hydro static stress and then the volumetric strain will be the strain parts. So, that will be the ratio of K will be the ratio of the immune stress and ah. So, it is also known as the bulk modulus K. K is also known as bulk modulus and it will be a ratio of the hydro static stress that is sigma m or mean stress divided by the volumetric strain.

So, that is you know we have already discussed about it. And, so, sigma m divided by. So, this is K will be. So, as we have a already defined the volumetric strain. So, that is sigma m divided by this and then, that will be. So, sigma m will be minus p that is your hydro static pressure. And, so, this is basically defined by another parameter that is 1 by beta. So, beta is basically the compressibility. Beta is compressibility that will be opposite to the; so, inverse of the K, bulk modulus of elasticity or volumetric modules of elasticity.

So, as we know that it should it will be the inverse of that if we K is more, beta is less or if the beta is more, K will be less. So, basically p is the hydro static pressure and being compressive in nature being try to define it as minus p and then, you define this beta and that is why we define it as the volumetric modules of elasticity. Now from these you know constants you have E, G, nu and K; here from you can have the different types of relationships between these elastic constants.

So, you can have this relationship between E, G, nu and K. So, we have already got these expressions. Now the thing is that if you try to find from the these table what we saw this table which we got epsilon x, epsilon y and epsilon z.

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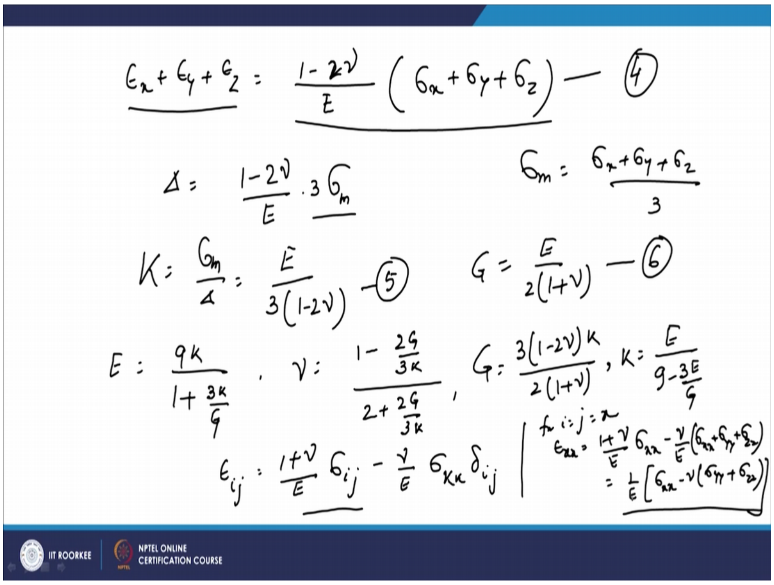


So, if you add them what we get is epsilon x plus epsilon y plus epsilon z, you can get as 1 minus 2 nu; 1 minus 2 times nu divided by E into sigma x plus sigma y plus sigma z.

So, this can we you know we can if we add these this is three equations, if you these if you add this 1 and this 2 and this 3, if you add these three equations what we get is sigma epsilon x plus epsilon y plus epsilon z will be 1 by E times and then, sigma x plus sigma y plus sigma z minus nu times, every this is coming two times. So, that is why it will be 2 nu. So, that is why 1 minus 2 nu by E and sigma x plus sigma y plus sigma z. So, that is what one of the you know you are getting one of the expression for this and we can call it as equation 4.

So, if you add these 3 equations as we discussed that if you add these 3 in the the, that equation here. So, you get this particular equation. So, this is one of the equation which we get. Now the thing is that we can have other you know expressions and what we see is that you have delta.

And that will be 1 minus 2 nu by E into 3 sigma m. So, in this case, what we see this in this expression itself this is 3 times delta m. So, as we know that immune stress is coming as sigma x plus sigma y plus sigma z by 3.

So, that is why this comes as 3 times sigma m and we get delta as 1 minus 2 nu by E into 3 times delta m. So, what we see is we have seen that K is nothing but you have sigma m by delta and that you will get as E by 3 into 1 minus 2 sigma. So, this is this is ah K and this sigma m by delta will be k. So, K can be put here.

So, in that case what we get is K will be E by 3 into 1 minus 2 nu. So, this way you can have the different relationships between these stresses. You can have this is constants. You can have other you know other important expression have been found and other important expression which we get is you also get G equal to E by 2 into 1 plus nu.

There are many other important equations like you have you can have E equal to 9K upon 1 plus 3K upon G. Similarly you have nu as 1 minus 2G by 3K and then divided by 2 plus 2G by 3K.

So, this way you have another expression can be found you can have another expression which we get is 3 into 1 minus 2 nu times K divided by 2 into 1 plus nu. Furthermore we may have a expression like K equal to E upon 9 minus 3E by G.

So, this way, you get the different type of correlations or relations between theses constants and from here, if you know suppose you know here in this expression you know K and G; so, in that case you can find E or so, or you know nu. So, you have to find nu and you go know K and G. So, you can find the value of nu by knowing these K and G.

So, so that is how you find the correlation between them. Now the thing is that after this ah, we need to have basically the way how to find the stress is stresses from these elastic strains. So, before that we also see that you have you can write in the other form also Tonsorial notice an form also and there are different ways also to write these expressions in the tonsorial forms.

And some of these forms which we get suppose you want to write in the tonsorial form, you can write as epsilon ij will be 1 plus nu by E and then, sigma ij minus nu by E and then, you can write sigma Kk and delta ij. So, this way you can write in the tensor form because you know that this will be the normal this is sigma K; this is sigma x, sigma y or sigma z will come here and this will be talking about all the i and sigma x y or y z or so. So that way you may have.

So, if you do you can have this expression in the tonsorial form. Now if i or j or x, they all are you know equal; in that case you will have the expression like sigma epsilon ij. So, epsilon xx will be actually 1 plus nu by E and then this will be sigma xx. So, this if you sigma i or j or K all that we taken as x in that case you will have one plus nu by E sigma xx and then again minus nu by E and then sigma Kk. So, you will have sigma xx plus sigma yy plus sigma zz and then z that can be taken.

So, this way you will have again, so, 1 by E will be coming up and you will have a expression like sigma x minus sigma xx minus nu times sigma yy plus sigma zz. So, this type of expression; so, you can write it here. So, suppose you have for i equal to j equal to x you can write sigma xx as 1 plus nu by E. So, you can write put here and then, this will be sigma xx minus nu by E of sigma xx plus sigma yy plus sigma zz.

So, that can further we written as 1 by E and then that that sigma xx will be ah. So, which is coming out and you will have sigma xx minus nu times sigma yy plus sigma zz. So, because this minus nu sigma x and plus nu sigma x, xx that is cut down. So, you will have this expression sigma epsilon xx, you can find if you in the terms of E sigma xx sigma yy and sigma zz and nu.

So, ah further for suppose i is x and j is y in those cases also you can have the expressions; where the z component will not be there. So, this way you have different type of expression that can be found. Now the thing is that once you have these strains found; then, from these strains you have to calculate the value of these stresses and for that you can basically add so we have already seen.

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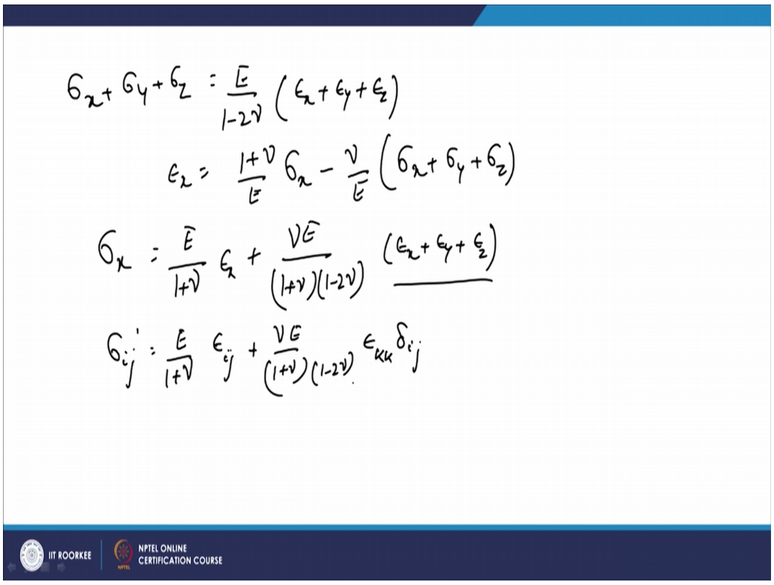


So, we have seen that what we have seen is sigma x plus sigma y plus sigma z. This we have seen from here. So, if you add this sigma x plus sigma y plus sigma z and that can we seen as E by 1 minus 2 nu and then epsilon x plus epsilon y plus epsilon z.

So,. So, this way you can find the value of epsilon x and ah now you have another equation on from there, you can have the expression for epsilon x and that will be 1 plus nu by E and then you have sigma x minus nu by E of sigma x plus sigma y plus sigma z.

So, further you can have you have we have discussed about these two equations and from these two equations, if you try to substitute the value from this into this. So, you can have the expression for sigma x and that you can get as E by one plus nu, then epsilon x and then plus nu E upon 1 plus nu into 1 minus 2 nu and then, you will have epsilon x plus epsilon y plus epsilon z.

So, that will be again you will have the immune strain. So, you can have this will be this can be further you know expressed in terms of immune strain. So, you can again further denote them in the tonsorial form and you can denote them as in the tonsorial form as E by 1 plus nu and epsilon ij and plus nu E by 1 plus nu into 1 minus 2 nu and then, you know this can be written as epsilon K k and then delta ij.

So, that is your immune strain. As we know this is the immune strain this can be further expression terms of immune strain. So, you can write them as that and basically this, this part this part is known as epsilon nu E upon 1 plus nu into this that is known as a Len's constant. So, we can further interpret it in other terms. So, this way you we have seen that the stress and the strain which are basically converted into the mean as well as stress mean and that deviator parts and from there you can find the different you know values depending upon the different values given.

So, this way I mean this is about you can further study about it more and more and you have two conditions which will be coming to us of for the considerations and they are the plane stress and the plane strain conditions when in one of the directions, the you know stress is 0; sigma 3 is 0 suppose then, it is a case of the plane strains and similarly you have the plane strain conditions also. So, that way you have plane stress condition is normally example of thin loaded sheet when the it is loaded in the you know thin sheet loaded in the plane of sheet.

So, that is example of the you know plane stress condition and also when we talk about the thin strain I mean plane strain condition. So, just like a long rod or cylinder you, so, where one dimension is quiet larger; then, the other two. So, that is example of the plane strain conditions you know cylinder which is has restrained ends.

So, that way you have the plane strain as well as plane strain conditions where either stress value or strain value in one of the direction is taken into 0. So, these are the cases which will we coming up for discussions or reference when we talk in our analysis in the of the forming parts. So, so that is about these you know this lecture.

Thank you very much.