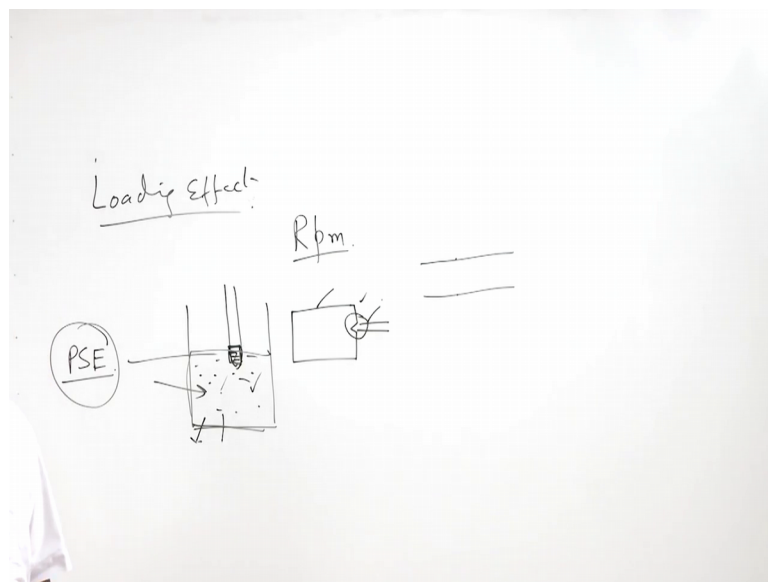


Mechanical Measurement Systems
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Lecture – 07
Loading Effect and Impedance Matching

I welcome you all in this course of Mechanical Measurement Systems, and today we will discuss loading effect and impedance matching. Earlier we have planned we have planned to discuss about the statistical methods, but before I thought that before switching over to the statistical methods we should discuss on loading effect and impedance matching, loading effect and impedance matching.

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So, in today's course we will be covering loading effect, impedance matching and we will be solving if you single numerical based on the loading effect.

Now, loading effect is witnessed in all measuring systems and it as I explained earlier also when we are measuring any quantity suppose we are measuring temperature of water and you are measuring temperature of water with the help of, with the help of a thermometer. So, the bulb of the thermometer is primary sensing element primary sensing element is an important part of any measuring instrument because primary sensing element is the part of the instrument which comes into contact with the measurement.

So, here when we are measuring temperature of the bulk of the fluid using a thermometer, the bulb of the thermometer is primary sensing element and when we are measuring the temperature of the bulk of the fluid and this primary sensing element is coming in contact with the fluid it is extracting energy from the fluid. So, the basic state of the measurement is disturbed right and it is possible that if the system is poorly designed poorly designed means you make a primary sensing element of a thermometer in such a manner that its thermal mass is very high, in that case, in that case it will absorb a lot of energy from the measurement and that is also possible that the temperature of this measurement may fall.

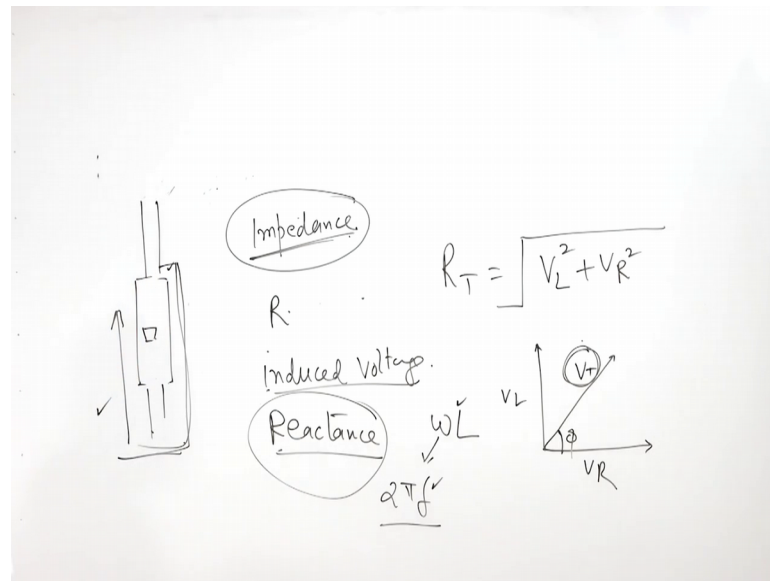
So, scientifically design instrumental in the scientifically designed measuring instrument the primary sensing element should draw minimum amount of power. I will give you another example in addition to the temperature. Measurement of rpm rotation of a shaft measure of its rotation of a shaft is done by the tachometer.

So, tachometer is a device it has also one sensor it has a knob which comes into the contact with the shaft. Shaft has a cavity where this head of the tachometer fits in and this shaft of the tachometer also starts rotating with the rpm of the shaft and that is how the rpm of the shaft is measured and in this case also energy is extracted from the shaft. So, this also causes a loading effect.

Now, another example I can give you, suppose we are measuring, diameter of a pipe with a micrometer diameter of a silicon pipe or pipe metallic pipe of very thin sheet in that case when the micrometer or vernier caliper when it will come in contact with the surface of the sheet it will change the diameter of the pipe. The surface of this tube it may change the diameter of the tube this is also a sort of loading effect. So, we can have many examples in mechanical engineering in this loading effect.

Another example can be flow measurement with the help of a flow meter.

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Suppose in a pipeline, suppose in a pipeline I fix rotameter, rotameter is a very popular device for measuring the flow rate. Suppose I want to measure water flow rate, so in the pipeline I will fix a rotameter rotameter has a float we will discuss rotameter in details when we will talk about the instruments.

So, this addition of this rotameter in the pipeline will also cause obstruction in the flow and due to this obstruction in the flow rate may or it happens to decrease, I mean if there is no case of possibility when we fix the rotameter in the pipeline it decreases the flow rate maybe of an insignificant level. But definitely flow is obstructed and this is also can be considered at this and also, this also can be considered as a case of loading effect.

So, loading effect is predominant when we are using any measuring instrument loading effect is there it may be insignificant I mean and that is the beauty how the instrument is design right it should have minimum loading effect. For example, I gave you the example of the liquid in glass type of thermometer. So, bulb of the thermometer it should be designed in such a way that it extracts the new energy from the measurement. So, the originally state of measurement is disturbed in a minimum manner.

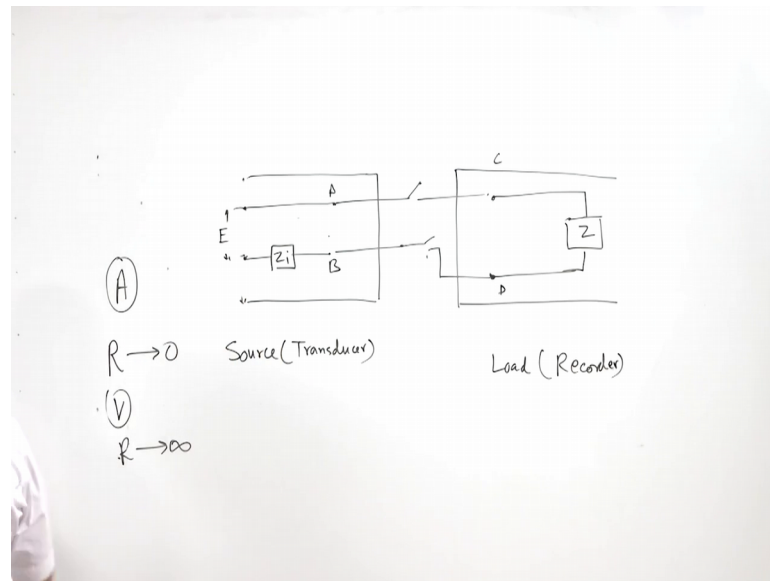
Now, I will give you an example electrical example when this loading effect is witness in electrical appliances it is known as impedance. Now impedance is resistance to flow of current right and it is not resistance, it is more than the resistance because suppose ac is flowing in of wire right. So, direction of the flow of the current is changing. So, this

direction of the flow of the current in the wire is resistance is resisted by the wire due to self inductance self induction and a term induced voltage, induced voltage comes into, comes into the picture and this induced voltage causes reactance or vice versa.

For example, there is a circuit which has the AC current is flowing in, AC is flowing in that circuit when AC is flowing in that circuit they due to reactance resistance will be offered and this resistance is going to be ωL , where ω is $2\pi f$, f is the frequency of ac right and L is a coefficient of induction inductance. So, the total resistance that is total impedance or the total voltage across the resistance is going to be R_T is going to be the vector sum or pythagorean sum of V_L^2 plus V_R^2 right. Or we can say it is if we take in terms of you want to depict in a vector form this is V_R and this is V_L and this is the final voltage V right, and this final voltage V and this is ϕ this final voltage V will be taken into account when we use the impedance for a given current.

So, impedance is the total resistance offered by the electrical circuit including ohmic resistance and due to reactance of the circuit right. So, now, this impedance has to be matched. If we do not match the impedance then this will affect the transmission of energy and in that case proper measurement will not be done. So, impedance matching in electrical machines in done is done for many reasons as far as the measurement is concerned. Now, measurement is concerned for example, you must have studied at the school level that a meter should have minimum resistance.

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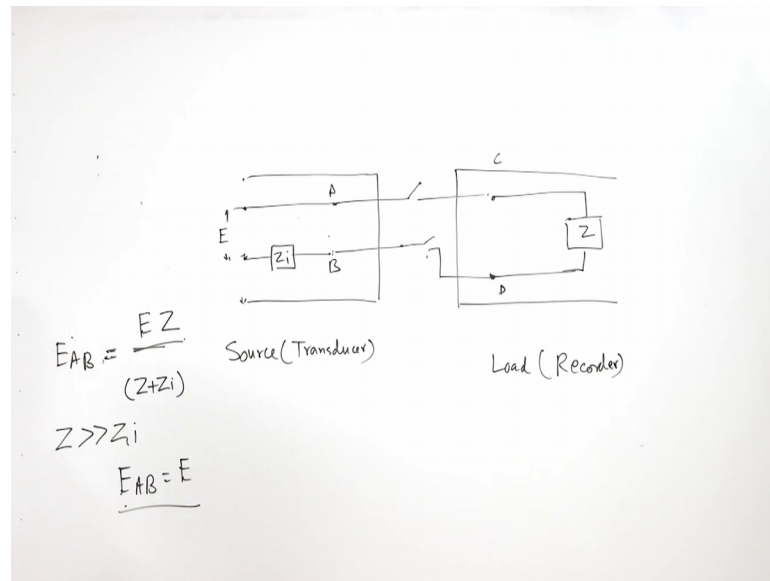


The resistance of the ammeter should tend to 0, and resistance of a voltmeter should tend to infinity. I take one example where there is a source, there is a source transducer, and source transducer has E has mf, and sorry this is a transducer let us this is housing of the transducer. So, this is E across these terminals, these terminals are extended here and this is A and B because the transducer this is a transduce source transducer now it has some internal resistance also and this internal resistance is depicted by Z i. This is some internal impedance of the transducer.

And it is connected to the load. So, there is another load here and this load is recorder right it has also terminals C and B and this recorder has also impedance of the order of Z . So, there is one source transducer there is a load it has terminal C and D and this has terminal A and B and these terminals are connected, because these terminals are connected, right.

Now E A B, what is going to be the value of E A B?

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E A B because here also there is a resistance, here also there is a resistance. So, E A B is going to be total E right, divided by total resistance this will give the current right.

So, total E divided by total impedance and the impedances are in series. So, Z plus Z i and so this will give the current which is flowing in the circuit. So, let us forget about this A B, A B this is a circuit. So, E if the turret is moving in the circuit and Z and Z i are in series. So, current in the circuit is E divided by Z plus Z i. Now we want to have, now this is not E A B. So, yes, this is E A B is equal to this current multiplied by resistance, so resistance is Z.

So, E A B is equal to Z multiplied by E total emf divided by Z plus Z i, right. When Z is much greater than Z i Z is in mega ohms and Z i in milliohms in that case E A B is equal to E that is what happens in the voltmeter. We say the voltmeters should, suppose these a recorder or voltmeter. So, voltmeter the impedance is infinity or resistance is infinity in that case what we are measuring the what E we are measuring here is same as what we E we are measuring here. So, that is one case right.

Another case is, the when we want to transmit maximum energy from source to the load. Now we have to take another stream where now that is required in the case of for example, in case of amplifiers where transmission of maximum amount of energy from source to load is required. Now, in that case if you want to transmit the maximum energy

then will again will start with energy equation P is equal to E A B square by Z that is the energy which is transmitted to the load.

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The image shows a handwritten derivation for maximum power transfer. At the top, the equation $0 = [1 - 2Z(Z + Z_i)^{-1}]$ is written. Below it is a circuit diagram with two parts. The left part, labeled 'Source (power amplifier)', shows a voltage source E in series with a resistor Z_i and terminals A and B. The right part, labeled 'Load (ED vibrator)', shows a load resistor Z connected between terminals C and D. The circuit is connected such that A is connected to C and B is connected to D. Below the diagram, the power P is given as $P = \frac{E^2 Z^2}{(Z + Z_i)^2}$. This is then simplified to $P = Z(Z + Z_i)^{-2} E^2$. The derivative $\frac{dP}{dZ}$ is calculated as $\frac{dP}{dZ} = E^2 \left[(Z + Z_i)^{-2} + Z(-2)(Z + Z_i)^{-3} \right] = 0$. The final line shows $0 = [$.

Now, they will also change source will be a power amplifier, power amplifier and recorder, a recorder maybe a electro dynamic vibrator, electro dynamic vibrator right. So, the instruments have also changed. So, this is the maximum power which is either the power which is transmitted to the load.

Now, E A B we can always replaced by E square Z square divided by Z plus Z i square multiple divided by again Z because this E A B is the previous formula we have taken E Z, Z plus Z i fine. Now, this power is nothing, but Z Z plus Z i raise to power minus 2. Now, we have power as a function of Z. So, what we can do this is simple mathematical technique let us take dp by dz, when we take dp by dz then it comes to be Z plus Z i minus two differentiation of this plus differentiation of this plus multiplied by this now differentiation of this multiplied by this. So, Z multiplied by minus 2 Z plus Z i multiplied by raise to power minus 3, this is 2 right. Differentiation of first term, differentiation then multiplied by the second term, now first term, differentiation of second term.

Now this is being to be 0. So, once this is 0 then we can always write just a minute, E E is E is not there know E E has to be there because E is there. So, E, E P is equal to this also be multiplied by E square, E square has to be there and, right. So, this will give 0 as

I should write somewhere here 0 as $1 - 2Z / (Z + Z_i)$ because this E will when this side is 0 and $Z - Z_i - 2Z$ can be taken out. So, ultimately if we simplify this equation we will be getting this equation.

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The image shows a handwritten derivation of the condition for maximum power transfer. The equations are as follows:

$$0 = \left[1 - \frac{2Z}{Z + Z_i} \right]$$

$$0 = \left[1 - \frac{2Z}{Z + Z_i} \right]$$

$$= \frac{Z + Z_i - 2Z}{Z + Z_i} = 0$$

$$Z_i - Z = 0$$

$$\boxed{Z = Z_i}$$

$$\frac{dp}{dZ} = \left[\frac{d}{dZ} \left(\frac{Z}{Z + Z_i} \right) + Z \left(-\frac{2}{(Z + Z_i)^2} \right) \right] = 0$$

$$0 = \left[\right]$$



To the right of the equations is a circuit diagram. It consists of a 'Source (power amplifier)' represented by a box with terminals A, B, C, and D. Inside the box, there is a voltage source 'E' and an impedance 'Z_i'. The source is connected to a 'Load (FD vibrator)' represented by a box with impedance 'Z'. The load is connected between terminals C and D. The circuit is shown with a switch between the source and the load.

So, once we have received this equation we have derived this equation then we can further simplify it as $1 - 2Z / (Z + Z_i)$ or $Z + Z_i - 2Z$ divided by $Z + Z_i$ is equal to 0 or we can say $Z - Z_i - Z$ is equal to 0 or Z is equal to Z_i . It means impedance of source should be equal to this impedance of the load. In that case energy transmission is going to be the maximum. In this regard we can solve a numerical it is given here.

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Numerical

A voltmeter with internal resistance $250\text{ k}\Omega$ is connected across an unknown resistance. It reads 225 V and the ammeter connected in series with resistance read 15 mA . Determine apparent resistance, actual resistance, and loading error. If the same devices connected to another resistance read 100 V and 5 A respectively, determine the loading error.

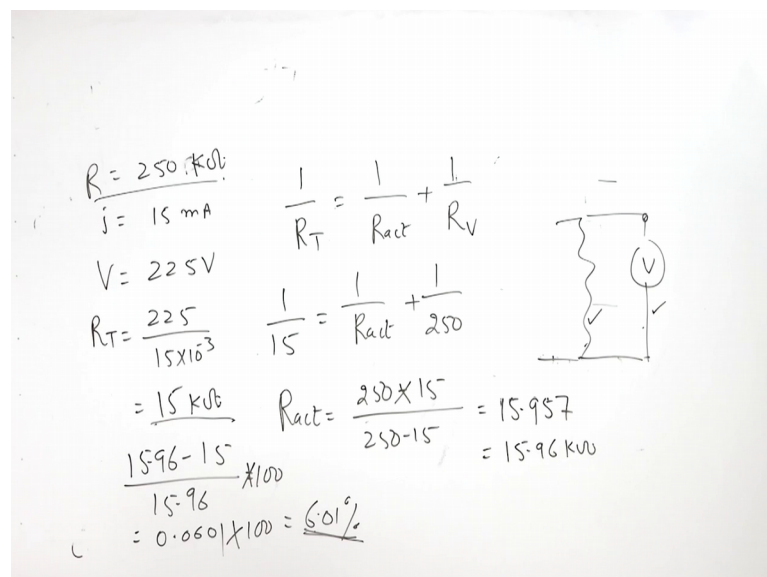
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A voltmeter with internal resistance 250 ohms , internal resistance is 250 kilo ohms is connected across an unknown resistance. It reads 225 volts and the ammeter connected in series with the resistance read reads 15 milliampere . So, the first case voltmeter resistance is 250 mega what is kilo ohms , kilo ohms and current is what is that current 15 milliampere , right.

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Handwritten calculations for the numerical problem:

$$\begin{aligned}
 R &= 250\text{ k}\Omega \\
 i &= 15\text{ mA} \\
 V &= 225\text{ V} \\
 R_T &= \frac{225}{15 \times 10^{-3}} = 15\text{ k}\Omega \\
 \frac{1}{R_T} &= \frac{1}{R_{act}} + \frac{1}{R_V} \\
 \frac{1}{15} &= \frac{1}{R_{act}} + \frac{1}{250} \\
 R_{act} &= \frac{250 \times 15}{250 - 15} = 15.957 = 15.96\text{ k}\Omega \\
 \frac{15.96 - 15}{15.96} \times 100 &= 0.0601 \times 100 = 6.01\%
 \end{aligned}$$

A circuit diagram is shown to the right of the calculations, depicting a voltage source connected in series with an unknown resistance (represented by a zigzag line) and a voltmeter (represented by a circle with a 'V' inside). The voltmeter is connected in parallel across the unknown resistance.

Now, now this is voltage this is sorry, now voltage is; what is the voltage? Voltage is 225 volts right. So, voltage is 225 volts , current is 15 . Total resistance is 225 divided by 15

into 10 to power minus 3 is equal to 9 kilo ohms sorry not 9, 15, 15 kilo ohms 225 divided by 15 is going to be 15 kilo ohms. So, this is the total resistance right and this is resistance of voltmeter.

Now, total resistance is because they are connected in parallel. So, total resistance is actual resistance plus resistance of voltmeter where they are connected in parallel because voltmeter is always connected parallel to the load suppose this is load right. So, voltmeter will be connected parallel to the volt.

So, resistances of I mean the voltmeter and the load they are in parallel they are connected in parallel. So, total resistance is 1 by 15 kilo ohm is 1 by R actual this is actual resistance plus voltmeter resistance is 250. Now, from this if we calculate actual resistance it is going to be 250 multiplied by 15 divided by 250 minus 15. So, actual resistance is going to be if we calculate this. So, here if we simplify this we will be getting into 50 divided by 235, 15.957 or 15.96 kilo ohms ok.

And now if we wish to calculate the error of the measurement because this is the actual resistance of this load, but the voltmeter is giving only 50 kilo ohms. So, that is error is going to be the actual value true value 15 minus 9, 6 measured value divided by 15.96 and this is going to be equal to 0.0601 is multiplied by 100. So, it is going to be equal to 6.01 percent. So, this is the error in measurement due to loading effect.

Now, in another case the case 2 is given, in the case 2 the same device is connected to another resistance read 100 volts and 5 ampere. Now, when the another device reads 100 volts, remove this otherwise get confusing it,

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Handwritten calculations and a circuit diagram:

$$R = 250 \text{ k}\Omega$$

$$I = 5 \text{ A}$$

$$V = 100 \text{ V}$$

$$R_T = \frac{100}{5} = 20 \Omega = 20 \times 10^{-3} \text{ k}\Omega = 0.02 \text{ k}\Omega$$

$$\frac{1}{R_T} = \frac{1}{R_V} + \frac{1}{R_{act}}$$

$$\frac{1}{0.02} = \frac{1}{250} + \frac{1}{R_{act}}$$

$$R_{act} = \frac{250 \times 0.02}{250 - 0.02}$$

$$= \frac{20.0016 - 20}{20.0016} \times 100 = 0.008\%$$

The circuit diagram shows a voltage source connected to a load resistor (represented by a zigzag line) and a voltmeter (represented by a circle with a 'V' inside) in parallel.

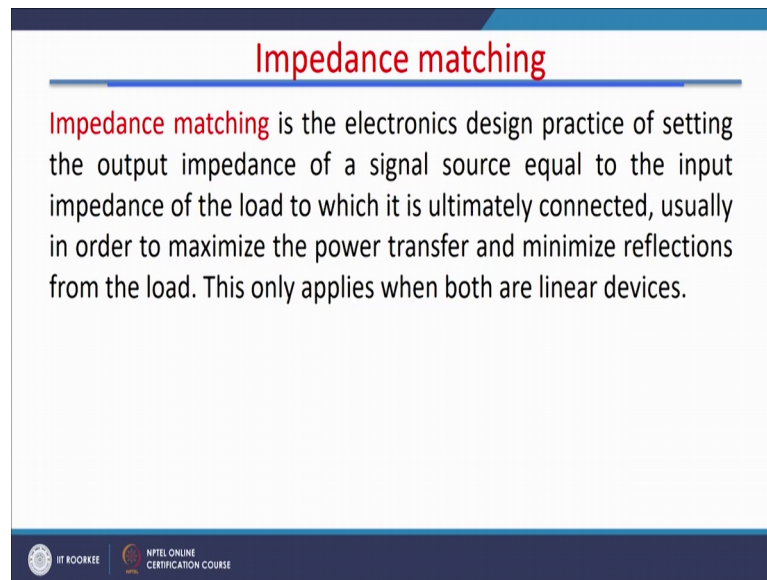
Now 100 volts and current is 5 ampere not 15 milliamperes it is 5 ampere. So, total resistance is 100 volts divided by 5 ampere is equal to 20 ohm or 20 into 10 to power minus 3 kilo ohm.

Again we will adopt the same method $\frac{1}{R_T}$ is equal to $\frac{1}{R_V}$ sorry $R_{\text{voltmeter}}$ plus $\frac{1}{R_{\text{actual}}}$ the total we are getting from here it is 0.02 kilo ohms right. So, $\frac{1}{0.02}$ is equal to $\frac{1}{250}$ plus $\frac{1}{R_{\text{actual}}}$. Now, from here also if we calculate R_{actual} it is going to be 250 multiplied by 0.02 divided by 250 minus 0.02 and from here will get the R_{actual} as 20.0016 ohms. This is the actual value of 20.0016, here it is 20 ohms is here 20.0016 ohm.

So, the percentage error if we want to calculate $\frac{20.0016 - 20}{20.0016}$ multiplied by 100 and it is going to be 0.008 percent very low. But in previous case the percentage was approximately 6.01 percent because in that case the resistance of this measurement was comparable with the resistance of the voltmeter because we are getting currents in milliamperes.

Now, here the this resistance has reduced drastically or the resistance of the voltmeter is much higher than the resistance of the load. So, in that case the error is minimized and it has become almost insignificant or 0.08 percentage. Now, so, impedance matching is a very important phenomena I mean it has to be ensured in many of the applications in day to days life.

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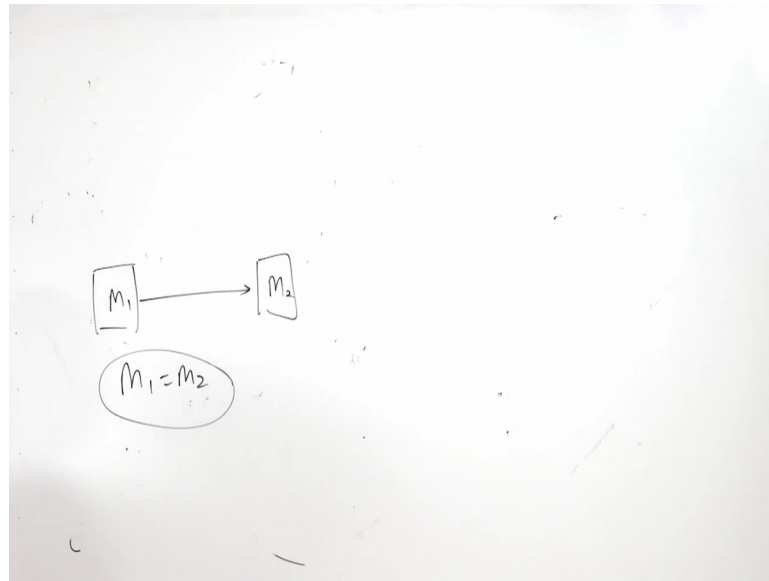


In electrically giving it has variety of applications, I mean in variety of applications it is being used, but I will like to give you some mechanical examples. For example, when the ultra sonography is done, when medical ultrasonography is done right you must have observed that a transducer is slid over the body right and before they a pasties applied on the skin. The purpose of the paste is not to provide the lubrication, purpose of the pasties for the maximum transmission of the sonic energy to the body for this purpose that paste is used.

So, that is the case of impedance mating with the impedance of the body and the transducer should match otherwise there will not be a full transmission of the sound waves and the image will not be very clear. Now, bones in the middle year in our body also there is impedance matching. The bone of middle year I mean middle year it facilitates the impedance matching between the eardrum and the fluid behind the if eardrum, because the where the sonic waves they strike the eardrum the eardrum starts vibrating and these vibrations are transmitted to the fluid on the other side of the eardrum and this impedance matching is done by the a middle year of the human being. So, in nature also impedance matching is ensure to realize certain processes.

And I will give you very simple mechanical example. This is the last example.

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When suppose there is a stationary body of mass M , and another body of mass M , there was this is M_1 and this is M_2 . And it strikes there is a elastic collision between M_1 and M_2 . The case when maximum energy transmission will take place when M_1 is equal to M_2 , this is the simplest example of impedance matching. The energy from M_1 can be transferred to M_2 only can be maximum I mean to the maximum order when mass of the this body 1 is equal to the mass of the body 2. This is all for today. In the next class we will start with statistical methods.